

$$(D^5 - 1)y = 0$$

As the auxiliary equation is  $m^5 - 1 = 0$

$$\text{or } (m^2 + 1)(m^4 + m^2 + 1) = 0$$

$$\text{or } (m^2 - 1)(m^4 + m^2 + 1) = 0$$

$$\text{or } (m-1)(m+1)(m^2 - m + 1)(m^2 + m + 1) = 0$$

$$\therefore m = 1, -1, \frac{1}{2} \pm i\frac{\sqrt{3}}{2} \text{ and } -\frac{1}{2} \pm i\frac{\sqrt{3}}{2}$$

Hence the solution is

$$y = C_1 e^x + C_2 e^{-x} + e^{x/2} \{ C_3 \cos\left(\frac{1}{2}x\sqrt{3}\right)$$

$$+ C_4 \sin\left(\frac{1}{2}x\sqrt{3}\right) \}$$

$$+ e^{-x/2} \{ C_5 \cos\left(-\frac{1}{2}x\sqrt{3}\right) + C_6 \sin$$

$$\left(-\frac{1}{2}x\sqrt{3}\right) \}$$