Equal area criterion-

By the swing equation-

$$\frac{d^2\delta}{dt^2} = \frac{1}{M}(P_m - P_e) = \frac{P_a}{M} \quad \text{—(1)}$$

$$\& \quad M = \frac{H}{\pi f} \text{ pu.}$$

Multiply both sides by $\left(2\frac{d\delta}{dt}\right)$ in equ. (1)

$$2 \cdot \frac{d\delta}{dt} \cdot \frac{d^2\delta}{dt^2} = \frac{2P_a}{M} \cdot \frac{d\delta}{dt}$$

$$\Rightarrow \frac{d}{dt}\left(\frac{d\delta}{dt}\right)^2 = \frac{2P_a}{M} \cdot \frac{d\delta}{dt}$$

$$\Rightarrow \int \frac{d}{dt}\left(\frac{d\delta}{dt}\right)^2 = \int \frac{2P_a}{M} \cdot \frac{d\delta}{dt} \cdot dt$$

$$\Rightarrow \boxed{\frac{d\delta}{dt} = \sqrt{\frac{2}{M}\int P_a \cdot d\delta}}$$

$$\frac{d\delta}{dt} = \sqrt{\frac{2}{M} \cdot \int (P_m - P_{max} \cdot \sin\delta)\, d\delta}$$

Under steady state condition,

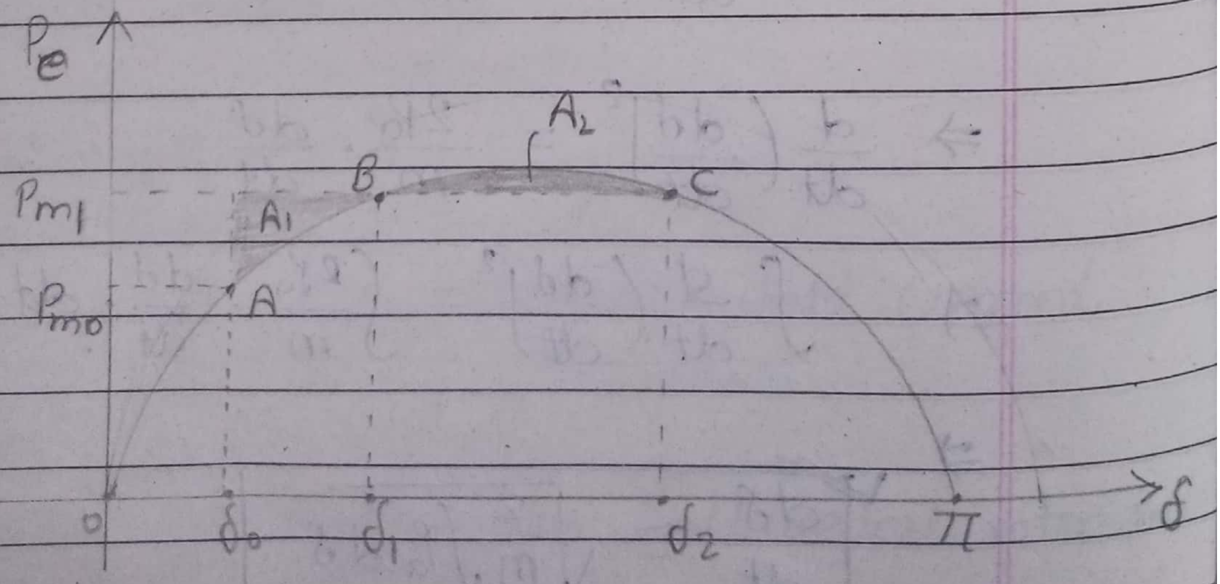
$$\frac{d\delta}{dt} = 0$$

$$\therefore \boxed{\int P_a \cdot d\delta = 0}$$

In equal area criterion, area under accelerating power is equal to the area under deaccelerating power.

Transient stability analysis by increasing or sudden change in mechanical input—

let assume a single synchronous machine connected to load is working continuously.



Under steady state condition (At point A),

$$P_{m0} = P_{e0} = P_{max} \cdot \sin\delta_0$$

Let, the mechanical input suddenly increase to P_{m1}.

$$P_a = P_{m1} - P_e \quad \& \quad P_a > 0$$

Here, electrical power cann't change suddenly because it depends on load while we are not changing the load.

Since, $P_a > 0$; the rotor is accelerating.

From point A to B,

$P_m > P_e$ i.e, $P_a > 0$. and rotor speed increase and becomes more than synchronous speed. Also rotor angle increasing.

$$\frac{d\delta}{dt} > 0$$

At point B,

$P_{m1} = P_e$ i.e, $P_a = 0$ & no acceleration but due to inertia in the rotor, rotor angle still increase

$$\frac{d\delta}{dt} > 0$$

From point B to C,

$P_e > P_{m1}$, $P_a < 0$, deaccelerating, the rotor speed begins to reduce but the angle continues to increase till δ_2.

$$\omega > \omega_s.$$

At point C,

$$P_e > P_{m1}$$

$P_a < 0$, decelerating.

$\omega < \omega_s$ & rotor angle begins to reduce.

By equal area criterion,

$$A_1 = A_2$$

$$\int P_a \cdot d\delta = 0$$

$$\Rightarrow \int (P_m - P_e) \cdot d\delta = 0$$

$$\Rightarrow \int_{\delta_0}^{\delta_1} (P_m - P_e) \cdot d\delta = \int_{\delta_1}^{\delta_2} (P_e - P_m)\, d\delta$$

$$\Rightarrow \int_{\delta_0}^{\delta_1} (P_m - P_{max} \cdot \sin\delta) \cdot d\delta = \int_{\delta_1}^{\delta_2} (P_{max} \sin\delta - P_m) \cdot d\delta$$

$$\Rightarrow P_m(\delta_1 - \delta_0) - P_{max} \cdot (\cos\delta_0 - \cos\delta_1) = P_{max}(\cos\delta_1 - \cos\delta_0) - P_m(\delta_2 - \delta_1)$$

The ~~A~~ maximum value of δ_2 be –

$$\boxed{\delta_2 = \delta_{max} = \pi - \delta_1 = \pi - \sin^{-1}\left(\frac{P_{m1}}{P_{max}}\right)}$$