Section-3

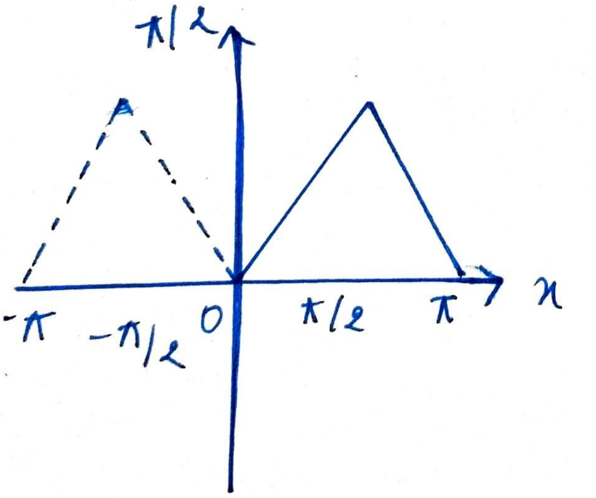
**Ans-1** Half Range series with example:-

Sometimes it is required to expand a function $f(x)$ in the range $(0, \pi)$ in the fourier series of period $2\pi$ or more generally in the range $(0, l)$ in a fourier series of period $2l$.

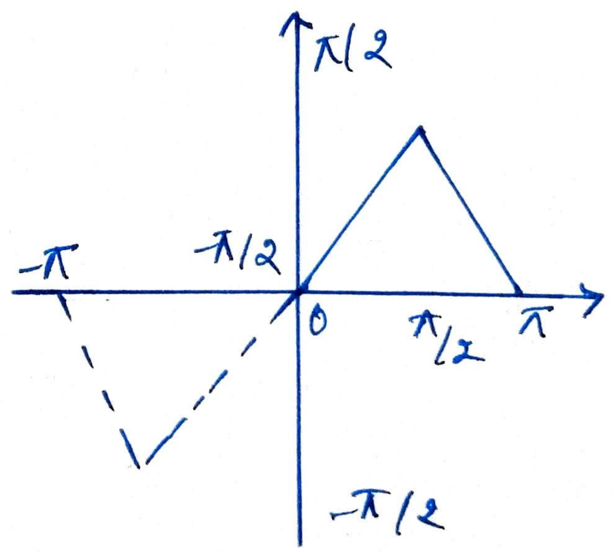
It is required to expand $f(x)$ by reflecting it in the y axis so that $f(-x) = f(x)$ then the extended function is even for which $b_n = 0$.

For example, consider a function

$$f(x) = \begin{cases} x, & 0 < x < \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} < x < \pi \end{cases}$$




(Reflection in y axis)




(Reflection in origin)

Here, a function $f(x)$ defined over the interval $0<x<l$ is capable of two distinct half-range series.

half range series of cosine is

$$\boxed{f x = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l}}$$

where $a_0 = \frac{2}{l}\int_0^l f(x)\,dx$

$a_n = \frac{2}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}\,dx$

half range sine series is

$$\boxed{f(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}}$$

where $b_n = \frac{2}{l}\int_0^l f(x) \cdot \sin\frac{n\pi x}{l}\,dx$

If the range is $0<x<\pi$, then,

(i) the half range cosine series is

$$\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx}$$

where

$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx$;

$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx$

(ii) the half range sine series is

$$\boxed{f(x) = \sum_{n=1}^{\infty} b_n \sin nx}$$

where,

$b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx$