CLASS

NOTES

ON

CONTROL SYSTEM (NEE-503)

A COURCE IN 5 TH SEMESTER OF

ELECTRICAL ENGINEERING AND ELECTRICAL AND ELECTRONICS ENGINEERING

DEPARTMENT OF ELECTRICAL ENGINEERING

SHAMBHU NATH INSTITUTE OF ENGINEERING AND TECHNOLOGY, JHALWA ALLAHABAD

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Lucknow, Uttar Pradesh.

NEE-503: CONTROL SYSTEM

UNIT-I

The Control System:

Open loop & closed control; servomechanism, Physical examples. Transfer functions, Block diagram algebra, and Signal flow graph, Mason's gain formula Reduction of parameter variation and effects of disturbance by using negative feedback.

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1.1 History

Systems: A system is a combination of components that act together and perform a certain objective. The system may be physical, biological, economical, etc.

A control system is an arrangement of physical components connected or related in such a manner as to command, direct, or regulate itself or another system, or is that means by which any quantity of interest in a system is maintained or altered in accordance with a desired manner.

Any control system consists of three essential components namely input, system and output. The input is the stimulus or excitation applied to a system from an external energy source. A system is the arrangement of physical components and output is the actual response obtained from the system. The control system may be one of the following type.

1) man made
2) natural and / or biological and
3) hybrid consisting of man made and natural or biological.

Examples:

1) **An electric switch is man made control system, controlling flow of electricity.**

    input : flipping the switch on/off
    system : electric switch
    output : flow or no flow of current

2) **Pointing a finger at an object is a biological control system.**

    input : direction of the object with respect to some direction
    system : consists of eyes, arm, hand, finger and brain of a man
    output : actual pointed direction with respect to same direction

3) **Man driving an automobile is a hybrid system.**

    input : direction or lane
    system : drivers hand, eyes, brain and vehicle
    output : heading of the automobile.

1.2 Classification of Control Systems

Control systems are classified into two general categories based upon the control action which is responsible to activate the system to produce the output viz.

1) Open loop control system in which the control action is independent of the out put.

2) Closed loop control system in which the control action is some how dependent upon the output and are generally called as feedback control systems.

1.3 Open Loop Control System

Open Loop System is a system in which control action is independent of output. To each reference input there is a corresponding output which depends upon the system and its operating conditions. The accuracy of the system depends on the calibration of the system. In the presence of noise or disturbances open loop control will not perform satisfactorily.

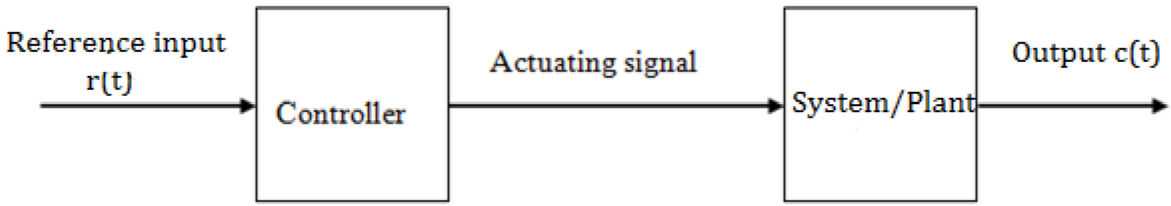


Figure 1. 1 Open Loop Control System

Example: Automatic hand driver, automatic washing machine, bread toaster, electric lift, traffic signals, coffee server, theatre lamp etc.

1.3.1 Advantages of open loop system:

1) They are simple in construction and design.
2) They are economic.
3) Easy for maintenance.
4) Not much problem of stability.
5) Convenient to use when output is difficult to measure.

1.3.2 Disadvantages of open loop system

1) Inaccurate and unreliable because accuracy is dependent on accuracy of calibration.
2) Inaccurate results are obtained with parameter variations, internal disturbances.
3) To maintain quality and accuracy, recalibration of controller is necessary from time to time.

1.4 Closed Loop Control System

A closed loop control system is one in which the control action depends on the output. In closed loop control system the actuating error signal, which is the difference between the input signal and the feed back signal (out put signal or its function) is fed to the controller. The elements of closed loop system are command, reference input, error detector, control element controlled system and feedback element.

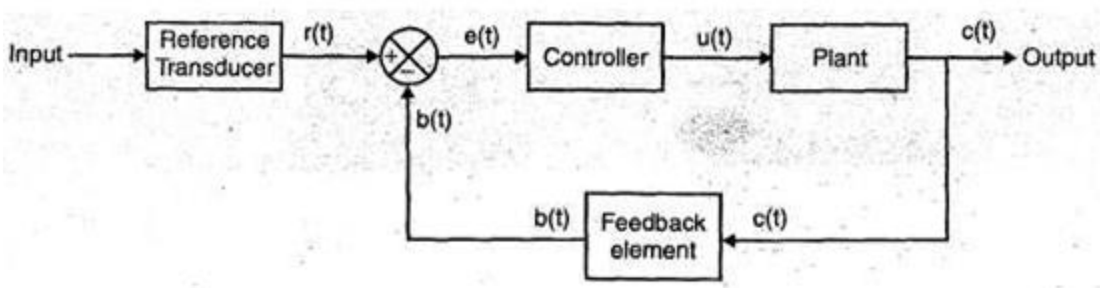


Figure 1. 2 Closed Loop Control System

Elements of closed loop system are:

1. Command : The command is the externally produced input and independent of the feedback control system.

2. Reference Input Element: It is used to produce the standard signals proportional to the command.

3. Error Detector : The error detector receives the measured signal and compare it with reference input. The difference of two signals produces error signal.

4. Control Element : This regulates the output according to the signal obtained from error detector.

5. Controlled System : This represents what we are controlling by feedback loop.

6. Feedback Element : This element feedback the output to the error detector for comparison with the reference input.

Example: Automatic electric iron, servo voltage stabilizer, sun-seeker solar system, water level controller, human perspiration system.

1.4.1 Advantages of closed loop system:

1) Accuracy is very high as any error arising is corrected.
2) It senses changes -in output due to environmental or parametric change, internal disturbance
3) etc. and corrects the same.
4) Reduce effect of non-linearities.
5) High bandwidth.
6) Facilitates automation.

1.4.2 Disadvantages

1) Complicated in design and maintenance costlier.
2) System may become unstable.

1.5 Servomechanism

A servomechanism is a power amplifying feedback control system in which the controlled variable is mechanical position or its time derivative such as velocity, acceleration. automatic device used to correct the performance of a mechanism by means of an error-sensing feedback. The term servomechanism properly applies only to systems in which the feedback and error-correction signals control mechanical position or one of its derivatives such as velocity or acceleration. Servomechanisms were first used in gunlaying (aiming) and in fire-control and marine-navigation equipment. Today, applications of servomechanisms include their use in automatic machine tools, satellite-tracking antennas, celestial-tracking systems on telescopes, automatic navigation systems, and antiaircraft-gun control systems.

All servomechanisms have at least these basic components: a controlled device, a command device, an error detector, an error-signal amplifier, and a device to perform any necessary error corrections (the servomotor).

1.6 Transfer Functions

In control systems, transfer function characterizes the input output relationship of components or systems that can be described by Liner Time Invariant Differential Equation(LTIV).

Transfer function of a LTIV system is defined as the ratio of the Laplace Transform of the output variable to the Laplace Transform of the input variable assuming all the initial condition as zero.

It is denoted by G(s).

$$G(s) = \frac{C(s)}{R(s)}$$

Where C(s) is laplace transform of output

R(s) is laplace transform of input

1.6.1 Importance:

Transfer function is highly important because of following reasons

1. It is used to give the gain of given block system.

2. The system poles/zeros can be found from transfer function.

3. Stability can be determined from characteristic equation.

4. The system differential equation can be obtained from transfer function by replacing.

s-variable with linear differential operator $\frac{d}{dt}$

1.6.2 Properties of Transfer Function

1. The transfer function is independent of the inputs to the system.
2. The transfer function of a system is the laplace transform of its impulse response for zero initial conditions.
3. The system poles/zeros can be found out from transfer function.
4. The transfer function is defined only for linear invariant systems. It is not defined for non-linear systems.

