Section - 3

A! Sometime it is required to expand a function $f(x)$ in the range $(0, \pi)$ in the fourier series of period $2\pi$ or more generally in the range $(0, l)$ in a fourier series of ~~range~~ period $2l$.

It is required to expand $f(x)$ in the interval $(0, l)$, then it is immaterial what the function may be outside the range $0 < x < l$. we are free to it arbitary in the interval $(-l, 0)$.

If we extend the function $f(x)$ by reflecting in the y-axis so that $f(-x) = f(x)$, then the extended function is even for which $b_n = 0$. The fourier expansion of $f(x)$ will contain only cosine terms.

If we extend the function $f(x)$ by reflecting it in the origin so that $f(-x) = f(x)$ Then the extended function $f(x)$ by reflecting it in the origin so that $f(-x) = f(x)$, then the extended function is odd for which $a_0 = a_n = 0$. The fourier expansion of $f(x)$ will contain only sine terms

Example: Consider a function $f(x) = \begin{cases} x, & 0 < x < \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} < x < \pi. \end{cases}$

Hence, a function $f(x)$ defined over the interval $0 < x < l$ is capable of two distinct half range series

* The half range cosine series is,

$$\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{l}}$$

* The half range sine series-

$$\boxed{f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l}}$$

if the range is $0 < x < \pi$ then,

(i) The half range cosine series is,

$$\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx}$$

(ii) The half-range sine series is

$$\boxed{f(x) = \sum_{n=1}^{\infty} b_n \sin nx}$$