(3) ~~(a) |z| < 1~~ Expand $\frac{1}{z^2 - 3z + 2}$ in the region

Sol^n Here $f(z) = \frac{1}{z^2 - 3z + 2}$

[By Partial fraction] $= \frac{1}{(z-1)(z-2)} = \frac{1}{(z-2)} - \frac{1}{(z-1)}$

(a) $f(z) = \frac{1}{-2\left(1 - \frac{z}{2}\right)} + \frac{1}{1-z} = -\frac{1}{2}\left(1 - \frac{z}{2}\right)^{-1} + (1-z)^{-1}$

$\rightarrow -\frac{1}{2}\left[1+\frac{z}{2}+\left(\frac{z}{2}\right)^2+\cdots\right]+(1+z+z^2+\cdots)$

$\rightarrow -\frac{1}{2}\sum_{n=0}^{\infty}\left(\frac{z}{2}\right)^n+\sum_{n=0}^{\infty} z^n$

This is a series in +ve power of z, So, it is a expansion of $f(z)$ in Taylor series within the circle $|z|<1$.

(b) $1<|z|<2$

$\therefore f(z) = \frac{1}{-2\left(1-\frac{z}{2}\right)} - \frac{1}{z\left(1-\frac{1}{z}\right)} = -\frac{1}{2}\left(1-\frac{z}{2}\right)^{-1} - \frac{1}{z}\left(1-\frac{1}{z}\right)^{-1}$

$\rightarrow -\frac{1}{2}\left[1+\frac{z}{2}+\left(\frac{z}{2}\right)^2+\cdots\right] - \frac{1}{z}\left[1+\frac{1}{z}+\left(\frac{1}{z}\right)^2+\cdots\right]$

$$= -\frac{1}{2} \cdot \sum_{n=0}^{\infty} \left(\frac{z}{2}\right)^n - \frac{1}{2} \cdot \sum_{n=0}^{\infty} \left(\frac{1}{z}\right)^n$$

This is a series in +ve & -ve powers of z, so it is a expansion of f(z) in Laurent's Series with the annulus

$1 < |z| < 2$.