1.6.3 Limitations of transfer function are listed below

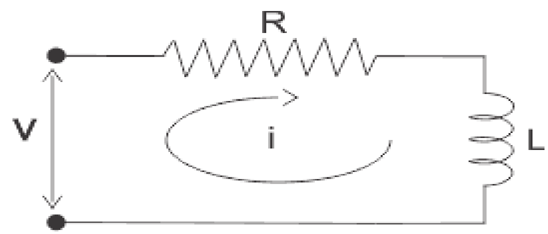
1 Transfer function is valid only for linear time invariant system.

2 It does not take into account the initial conditions initial conditions loose its significance.

3 It does not give any idea about how the present output is progressing.

EAXAMPLES:

Example. 1. Let us consider a system consists of a series connected resistance (R) and inductance (L) across a voltage source (V).



In this circuit, the current 'i' is the response due to applied voltage (V) as cause. Hence the voltage and current of the circuit can be considered as input and output of the system respectively.

From the circuit, **Applying KVL**

we get $$V = Ri + L\frac{\mathrm{d}i}{\mathrm{d}t}$$

Now applying Laplace Transform,

we get,

$$V(s) = RI(s) + L\left[sI(s) - i(0^+)\right]$$

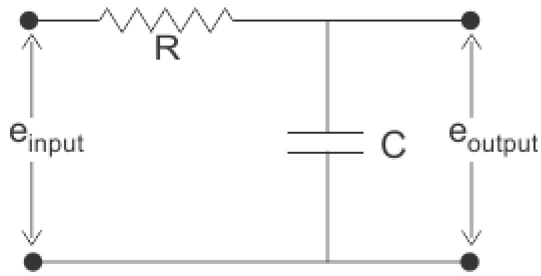
$$\left[\because \textit{Initially inductor behaves as open, hence}, i(0^+) = 0\right]$$

$$\Rightarrow V(s) = I(s)\left[R + Ls\right]$$

$$\Rightarrow \frac{I(s)}{V(s)} = \frac{1}{R + Ls} = \frac{1/L}{s + R/L}$$

The transfer function of the system, G(s) = I(s)/V(s), the ratio of output to input.

Example. 2. Let us consider a system consists of a parallel connected resistance (R) and capacitance (C) across a voltage source (e).



In the above network **Applying KVL**

$$e_{input} = Ri(t) + \frac{1}{C}\int i(t)dt$$

$$e_{output} = \frac{1}{C}\int i(t)dt$$

Let us assume,

$$\pounds\left[e_{input}(t)\right] = E_{input}(s),\ \pounds\left[e_{output}(t)\right] = E_{output}(s)\ ,\pounds\left[i(t)\right] = I(s)$$

$$And\ then,\ \pounds\left[\int i(t)dt\right] = \frac{I(s)}{s} + \int i(0)dt = \frac{I(s)}{s} + 0 = \frac{F(s)}{s}$$

$$\left[\int i(0)dt = 0, as\ there\ is\ no\ current\ initially\ through\ the\ capacitor\right]$$

Taking the Laplace transform of above equations with considering the initial condition as zero, we get,

$$E_{input}(s) = RI(s) + \frac{1}{C}\cdot\frac{I(s)}{s} = I(s)\left[R + \frac{1}{Cs}\right]$$

$$E_{output}(s) = \frac{1}{C}\cdot\frac{I(s)}{s} = I(s)\frac{1}{Cs}$$

$$Transfer\ function\ of\ the\ network,$$

$$\frac{E_{output}(s)}{E_{input}(s)} = \frac{I(s)\frac{1}{Cs}}{I(s)\left[R + \frac{1}{Cs}\right]} = \frac{\frac{1}{Cs}}{\left[R + \frac{1}{Cs}\right]} = \frac{1}{RCs + 1}$$

1.7 Block diagram algebra

Block diagram gives a pictorial representation of a control system by way of short handing the transfer function Signal flow graph further shortens the representation of a control system by eliminating summing symbol take-off point and block This elimination is achieved by way of representing the variables by points called ―nodes

- A pictorial representation of the relationship between input and output of a system is termed as block diagram.
- The direction of flow of signal from one block to other is indicated by an arrow.
- The point in a block diagram at which signal can be added or subtracted is termed as summing point.
- Gain is the ratio of laplace transform of output to laplace transform of input.
- Blocks in series are algebraically combined by multiplication.
- The lines drawn between the blocks to indicate the connections between the blocks are termed as branches.
- The point from which a signal is taken for the feedback purpose is called as take-off point.
- The order of summing point can be changed if two or more summing points are in series.

The arrow head pointing towards the block indicates the input and the arrow head away from the block represents the output. Such arrows are entered as signals.

$$X(s) \longrightarrow G(s \longrightarrow Y(s)$$

The followings components are involved in block diagram reduction method.
\

1.7.1Error detector

The error detector produces a signal which is the difference between the reference input and the feed back signal of the control system. Choice of the error detector is quite important and must be carefully decided. This is because any imperfections in the error detector will affect the performance of the entire system. The block diagram representation of the error detector is shown in figure 1.3

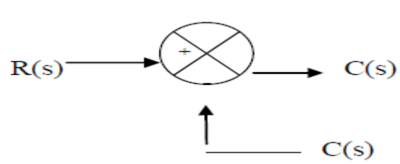


Figure 1. 3 Error detector

Note that a circle with a cross is the symbol which indicates a summing operation. The plus or minus sign at each arrow head indicates whether the signal is to be added or subtracted. Note that the quantities to be added or subtracted should have the same dimensions and the same units.

1.7.2 Block diagram of a closed loop system

The output C(s) is fed back to the summing point, where it is compared with reference input R(s).

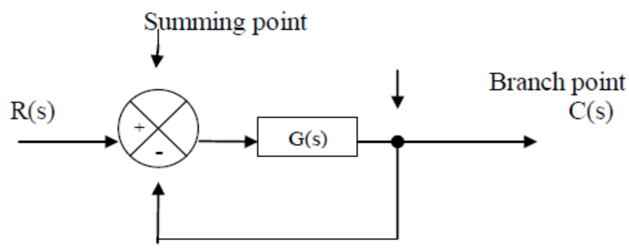


Figure 1. 4 Block diagram of a closed loop system

When the output is fed back to the summing point for comparison with the input, it is necessary to convert the form of output signal to that of he input signal. This conversion is followed by the feed back element whose transfer function is H(s) as shown in fig

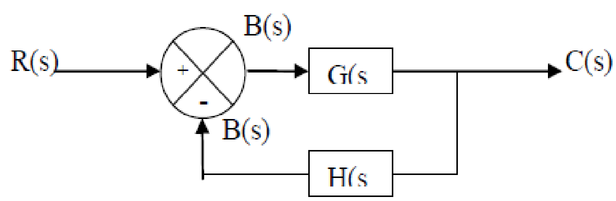


Figure 1. 5 Block diagram of a closed loop system with feed back element

The ratio of the feed back signal B(s) to the actuating error signal E(s) is called the open loop transfer function.

open loop transfer function = B(s)/E(s) = G(s)H(s)

The ratio of the output C(s) to the actuating error signal E(s) is called the feed forward transfer function .

