SECTION 4

Q1 (a) We prove this by induction on n.

Base case: For $n=1$, $11^{n+1} + 12^{2n-1} =$
$11^2 + 12^1 = 133$ which is divisible by
133.

Inductive step: Assume that the hypothesis holds for $n=k$, i.e.,
$11^{k+1} + 12^{2k-1} = 133A$ for some integer A.
Then for $n = k+1$,
$$11^{n+1} + 12^{2n-1} = 11^{k+1+1} + 12^{2(k+1)-1}$$
$$= 11^{k+2} + 12^{2k+1}$$
$$= 11 * 11^{k+1} + 144 * 12^{2k-1}$$
$$= 11 * 11^{k+1} + 11 * 12^{2k-1} + 133 * 12^{2k-1}$$
$$= 11[11^{k+1} + 12^{2k-1}] + 133 * 12^{2k-1}$$
$$= 11 * 133A + 133 * 12^{2k-1}$$
$$= 133[11A + 12^{2k-1}]$$

Thus if the hypothesis holds for $n=k$ it also hold for $n=k+1$. Therefore, the statement given in the equation is true.

(6) 1. Suppose G is a finite group whose order is a prime number p, then to prove that G is a cyclic group.

(2) An Integer p is said to be a prime number if $p \neq 0$, $p \neq \pm 1$, and if the only divisors of p are $\pm 1, \pm p$.

(3.) Some G is a group of prime order, therefore G must contain at least 2 element. Note that 2 is the least positive prime integer.

(4) Since a is not the identity element, therefore $o(a)$ is definitely $\geq 2$. Let $o(a) = m$.. If H is the cyclic subgroup of G generated by a then $o(H = o(a) = m)$.

5. By Lagrange's theorem m must be a divisor of p. But p is prime and $m \geq 2$. Hence $m = p$.

6. $\because H = G$. Since H is cyclic therefore G is cyclic and a is a generator of G.