→ Thermal Diffusivity :—

* The product ρC_p is called the heat capacity of a material. Both the specific heat C_p and the heat capacity ρC_p represent the heat storage capability of a material. But C_p expresses it per unit mass whereas ρC_p expresses it per unit volume.

* Another material property that appears in the transient heat conduction analysis is the thermal diffusivity, which represents how fast heat diffuses through a material and is defined as

$$\boxed{\alpha = \frac{\text{Heat Conducted}}{\text{Heat stored}} = \left(\frac{k}{\rho C_p}\right)}$$

* A material that has a high thermal conductivity or a low heat capacity will obviously have a large thermal diffusivity.
* The larger the thermal diffusivity, the faster the propagation of heat into the medium. A small value of thermal diffusivity means that heat is mostly absorbed by the material and a small amount of heat is conducted further.

2. Convection :—

* Convection is the mode of energy transfer between a solid surface and the adjacent liquid or gas that is in motion, and it involves the combined effects of conduction and fluid motion.

* The faster the fluid motion, the greater the convection heat transfer. In the absence of any bulk fluid motion, heat transfer between a solid surface and the adjacent fluid is by pure conduction.

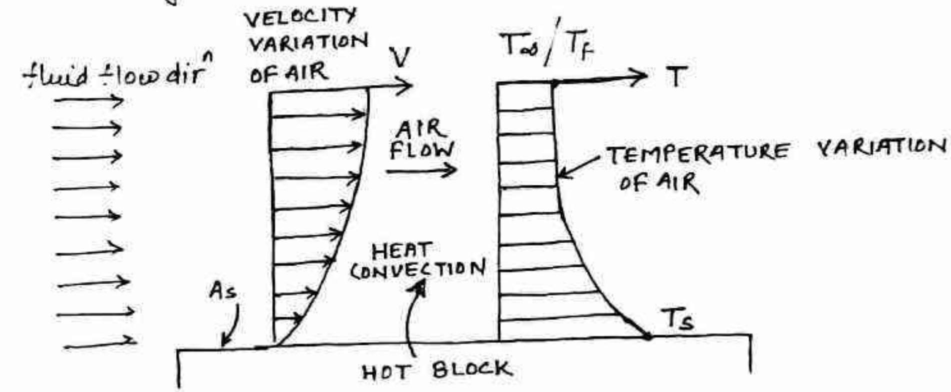


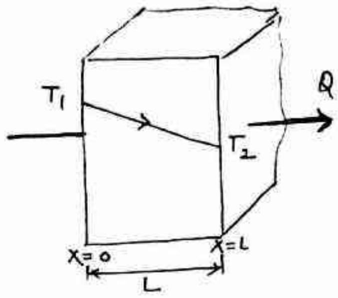
Fig : HEAT TRANSFER FROM A HOT SURFACE TO AIR BY CONVECTION

* Consider the cooling of a hot block by blowing cool air over it's top surface. Heat is first transferred to the air layer adjacent to the block by conduction. This heat is then carried away from the surface by convection, that is, by the combined effects of conduction within the air that is due to random motion of air molecules and the bulk or macroscopic motion of air that removes the heated air near the surface and replaces it by the cooler air.

* Convection is called forced convection if the fluid is forced to flow over the surface by external means such as a fan, pump, or the wind. In contrast convection is called natural or free convection if the motion is caused by buoyancy forces that are induced by density differences due to the variation of temperature in the fluid.

→ Analysis of One Dimensional Steady State Heat Conduction Without Internal Heat Generation :— ①

(A) Plane Wall / Infinite Slab :—



* One dimensional heat conduction can be given as (Laplace eqⁿ)

$$\frac{d^2T}{dx^2} = 0 \quad \text{—— (1)}$$

on integration

$$\frac{dT}{dx} = C_1 \quad \text{—— (2)}$$

on integrating again

$$T = C_1 x + C_2 \quad \text{—— (3)}$$

* Eqⁿ (2) represents the slope of temperature profile i.e. slop is constant and eqⁿ (3) represents Temperature profile which is linear.

* Boundary conditions are :

(a) at $x = 0$; $T = T_1$ and

(b) at $x = L$; $T = T_2$.