Feed forward transfer function = C(s)/E(s) = G(s)

If the feed back transfer function is unity, then the open loop and feed forward transfer function are the same. For the system shown in Fig1.4, the output C(s) and input R(s) are related as follows.

$$C(s) = G(s)\,E(s)$$

$$E(s) = R(s) - B(s)$$

$= R(s) - H(s)\,C(s)$ but $B(s) = H(s)C(s)$

Eliminating E(s) from these equations

$$C(s) = G(s)\,[R(s) - H(s)\,C(s)]$$

$$C(s) + G(s)\,[H(s)\,C(s)] = G(s)\,R(s)$$

$$C(s)[1 + G(s)H(s)] = G(s)R(s)$$

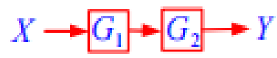
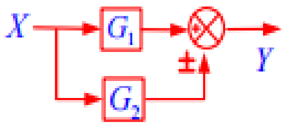
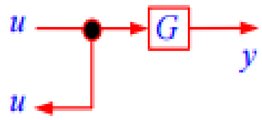
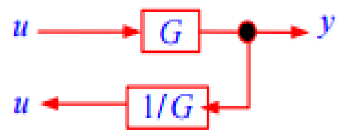
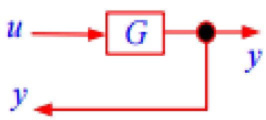
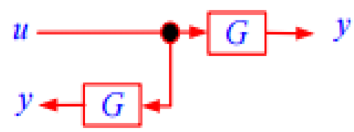
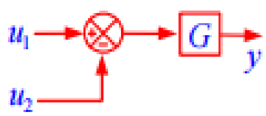
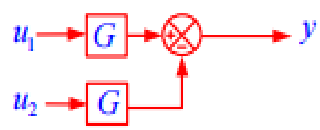
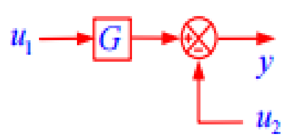
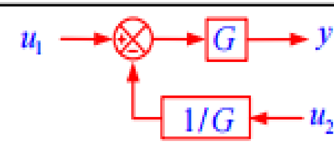
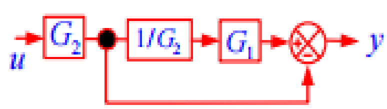
$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)\,H(s)}$$

C(s)/R(s) is called the closed loop transfer function.

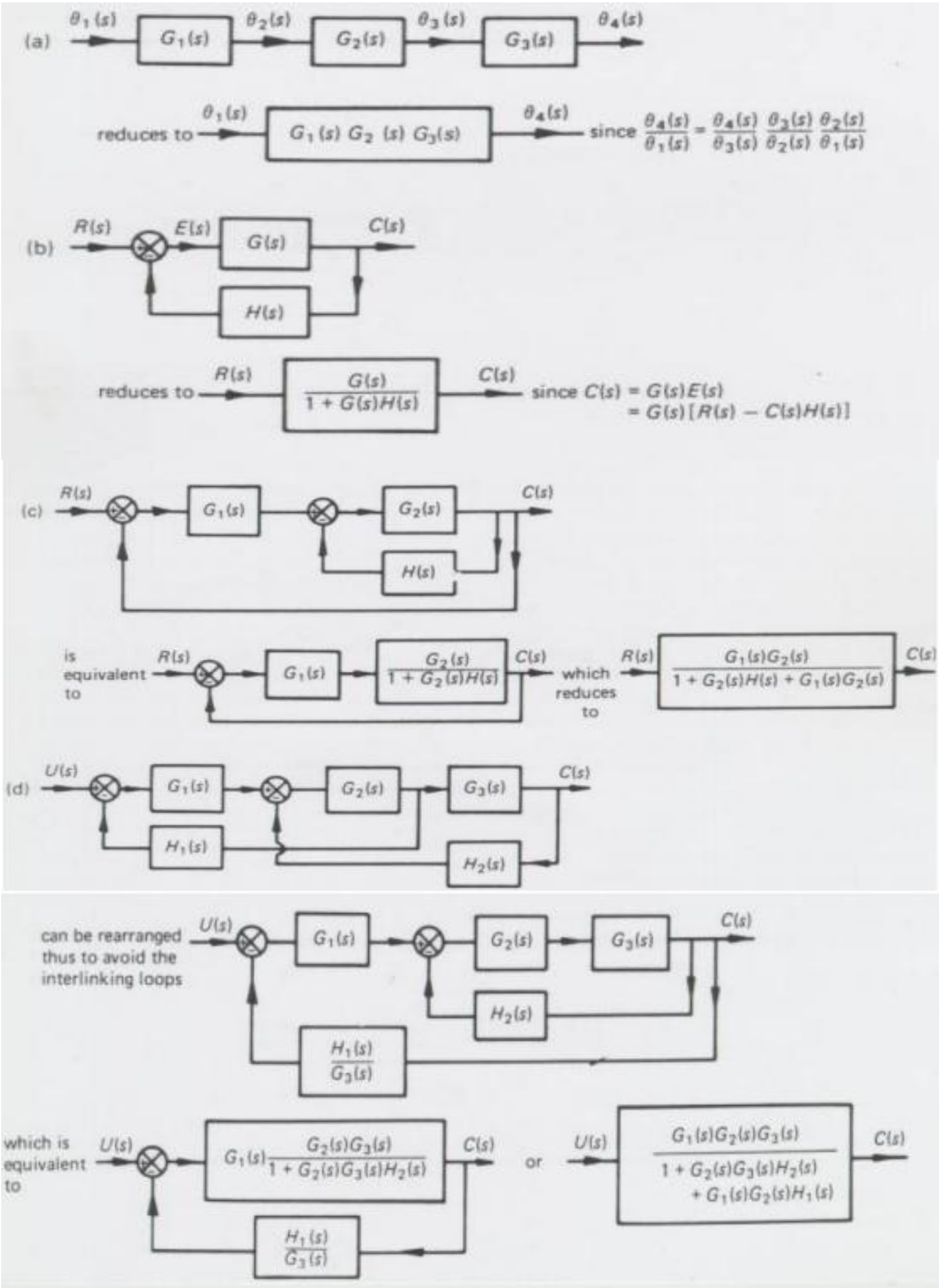
The output of the closed loop system clearly depends on both the closed loop transfer function and the nature of the input. If the feed back signal is positive, then

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 - G(s)\,H(s)}$$

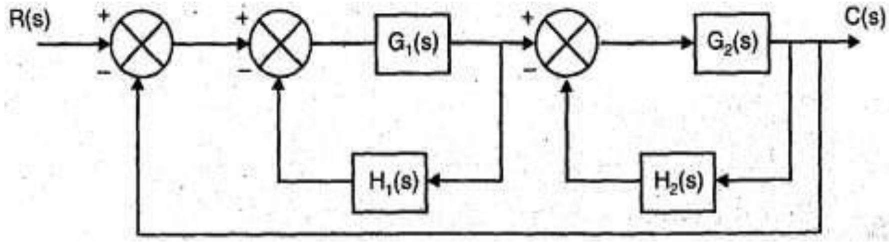
1.7.3 Some Basic Rules with Block Diagram Transformation

	Manipulation	Original Block Diagram	Equivalent Block Diagram	Equation
1	Combining Blocks in Cascade			$Y=(G_1G_2)X$
2	Combining Blocks in Parallel; or Eliminating a Forward Loop			$Y=(G_1\pm G_2)X$
3	Moving a pickoff point behind a block			$y = G u$ $u = \frac{1}{G} y$
4	Moving a pickoff point ahead of a block			$y=Gu$
5	Moving a summing point behind a block			$e_2=G(u_1-u_2)$
6	Moving a summing point ahead of a block		 	$y=Gu_1-u_2$ $y=(G_1-G_2)u$

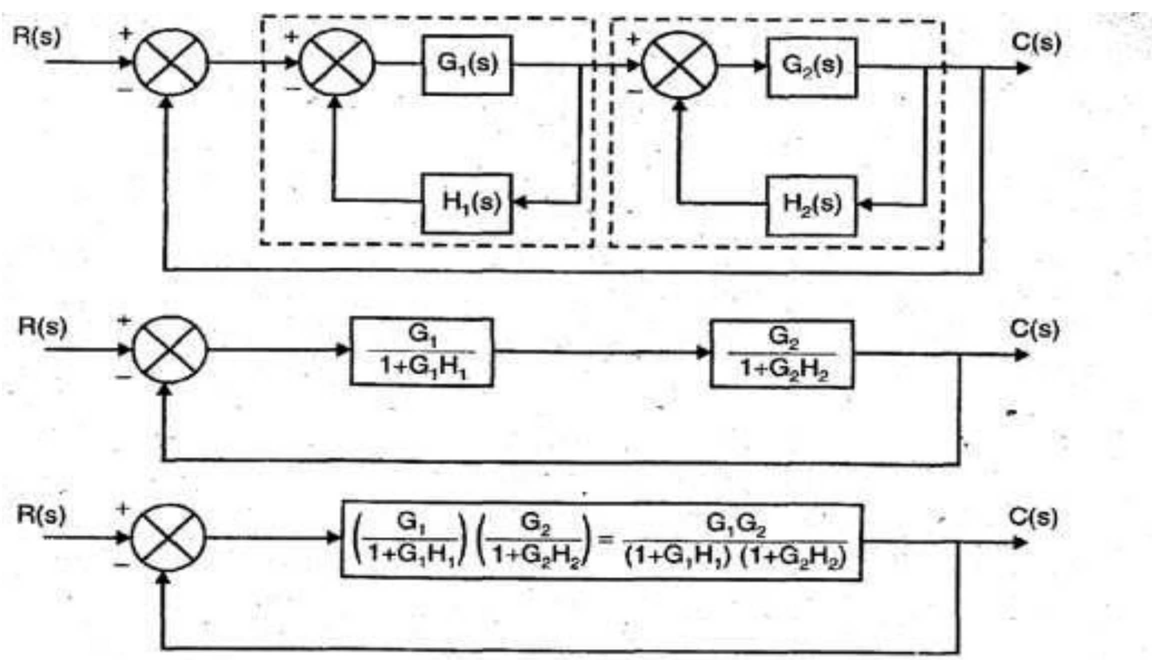
Examples: Q.1. Find the transfer functions using reduction technique.



