Q. $(1-x^2)(1-y)\,dx = xy(1+y)\,dy$

Ans Seperate the variables first

$$\frac{(1-x^2)}{x}dx = \frac{y(1+y)}{(1-y)}dy$$

$$\left(\frac{1}{x}-x\right)dx = \left[\frac{y^2+y}{1-y}\right]dy$$

$$\left(\frac{1}{x}-x\right)dx = \left[\frac{(1-y)(-y-2)+2}{1-y}\right]dy$$

$$\int\left(\frac{1}{x}-x\right)dx = \int\left[-(y+2)+\frac{2}{1-y}\right]dy$$

$$\begin{array}{l|l} & -y-2 \\ \hline -y+1) & y^2-y \\ & \underline{(-)\,y^2 \;\overset{(+)}{-}\; y} \\ & 2y \\ & \underline{(-)\,2y-2} \\ & (-2) \end{array}$$

Integrating on both sides

$$\ln|x| - \frac{x^2}{2} + c = -\left(\frac{y^2}{2}+2y\right)$$
$$= -2\ln|1-y|$$
$$\ln|x| - \frac{x^2}{2} + c = \frac{-y^2}{2} - 2y - 2\ln|1-y|$$

hence proved.