Shiva
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Section-3

Q-1

⇒ Expression for hydraulic efficiency of turbine. (Pelton wheel)

1. Work done per second = $\rho a V_1 (V_{w_1} + V_{w_2}) u$

2. KE of jet per/s $= \frac{1}{2} m V_1^2 = \frac{1}{2} (\rho a V_1) V_1^2$

3. For Pelton wheel turbine, hydraulic efficiency is given by

$$\eta_h = \frac{\text{Work done per/s}}{\text{K.E of jet per/s}}$$

$$\eta_h = \frac{\rho a V_1 [V_{w_1} + V_{w_2}] u}{\frac{1}{2} (\rho a V_1) V_1^2} \quad \text{— (1)}$$

(4) For a pelton wheel we have

$$V_{w_1} = V_1 \qquad V_{r_1} = V_1 - u_1 = V_1 - u$$

$$V_{r_2} = V_1 - u$$

$$V_{w_2} = V_{r_2} \cos\phi - u_2$$

$$= V_{r_2} \cos\phi - u$$

$$= (V_1 - u) \cos\phi - u$$

(5) Substitute the value of V_{w_1} and V_{w_2} in eq (1) we get

$$\eta_h = \frac{2[V_1 + (V_1 - u)\cos\phi - u]u}{V_1^2}$$

$$= \frac{2[V_1 - u + (V_1 - u)\cos\phi]u}{V_1^2}$$

$$= \frac{2(V_1 - u)(1 + \cos\phi)u}{V_1^2}$$

(6) For maximum efficiency

$$\frac{d}{du}(\eta_h) = 0$$

$$\frac{d}{du}\left[\frac{2(V_1 - u)(1 + \cos\phi)u}{V_1^2}\right] = 0$$

$$\frac{(1 + \cos\phi)}{V_1^2}\frac{d}{du}[2(V_1 - u)u] = 0$$

$$\frac{d}{du}[2V_1u - 2u^2] = 0$$

$$2V_1 - 4u = 0$$

$$\boxed{u = \frac{V_1}{2}}$$

(7) So, hydraulic efficiency of Pelton wheel turbine will be max when the velocity of wheel is half of the jet of water at inlet.