Q.2 . Determine the transfer function of the system given in fig.by block diagram reduction method.



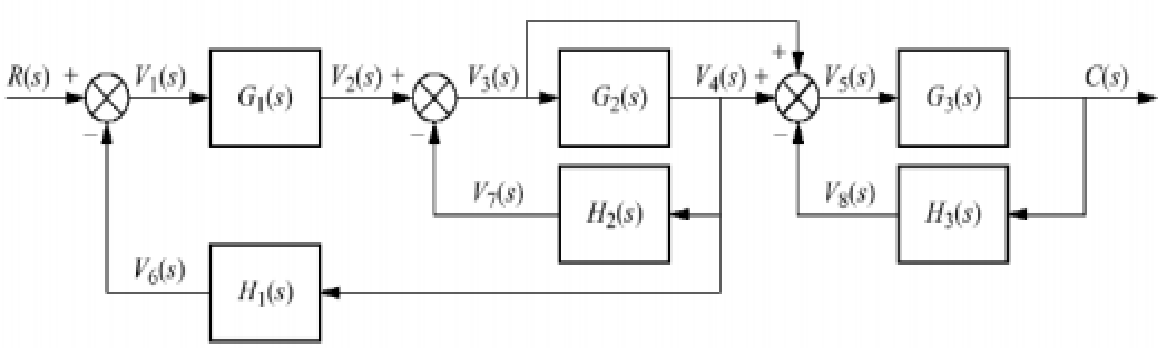
Ans.



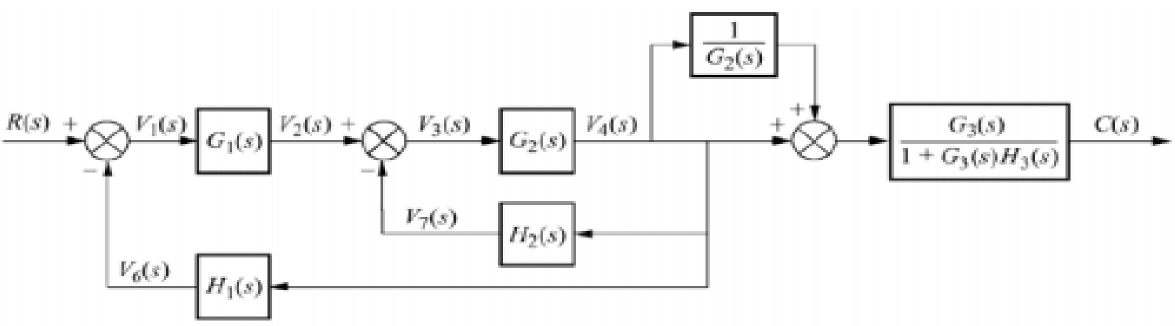
$$\frac{C(s)}{R(s)} = \frac{\dfrac{G_1 G_2}{(1+G_1 H_1)(1+G_2 H_2)}}{1+\dfrac{(G_1 G_2)}{(1+G_1 H_1)(1+G_2 H_2)}}$$

$$= \frac{\dfrac{G_1 G_2}{(1+G_1 H_1)(1+G_2 H_2)}}{\dfrac{(1+G_1 H_1)(1+G_2 H_2)+G_1 G_2}{(1+G_1 H_1)(1+G_2 H_2)}} = \frac{G_1 G_2}{(1+G_1 H_1)(1+G_2 H_2)+G_1 G_2}$$

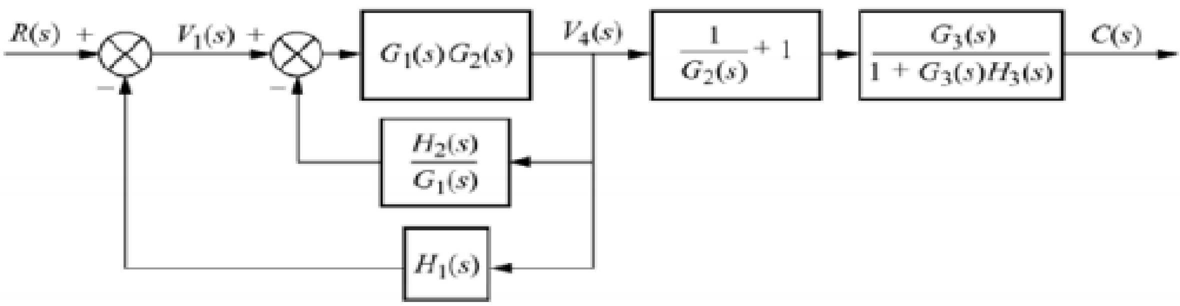
Q.3. Determine the transfer function of the system given in fig.by block diagram reduction method.



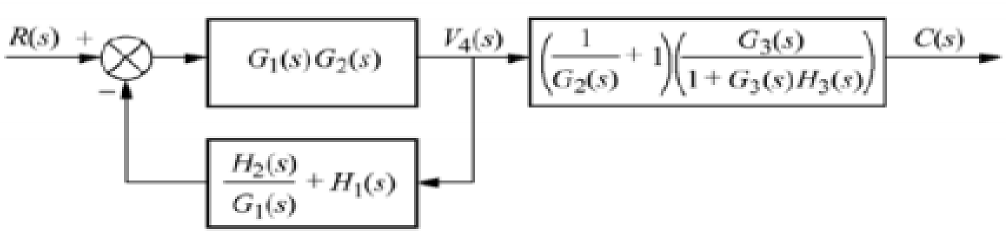
Ans:



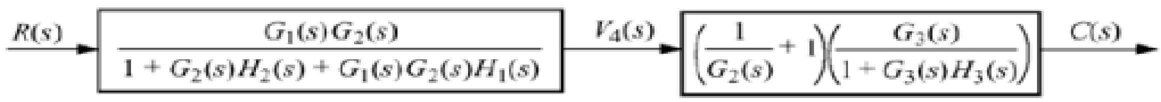
(a)



(b)



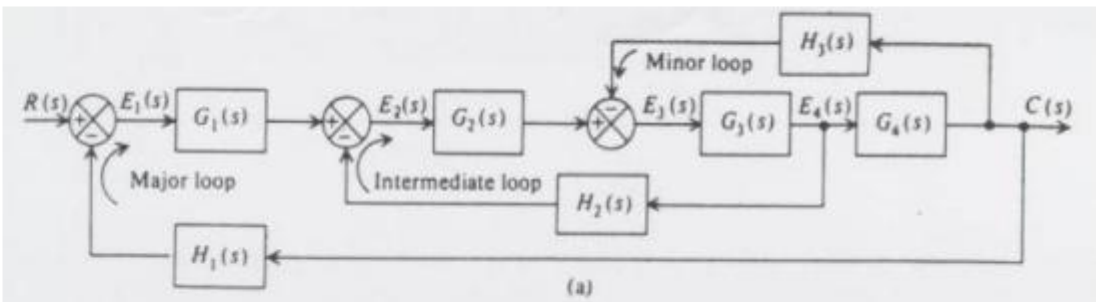
(c)