— Using boundary condition at $x = 0$, $T = T_1$ in eqⁿ (3) we get

$$T_1 = C_1(0) + C_2 \rightarrow C_2 = T_1 \quad \text{—— (4)}$$

— applying boundary condition at $x = L$, $T = T_2$ in eqⁿ (3)

$$T_2 = C_1(L) + C_2 = C_1(L) + T_1$$

$$C_1 = \frac{(T_2 - T_1)}{L} \quad \text{—— (5)}$$

— On substituting the values of C_1 & C_2 in eqⁿ (3)

$$T = \left(\frac{T_2 - T_1}{L}\right) x + T_1$$

or $\boxed{\frac{T - T_1}{T_2 - T_1} = \frac{x}{L}}$ This eqⁿ gives temperature distribution in the slab.

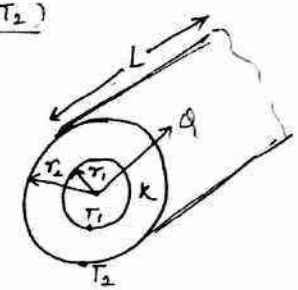
From Fourier's law of heat conduction

$$Q = -KA\frac{dT}{dx} \quad \text{but from eq}^n \text{ (2)} \quad \frac{dT}{dx} = C_1 = \frac{(T_2 - T_1)}{L}$$

(2) Hence, $Q = -KA\left(\frac{T_2 - T_1}{L}\right) = KA\frac{(T_1 - T_2)}{L}$

where $R = (L/KA)$ and $q = \frac{Q}{A} = \frac{K(T_1 - T_2)}{L}$

(B) Hollow Cylinder :—



Conduction eqⁿ for radial (1D) dirⁿ without heat generation is given as

$$\frac{1}{r}\frac{d}{dr}\left[r \cdot \frac{dT}{dr}\right] = 0$$

or $$\frac{d}{dr}\left[r \cdot \frac{dT}{dr}\right] = 0 \quad \text{—— (1)}$$

* On integrating the above eqⁿ w.r.t. 'r'.

$$r \cdot \frac{dT}{dr} = C_1 \quad \text{or} \quad \frac{dT}{dr} = \frac{C_1}{r} \quad \text{—— (2)}$$

* On integrating again

$$T = C_1 \ln r + C_2 \quad \text{—— (3)}$$

Boundary conditions are:

(a) At $r = r_1$; $T = T_1$ and

(b) At $r = r_2$; $T = T_2$

Now substituting the boundary conditions in eqⁿ (3)

$$T_1 = C_1 \ln r_1 + C_2 \quad \text{—— (4)}$$

and $$T_2 = C_1 \ln r_2 + C_2 \quad \text{—— (5)}$$

Subtracting eqⁿ (4) from (5)

$$(T_2 - T_1) = C_1(\ln r_2 - \ln r_1) = C_1 \ln r_2/r_1$$

$$\text{or } C_1 = \left[\frac{T_2 - T_1}{\ln(r_2/r_1)}\right] \quad \text{—— (6)}$$

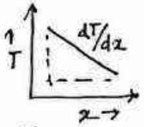
Substituting the value of C_1 in eqⁿ (4)

$$T_1 = \frac{(T_2 - T_1)}{\ln(r_2/r_1)}\ln r_1 + C_2 \rightarrow C_2 = T_1 - \frac{(T_2 - T_1)}{\ln(r_2/r_1)}\ln r_1 \quad \text{—— (7)}$$

where the constant of proportionality 'k' is the Thermal conductivity of the material, which is a measure of the ability of a material to conduct heat.

Note: In the limiting case of $\Delta x \to 0$, the eq^n reduces to the differential form

$$\boxed{\dot{Q}_{conducted} = -kA_s \frac{dT}{dx}}$$



Which is called Fourier's law of heat conduction.

