

## SECTION - 4

Q2  
Ans

(a) 1. Number of bit strings of length eight that start with a 1 bit:  
 $2^7 = 128.$

2) Number of bit strings of length eight that end with bit's 00:  $2^6 = 64.$

3.) Number of bit strings of length eight that start with a 1 bit and end with bits 00:  $2^5 = 32$

Hence, the number is  $128 + 64 - 32 = 160.$

(b).

1. One-to-one function (Injective function or injection): Let  $f: X \rightarrow Y$  then  $f$  is called one to one function if for distinct elements of  $X$  there are distinct image in  $Y$  i.e.,  $f$  is one-to-one iff

$$f(x_1) = f(x_2) \text{ implies } x_1 = x_2 \quad \forall x_1, x_2 \in X$$

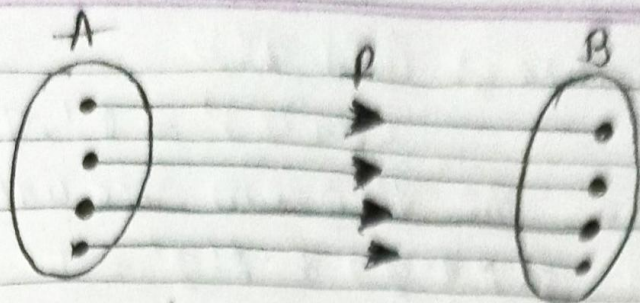


Fig - 1 (one to one)

2. Onto function (Surjection or surjection function):  
 Let  $f: X \rightarrow Y$  then  $f$  is called onto function iff  
 for every element  $y \in Y$   
 there is an element  $x \in X$  with  $(f(x) = y)$  or  
 $f$  is onto iff  $\text{Range}(f) = Y$ .

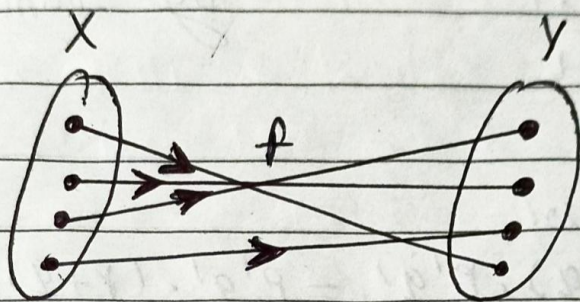
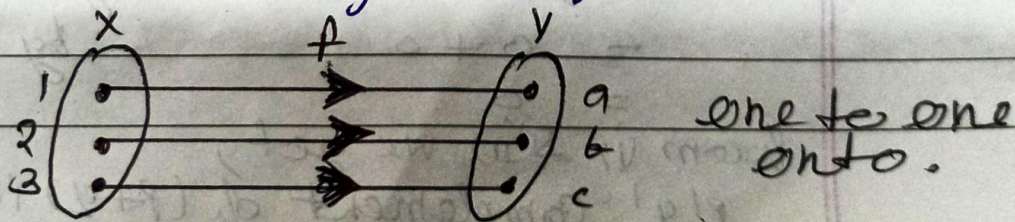


Fig - 2 (Onto).

3. One to one onto function (Bijective function or bijection):  
 A function which is both one to one and onto is called one to one onto function or bijective function.



one to one onto.

(c) To prove:  $(P+Q)'$

$$(P+Q)' = P' \cdot Q'$$

To prove the theorem we will show that

$$(P+Q) + P' \cdot Q' = 1$$

Consider  $(P+Q) + P' \cdot Q' = \xi(P+Q) + P' \cdot \xi \cdot Q'$  by distributive law

$$= \xi(Q+P) + P' \cdot \xi \cdot (P+Q) + Q'$$
 by commutative law

$$= \xi(Q + (P+P')) \cdot \xi(P + (Q+Q'))$$
 by associative law

$$= (Q+1) \cdot (P+1)$$
 by complement law

$$= 1 \cdot 1$$
 by dominance law

$$= 1 \quad \dots (i)$$

Also consider

$$(P+Q) \cdot P' \cdot Q' = P' \cdot Q' \cdot (P+Q) \text{ by commutative law}$$

$$= P' \cdot Q' \cdot P + P' \cdot Q' \cdot Q \text{ by distributive law}$$

$$= P \cdot (P' \cdot Q') + P' \cdot (Q \cdot Q') \text{ by associative law}$$

$$= (P \cdot P') \cdot Q' + P' \cdot (Q \cdot Q') \text{ by commutative law}$$

$$= 0 \cdot Q' + P' \cdot 0 \text{ by complement law}$$

$$= Q' \cdot 0 + P' \cdot 0 \text{ by commutative law}$$

$$= 0 + 0 \text{ by dominance law}$$

$$= 0$$

From (i) & (ii) we get,

$P' \cdot Q'$  complement of  $(P+Q)$  i.e.  $(P+Q)'$