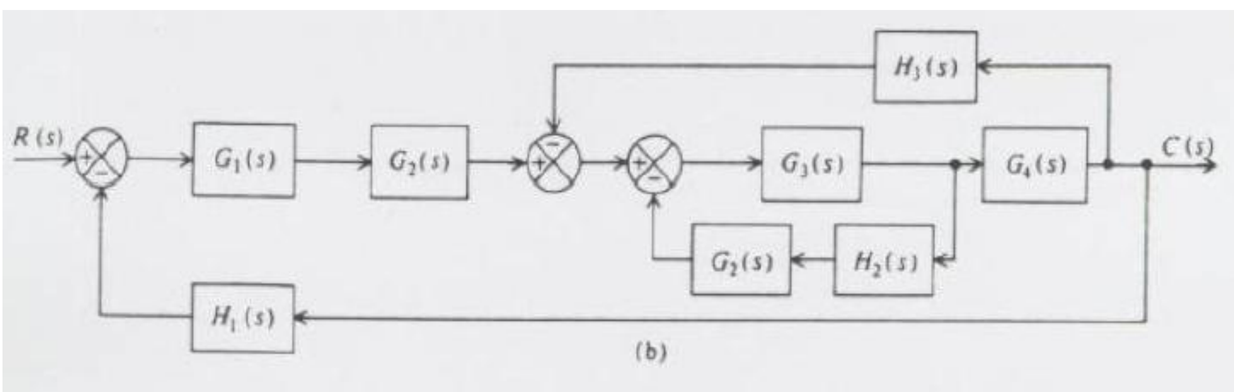
(*d*)

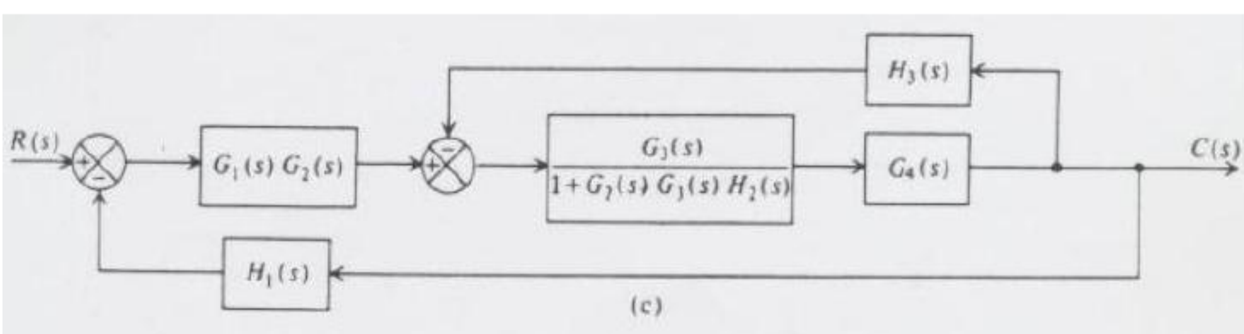
(*e*)

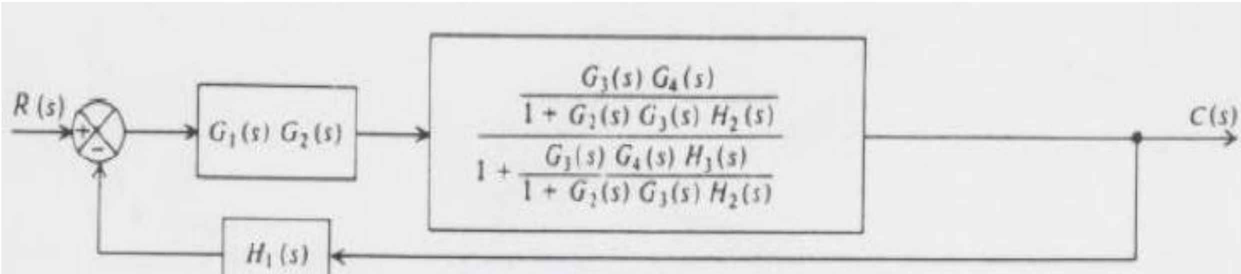
Q.4. Determine the transfer function of the system given in fig.by block diagram reduction method



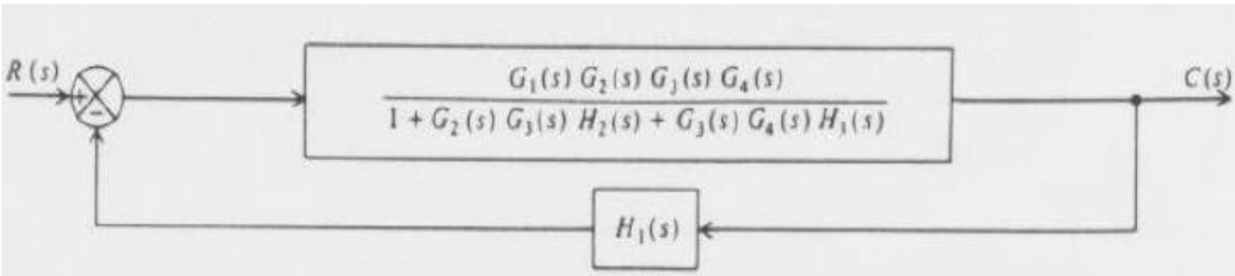
Ans:



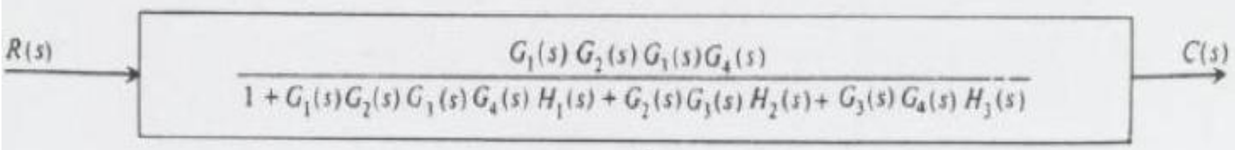



$R(s)$
$G_1(s)\,G_2(s)$
$\dfrac{\dfrac{G_3(s)\,G_4(s)}{1+G_2(s)\,G_3(s)\,H_2(s)}}{1+\dfrac{G_3(s)\,G_4(s)\,H_3(s)}{1+G_2(s)\,G_3(s)\,H_2(s)}}$
$C(s)$
$H_1(s)$

(d)


$R(s)$
$\dfrac{G_1(s)\,G_2(s)\,G_3(s)\,G_4(s)}{1+G_2(s)\,G_3(s)\,H_2(s)+G_3(s)\,G_4(s)\,H_3(s)}$
$C(s)$
$H_1(s)$

(e)


$R(s)$
$\dfrac{G_1(s)\,G_2(s)\,G_1(s)G_4(s)}{1+G_1(s)G_2(s)\,G_1(s)\,G_4(s)\,H_1(s)+G_2(s)G_3(s)\,H_2(s)+G_3(s)\,G_4(s)\,H_3(s)}$
$C(s)$

(f)

1.8 Signal Flow Graphs

An alternate to block diagram is the signal flow graph due to S. J. Mason. A signal flow graph is a diagram that represents a set of simultaneous linear algebraic equations. Each signal flow graph consists of a network in which nodes are connected by directed branches. Each node represents a system variable, and each branch acts as a signal multiplier. The signal flows in the direction indicated by the arrow.

Definitions:

Node: A node is a point representing a variable or signal.

Branch: A branch is a directed line segment joining two nodes.

Transmittance: It is the gain between two nodes.

Input node: A node that has only outgoing branche(s). It is also, called as source and corresponds to independent variable.

Output node: A node that has only incoming branches. This is also called as sink and corresponds to dependent variable.

Mixed node: A node that has incoming and out going branches.

Path: A path is a traversal of connected branches in the direction of branch arrow.

Loop: A loop is a closed path.

Self loop: It is a feedback loop consisting of single branch.

Loop gain: The loop gain is the product of branch transmittances of the loop.

Non-touching loops: Loops that do not posses a common node.

Forward path: A path from source to sink without traversing an node more than once.

Feedback path: A path which originates and terminates at the same node.

