Section -3

Ans-3 $\frac{1}{z^2-3z+2}$

(a.) $|z|<1$

Here $f(z)=\frac{1}{z^2-3z+2}=\frac{1}{(z-1)(z-2)}=\frac{1}{(z-2)}-\frac{1}{z-1}$

(By Partial fraction)

for $|z|<1$:

$\therefore f(z)=\frac{1}{-2(1-\frac{z}{2})}+\frac{1}{1-z}$

$$=\frac{-1}{2}\left(1-\frac{z}{2}\right)^{-1}+(1-z)^{-1}$$

$$=\frac{-1}{2}\left[1+\frac{z}{2}+\left(\frac{z}{2}\right)^2+\dots\right]+(1+z+z^2+\dots)$$

$$=\frac{-1}{2}\cdot\sum_{n=0}^{\infty}\left(\frac{z}{2}\right)^n+\sum_{n=0}^{\infty}z^n$$

This is a series of positive power of z, so it is an expansion of f(z) in Taylor's series within the circle $|z|<1$.

(b.) when $1 < |z| < 2$

$$\therefore f(z) = \frac{1}{-2\left(1-\frac{z}{2}\right)} - \frac{1}{z\left(1-\frac{1}{z}\right)} = \frac{-1}{2}\left(1-\frac{z}{2}\right)^{-1} - \frac{1}{z}\left(1-\frac{1}{z}\right)^{-1}$$

$$= \frac{-1}{2}\left[1+\frac{z}{2}+\left(\frac{z}{2}\right)^2+\ldots\right] - \frac{1}{z}\left[1+\frac{1}{z}+\left(\frac{1}{z}\right)^2+\ldots\right]$$

$$= \frac{-1}{2}\sum_{n=0}^{\infty}\left(\frac{z}{2}\right)^n - \frac{1}{z}\sum_{n=0}^{\infty}\left(\frac{1}{z}\right)^n$$

There is a series in positive & negative power of z so, it is an expansion of f(z) in Laurent's series within the annulus $1<|z|<2$.