② $f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$

Here, $a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx = \frac{1}{\pi}\int_0^{2\pi}\left(\frac{\pi - x}{2}\right)dx$

$= \frac{1}{2\pi}\left[\pi x - \frac{x^2}{2}\right]_0^{2\pi} = \frac{1}{2\pi}\left[2\pi^2 - 2\pi^2\right] = 0$

$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x) \cos nx\,dx = \frac{1}{2\pi}\int_0^{2\pi} (\pi - x)\cos nx\,dx$

$= \frac{1}{2\pi}\left[(\pi - x)\frac{\sin nx}{n} - (-1)\left\{-\frac{\cos nx}{n^2}\right\}\right]_0^{2\pi} = 0.$

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$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,dx = \frac{1}{2\pi}\int_0^{2\pi}(\pi - x)\sin nx\,dx.$$

$$= \frac{1}{2\pi}\left[-(\pi - x)\frac{\cos nx}{n} - \frac{\sin nx}{n^2}\right]_0^{2\pi} = \frac{1}{n}$$

$$\therefore f(x) = \frac{\pi - x}{2} = 0 + 0 + \sum_{n=1}^{\infty}\frac{1}{n}\sin nx$$

$$= \sin x + \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x + \cdots$$