Forward path gain: Product of branch transmittances of a forward path

1.8.1 Properties of Signal Flow Graphs:

1) Signal flow applies only to linear systems.

2) The equations based on which a signal flow graph is drawn must be algebraic equations in the form of effects as a function of causes. Nodes are used to represent variables. Normally the nodes are arranged left to right, following a succession of causes and effects through the system.

3) Signals travel along the branches only in the direction described by the arrows of the branches.

4) The branch directing from node X_k to X_j represents dependence of the variable Xj on X_k

but not the reverse.

5) The signal traveling along the branch X_k and X_j is multiplied by branch gain a_{kj} and signal $a_{kj}X_k$ is delivered at node X_j.

1.8.2 Guidelines to Construct the Signal Flow Graphs:

The signal flow graph of a system is constructed from its describing equations, or by direct reference to block diagram of the system. Each variable of the block diagram becomes a node and each block becomes a branch. The general procedure is

1) Arrange the input to output nodes from left to right.
2) Connect the nodes by appropriate branches.
3) If the desired output node has outgoing branches, add a dummy node and a unity gain branch.
4) Rearrange the nodes and/or loops in the graph to achieve pictorial clarity

1.9 Mason‟s Gain Formula

The relationship between an input variable and an output variable of a signal flow graph is given by the net gain between input and output nodes and is known as overall gain of the system. Masons gain formula is used to obtain the over all gain (transfer function) of signal flow graphs.

Gain P is given by

$$P = \frac{1}{\Delta}\sum_k P_k \Delta_k$$

Where, P_k is gain of k^{th} forward path,

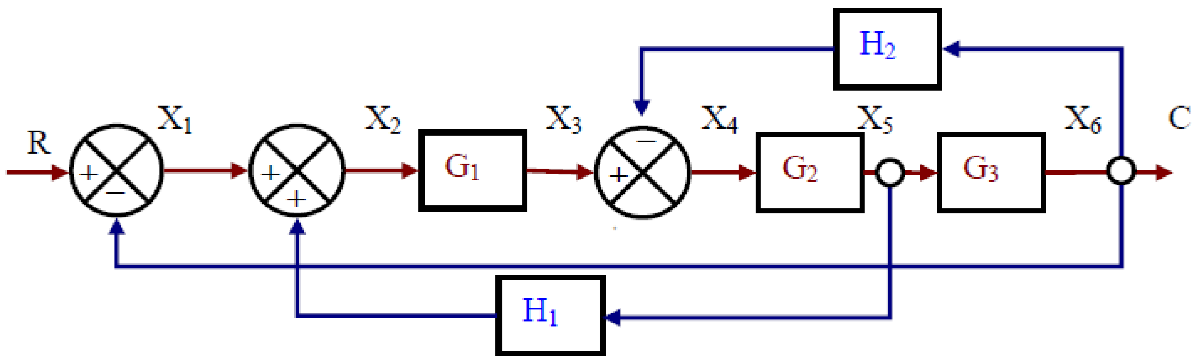
Δ is determinant of graph

Δ=1-(sum of all individual loop gains)+(sum of gain products of all possible combinations of two nontouching loops – sum of gain products of all possible combination of three nontouching loops) + ···

Δ_k is cofactor of k^{th} forward path determinant of graph with loops touching k^{th} forward path. It is obtained from Δ by removing the loops touching the path P_k.

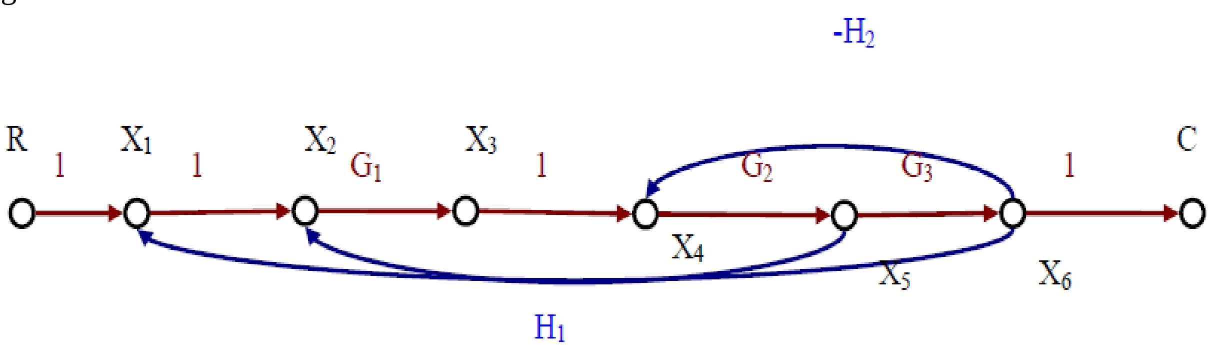
Examples :

Q.1. Draw the signal flow graph of the block diagram shown in Figure

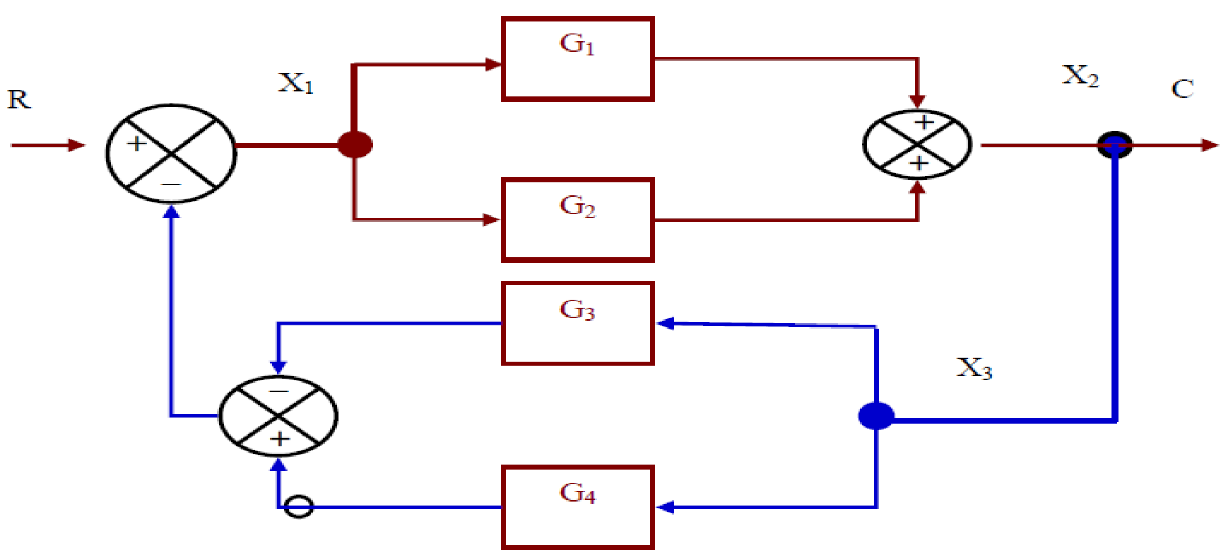


Ans: Choose the nodes to represent the variables say X_1 .. X_6 as shown in the block diagram..

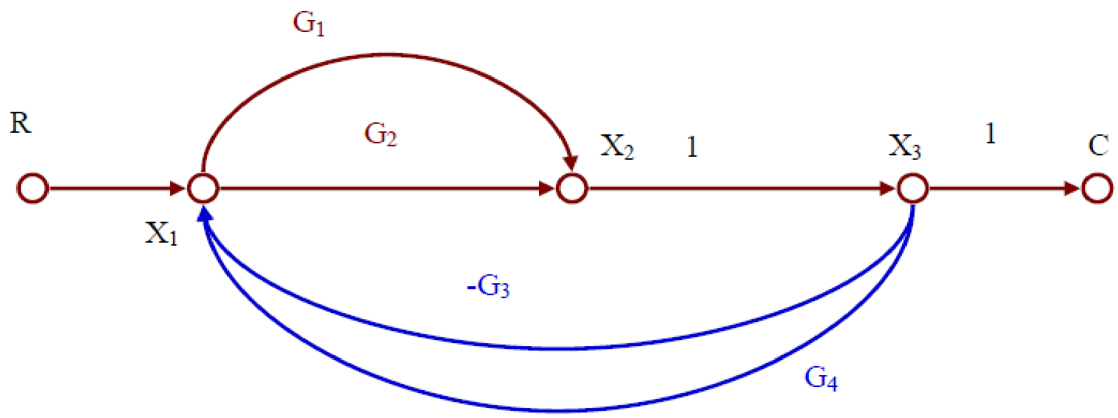
Connect the nodes with appropriate gain along the branch. The signal flow graph is shown in Fig.



