Pg.:

Q Expand $\frac{1}{z^2-3z+2}$ in the region:

As Here $f(z) = \frac{1}{z^2-3z+2} = \frac{1}{(z-1)(z-2)} = \frac{1}{z-2} - \frac{1}{z-1}$

a) when $|z| < 1$:

$\therefore F(z) = \frac{1}{-2\left(1-\frac{z}{2}\right)} + \frac{1}{1-z} = \frac{-1}{2}\left(1-\frac{z}{2}\right)^{-1} + (1-z)^{-1}$

$= \frac{1}{2}\left[1+\frac{z}{2}+\left(\frac{z}{2}\right)^2+\dots\right] + \left(1+z+z^2+\dots\right)$

$= -\frac{1}{2}\sum_{n=0}^{\infty}\left(\frac{z}{2}\right)^n + \sum_{n=0}^{\infty} z^n$

b) when $1 < |z| < 2$:

$\therefore F(z) = \frac{1}{-2\left(1-\frac{z}{2}\right)} - \frac{1}{z\left(1-\frac{1}{z}\right)} = \frac{1}{2}\left(1-\frac{z}{2}\right)^{-1} - \frac{1}{z}\left(1-\frac{1}{z}\right)^{-1}$

$= -\frac{1}{2}\left[1+\frac{z}{2}+\left(\frac{z}{2}\right)^2+\dots\right] - \frac{1}{z}\left[1+\frac{1}{z}+\left(\frac{1}{z}\right)^2 + \dots\right]$

$= -\frac{1}{2}\sum_{n=0}^{\infty}\left(\frac{z}{2}\right)^n - \frac{1}{z}\sum_{n=0}^{\infty}\left(\frac{1}{z}\right)^n$