

SECTION - 3

Q1

Ans (a.) The Composition table of G is.

*	1	-1	i	-i
1	1	-1	i	-i
-1	-1	1	-i	i
i	i	-i	1	1
-i	i	i	-1	1

1. Closure property: Since all the entries of the Composition table are the elements of the given set, the set G is closed under multiplication.
2. Associativity: The elements of G are Complex numbers, and we know that multiplication of Complex number is associative.
3. Identity: Here, 1 is the identity element.
4. Inverse: From the Composition table, we see that the inverse elements of 1, -1, i, -i are 1, -1, -i, i respectively.

Date: _____
Page No.: _____

5. Commutativity: The corresponding rows & columns of the table are identical. Therefore $(\mathbb{Z}, *)$ is an abelian group.

(b) Ring: A ring $(R, +, \cdot)$ is a set R together with two binary operations $+$ (Addition) and \cdot (multiplication) defined on R such that the following axioms are satisfied:

(R₁) $(a+b)+c = a+(b+c)$ for all $a, b, c \in R$.

(R₂) $a+b = b+a$ for all $a, b \in R$.

(R₃) There exists an element 0 in R such that $a+0 = a$ for all $a \in R$.

(R₄) For all $a \in R$, there exist an element $-a \in R$ such that

$$a+(-a) = 0.$$

(R₅) $(a \cdot b) \cdot c = a \cdot (b \cdot c)$ for all $a, b, c \in R$.

(R₆) $a \cdot (b+c) = (a \cdot b) + (a \cdot c)$ for all $a, b, c \in R$.
(Left distributive law)

(R₇) $(b+c) \cdot a = (b \cdot a) + (c \cdot a)$ for all $a, b, c \in R$.
(Right distributive law).