Q.2.Draw the signal flow graph of the block diagram shown in Fig. and also Obtain the transfer function of C/R of the system



Ans: First draw the signal flow graph.The nodal variables are X_1, X_2, and X_3. The signal flow graph is shown in figure below.



Step I Obtain total number of forward paths

There are two forward paths:

Gain of path 1 : $P_1=G_1$
Gain of path 2 : $P_2=G_2$

Step II. Total number of single loop. There are four loops with loop gains:

$$L_1=-G_1G_3,\ L_2=G_1G_4,\ L_3=-G_2G_3,\ L_4=G_2G_4$$

Step III. Value of Δ

$$\Delta = 1+G_1G_3-G_1G_4+G_2G_3-G_2G_4$$

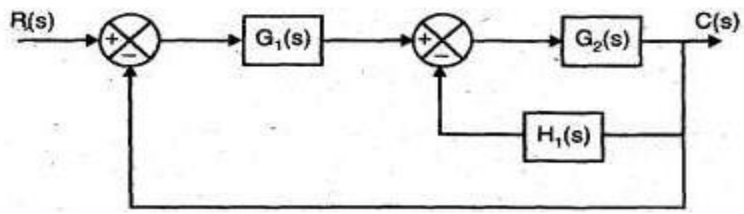
There are no non-touching loops

Forward paths 1 and 2 touch all the loops. Therefore, $\Delta_1= 1$, $\Delta_2= 1$

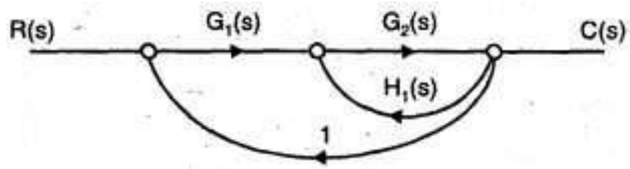
Step IV Obtain transfer function

$$\text{The transfer function } T = \frac{C(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1 + G_2}{1 + G_1G_3 - G_1G_4 + G_2G_3 - G_2G_4}$$

Q. 3. Find out the $\frac{C(s)}{R(s)}$ for the system shown in the following block diagram.

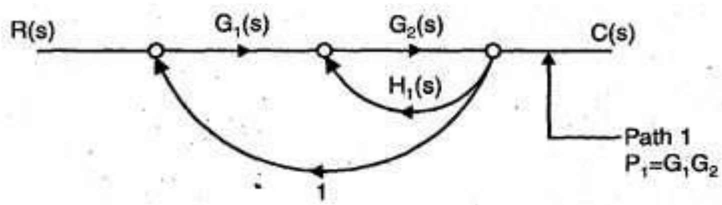


Ans. First draw the signal flow graph.



Step I Obtain total number of forward paths

There is only one forward path



Step II. Total number of single loop There are two loops. Thus

$$P_{11} = -G_2 H_1$$
$$P_{21} = -G_1 G_2$$

Step III. Value of Δ

$$\begin{aligned}\Delta &= 1 - (P_{11} + P_{21})\\ &= 1 - (-G_2 H_1 - G_1 G_2)\\ &= 1 + G_2 H_1 + G_1 G_2\end{aligned}$$

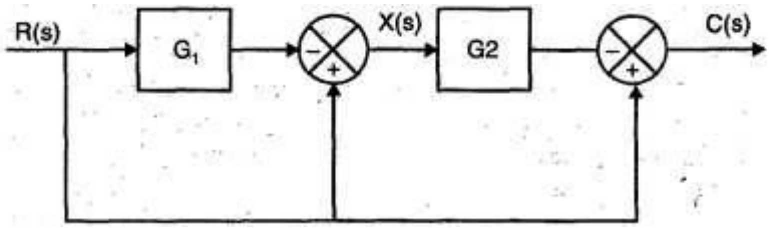
As there is one forward path which touch all the loops

$$\Delta_1 = 1$$

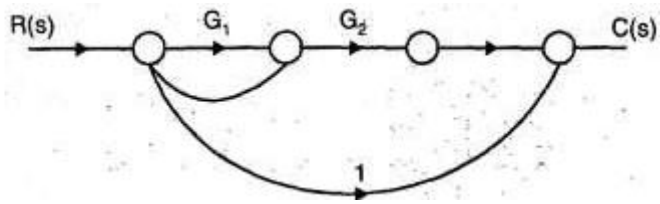
Step IV. Obtain transfer function

$$\frac{C(s)}{R(s)} = \frac{P_1 \Delta_1}{\Delta} = \frac{G_1 G_2}{1 + G_2 H_1 + G_1 G_2}$$

Q. 4. Simplify the block diagram in fig and obtain the transfer function relating C(s) and R(s).



Ans. First draw the signal flow graph



Step I Obtain total number of forward paths

$$P_1 = G_1 G_2$$
$$P_2 = G_2$$
$$P_3 = 1$$

Step II. Total number of single loop: There are no loops.

Step III. Value of Δ

$$\Delta_1 = 1 \quad \Delta_2 = 1 \quad \Delta_3 = 1$$

Step IV. Obtain transfer function

$$\frac{C(s)}{R(s)} = \frac{G_1G_2 + G_2 + 1}{1} = G_1 G_2 + G_2 + 1.$$

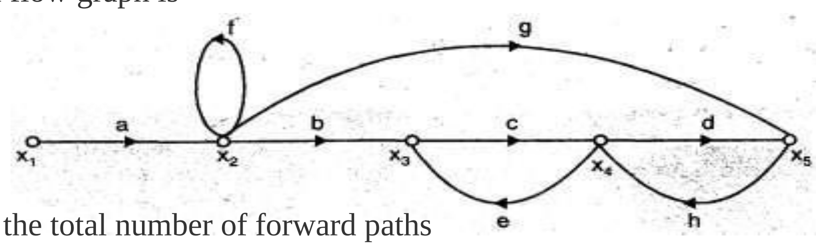
Q. 5. Represent the following set of equations by a signal flow graph and determine the overall gain relating x_5 and x_1.

$$x_2 = ax_1 + fx_2$$
$$x_3 = bx_2 + ex_4$$
$$x_4 = cx_3 + hx_5$$
$$x_5 = dx_4 + gx_2$$

Ans. Given equations are :

$$x_2 = ax_1 + fx_2$$
$$x_3 = bx_2 + ex_4$$
$$x_4 = cx_3 + hx_5$$
$$x_5 = dx_4 + gx_2$$

required signal flow graph is



Step I. Obtain the total number of forward paths

$$P_1 = abcd$$
$$P_2 = ag$$

Step II. Obtain the number of single loops

$$L_1 = f$$
$$L_2 = ce$$
$$L_3 = dh$$

Step lll. Obtain the number of two non-touching loop

$$L_1L_2 = fce$$
$$L_1L_3 = fdh$$

Step IV. Number of three non-touching loops --no

Step V. Find the value of Δ

$$\Delta = 1 - (L_1 + L_2 + L_3) + (L_1 L_2 + L_1 L_3)$$
$$= 1 - (f + ce + dh) + (fce + fdh)$$
$$= 1 - f - ce - dh + fce + fdh$$

