SECTION-5

Q1 Ans A. Assumptions of lame's Theory:

1. The material is homogeneous and isotropic
2. The material is stressed within elastic limit.
3. Plane section perpendicular to the longitudinal axis of the cylinder remain plane after the application of internal pressure.
4. All the fibers of material are free to expand or contract indendently without being constrained by adjacent fibers.

B. Derivation:

1. Consider a thick subject to internal and external radial stress (pressure) is known in fig. 5.22.1. Consider an element ring of internal radius r and thickness dr.

2. Let,

r_1 = Internal radius of the cylinder.
r_2 = External " " " ".
l = Length of the cylinder.
P_1 = Pressure on inner surface of cylinder.
P_2 = Pressure on outer surface of ".
σ_r = Internal radial stress on the elemental ring.
$(\sigma_r + d\sigma)$ = External radial stress on the elemental ring.
σ_c = circumferential stress on elemental ring.

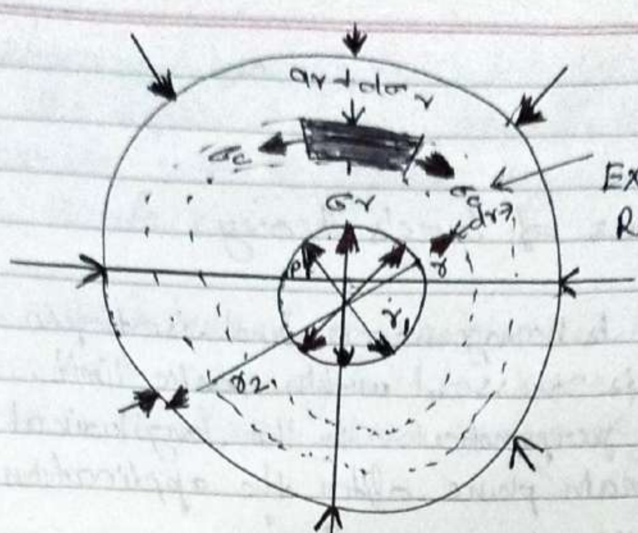

Fig. - 1

3. The Conditions for equilibrium on one half of the element ring (similar to those in the case of thin cylinder) are as follows:

Bursting force $= (\sigma_r \times 2rl) - [\sigma_r + d\sigma_r] \times 2(r \times dr) l]$

(Neglecting the product of small quantities).

Resisting force $= 2\sigma_c \, l dr$

4. Equating the resisting force to bursting force (for equilibrium), we get.

$$2\sigma_c l \, dr = -2(\sigma_r dr + r \, d\sigma_r)$$

$$\dots (5.2)$$

$$\sigma = -\sigma_r \frac{d\sigma_r}{dr}$$

5. Now let us obtain another relation between the radial stress (pressure and Circumferential (or hoop) Stress by using the Condition that the longitudinal strain (ε_l) at any point in the Section is Same.

classmate

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The Longitudinal stress,

$$\sigma_l = \frac{P_1 \times \pi r_1^2}{\pi(r_2^2 - r_1^2)} = \frac{P_1 r_1^2}{(r_2^2 - r_1^2)}$$

6. Hence at any point in the section of the element ring considered above, the following three principal stress, exist.

$$\sigma_l = \frac{P_1 \times \pi r_1^2}{\pi(r_2^2 - r_1^2)} = \frac{P_1 r_1^2}{(r_2^2 - r_1^2)}$$

(i) The radial Stress (Pressure). σ_r.

(ii) The circumference Stress, σ_c.

(iii) The longitudinal tensile stress

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7. Since the longitudinal strain (ε_l) is constant, we have,

$$\varepsilon_l = \frac{\sigma_l}{E} - \frac{\mu\sigma_c}{E} - \frac{\mu\sigma_r}{E} = \text{Constant}$$

But, since σ_l, μ and E are constant

$$\sigma_c - \sigma_r = \text{Constant}$$

8. Let $\sigma_c - \sigma_r = 2a$

putting $\sigma_c = (\sigma_r + 2a)$ in eq. (6.1) we get

$$(\sigma_r + 2a) = -\sigma_r - r\frac{d\sigma_r}{dr} \; ; \; \frac{d\sigma_r}{dr} =$$

$$= -\frac{2(\sigma_r + a)}{r}$$

$$\frac{d\sigma_r}{\sigma_r + a} = -\frac{2dr}{r}$$

Integrating both sides, we get

$$\log_e(\sigma_r + a) = -2\log_e r + \log_e b$$

$$(\because \log_e b = \text{Constant})$$

$$\log_e(\sigma_r + a) = \log_e \frac{b}{r^2} \qquad \text{———— 5.2}$$

$$\sigma_r = \frac{b}{r^2} - a$$

$$\sigma_c = \frac{b}{r^2} + a \qquad \text{———— 5.3}$$

9. Similarly,
10. The eq.(5.2 and 5.3) are called Lame's equation.
11. The constant a and b evaluated from the known internal and external radial pressure and radius.
12. It may be noted that in the above equation σ_r is compressive and σ_c is tensile.