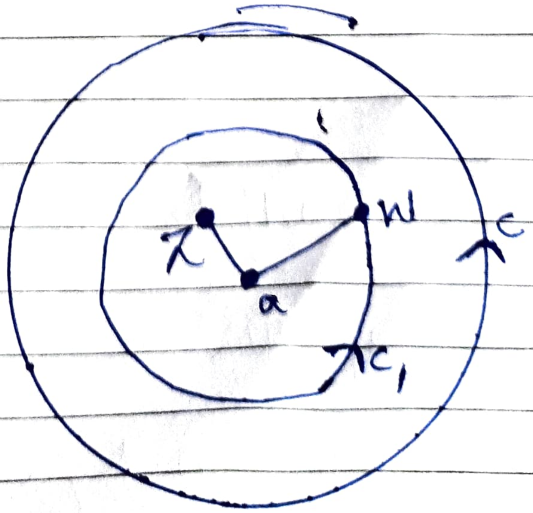
Q2 State and prove the taylor series for $F(z)$ about $z=a$.

Ans Statement: if $F(z)$ is analytic inside a circle c with centre at a, then for all z inside c:

$$F(z) = F(a) + (z-a)\cdot F'(a) + \frac{(z-a)^2}{2!}F''(a) + \dots + \frac{(z-a)^n}{n!}\cdot F^n(a) + \dots$$

or $F(z) = \sum_{n=0}^{\infty} a_n (z-a)^n$, where $a_n = \frac{F^n(a)}{n!}$

Proof: Let z be any point inside the circle c. Draw a circle c_1 with centre at a and radius smaller than that of c such that z is an interior point of c_1.



Delta

Let w be any point on C_1 then

$$|z-a| < |w-a| \text{ i.e } \left|\frac{z-a}{w-a}\right| < 1$$

Now,

$$\frac{1}{w-z} = \frac{1}{(w-a)-(z-a)} = \frac{1}{w-a}\left[1-\frac{z-a}{w-a}\right]^{-1}$$

Expanding RHS by bionomial theorm as $\left|\frac{z-a}{w-a}\right| < 1$

we get,

$$\frac{1}{w-z} = \frac{1}{w-a}\left[1+\frac{z-a}{w-a}+\left(\frac{z-a}{w-a}\right)^2+\dots+\left(\frac{z-a}{w-a}\right)^n+\dots\right]$$

Hence proved