Applying Mason's gain formula Overall transfer function is-

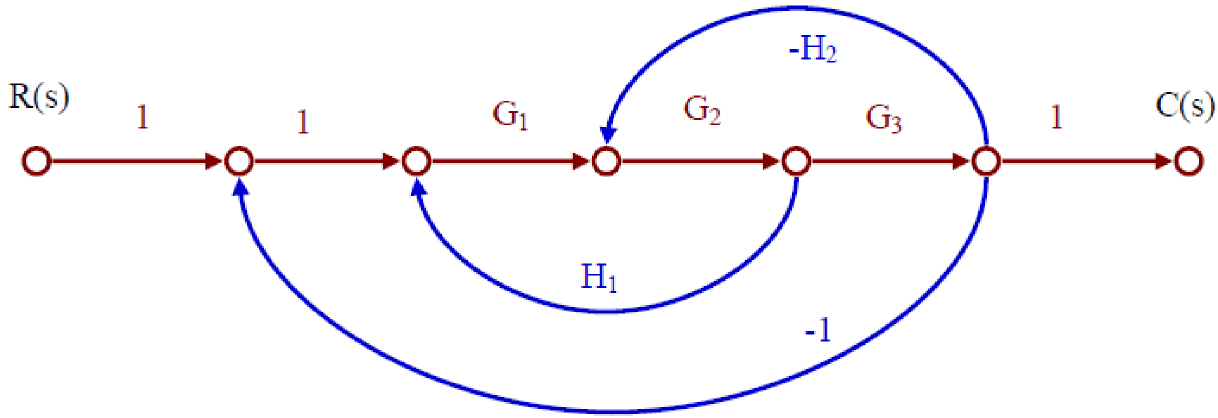
$$T.F = \frac{1}{\Delta} \Sigma P_i \Delta_i$$

$$T.F = \frac{1}{(1-f-ce-dh+fce+fdh)} \times [P_1\Delta_1 + P_2\Delta_2]$$

$$\frac{x_5}{x_1} = \frac{abcd.1 + ag(1-ce)}{1-f-ce-dh+fce+fdh}$$

$$= \frac{abcd + ag(1-ce)}{1-f-ce-dh+fce+fdh}.$$

Q.6. Obtain the transfer function of C(s)/R(s) of the system whose signal flow graph is shown in figure.



Ans:

Step I There is one forward path, whose gain is:

$$P_1=G_1G_2G_3$$

Step II There are three loops with loop gains:

$$L_1=-G_1G_2H_1,\ L2=G_2G_3H_2,\ L3=-G_1G_2G_3$$

Step III. Value of Δ

There are no non-touching loops.

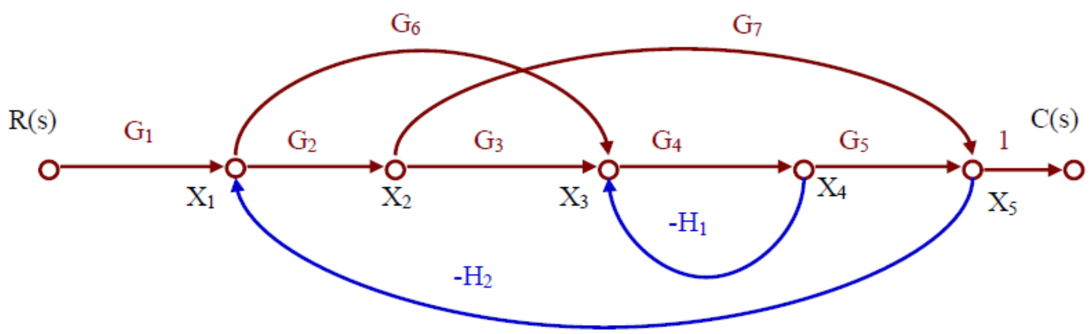
$$\Delta = 1-G_1G_2H_1+G_2G_3H_2+G_1G_2G_3$$

Forward path 1 touches all the loops. Therefore, $\Delta_1= 1$.

Step IV. Obtain transfer function

$$\text{The transfer function T} = \frac{C(s)}{R(s)} = \frac{P_1\Delta_1}{\Delta} = \frac{G_1G_2G_3}{1-G_1G_2H_1+G_1G_3H_2+G_1G_2G_3}$$

Q.7. Obtain the transfer function of C(s)/R(s) of the system whose signal flow graph is shown in figure.



Ans:

Step I There are three forward paths.

The gains of the forward path are:

$$P_1=G_1G_2G_3G_4G_5$$

$$P_2=G_1G_6G_4G_5$$

$$P_3= G_1G_2G_7$$

Step II There are four loops with loop gains:

$$L_1=-G_4H_1,\ L_2=-G_2G_7H_2,\ L3= -G_6G_4G_5H_2,\ \ L_4=-G_2G_3G_4G_5H_2$$

Step III. Value of Δ

There is one combination of Loops L_1 and L_2 which are non-touching with loop gain product

$$L_1L_2=G_2G_7H_2G_4H_1$$

$$\Delta = 1+G_4H_1+G_2G_7H_2+G_6G_4G_5H_2+G_2G_3G_4G_5H_2+ G_2G_7H_2G_4H_1$$

Forward path 1 and 2 touch all the four loops.

Therefore $\Delta_1= 1$, $\Delta_2= 1$.

Forward path 3 is not in touch with loop1.

Hence, $\Delta_3= 1+G_4H_1$

Step IV. Obtain transfer function

The transfer function

T= C(s) / R(s)

$$\frac{C(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3}{\Delta} = \frac{G_1G_2G_3G_4G_5 + G_1G_4G_5G_6 + G_1G_2G_7(1+G_4H_1)}{1+G_4H_1 + G_2G_7H_2 + G_6G_4G_5H_2 + G_2G_3G_4G_5H_2 + G_2G_4G_7H_1H_2}$$

2.0 Closed Loop System Subjected To a Disturbance

Fig2.5 shows a closed loop system subjected to a disturbance. When two inputs are present in a linear system, each input can be treated independently of the other and the outputs corresponding to each input alone can be added to give the complete output. The way in which each input is introduced into the system is shown at the summing point by either a plus or minus sign.

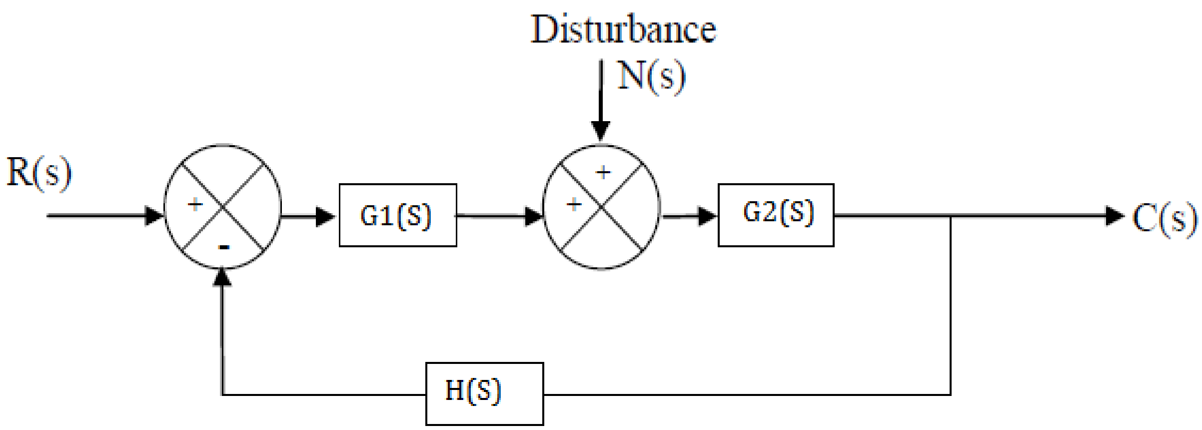


Figure 1. 6 Closed Loop System Subjected To A Disturbance

Consider the system shown in fig 2.5. We assume that the system is at rest initially with zero error. Calculate the response $C_N(s)$ to the disturbance only. Response is

$$\frac{C_N(s)}{R(s)} = \frac{G_2(s)}{1 + G_1(s)G_2(s)H(s)}$$

On the other hand, in considering the response to the reference input R(s), we may assume that the disturbance is zero.

Then the response CR(s) to the reference input R(s) is

$$\frac{CR(s)}{R(s)} = \frac{G_1(s)G_2(s)}{1 + G_1(s)G_2(s)H(s).}$$

The response C(s) due to the simultaneous application of the reference input R(s) and the disturbance N(s) is given by

$$C(s) = C_R(s) + C_N(s)$$

$$C(s) = \frac{G_2(s)}{1 + G_1(s)G_2(s)H(s)} [G_1(s)R(s) + N(s)]$$