* Here (dT/dx) is the temperature gradient, which is the ~~slope~~ ~~of the~~ slope of the temperature curve on a $(T-x)$ diagram
* The above relation indicates that the rate of heat conduction in a given direction is proportional to temp. gradient in that dir^n. Heat is conducted in dir^n of decreasing temperature, and the temp. gradient becomes negative when temperature decreases with increasing x. The negative sign in above eq^n ensures that heat transfer in a positive x-dir^n is a positive quantity. The heat transfer area is always perpendicular to the dir^n of heat flow.

* The heat flux $\dot{q}$ is the heat conducted per unit time per unit area is given by.

$$\dot{q} = \frac{\dot{Q}}{A_s} = -\frac{k \cancel{A_s}}{\cancel{A_s}}\frac{dT}{dx} = -k\frac{dT}{dx}$$

→ Thermal Conductivity :—

* Thermal conductivity of a material can be defined as the rate of heat transfer through a unit thickness of the material per unit area per unit temperature difference.
* The thermal conductivity k is a measure of a material's ability to conduct heat.
* A high value for thermal conductivity indicates that the material is a good heat conductor and vice-versa.

$$\boxed{k = \frac{\dot{Q}}{A_s} \times \frac{dx}{dT}}$$

The value of $k = 1$ when, $\dot{Q} = 1$, $A_s = 1$ and $\frac{dT}{dx} = 1$

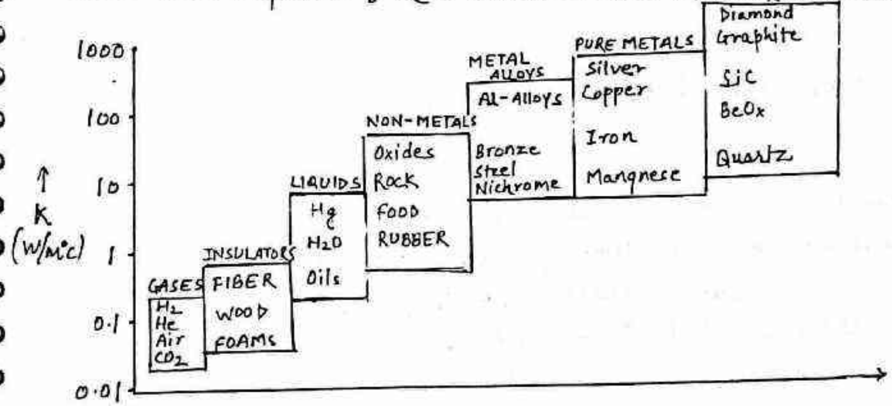
Unit of $k = \frac{W}{m^2} \times \frac{m}{°C\,(\text{or}\,K)} = (W/m°C)$

* Conduction of heat occurs most readily in pure metals, less so in alloys, and much less readily in non-metals.
* Thermal conductivity (a property of material) depends essentially upon following factors.

a) Material structure
b) Moisture content
c) Density of material
d) Pressure and temperature (Operating conditions)

→ Important points regarding conductivity are as follows:

1. Thermal conductivity in case of pure metals is the highest. It decreases with increase in impurity.
Metals : $k = 10-400$ W/m°C
Alloys : $k = 12-120$ W/m°C
Heat insulating & building materials : $k = 0.023 - 2.9$ W/m°C
Liquids : $k = 0.2-0.5$ W/m°C
Gases and vapours : $k = 0.006 - 0.05$ W/°C m

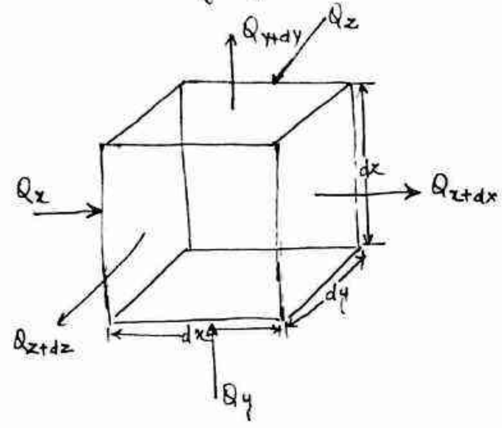


2. Thermal conductivity of most metals decreases with the increase in temperature. (Al & Ur are exception)
In most of liquids the value of thermal conductivity decreases with temperature (Water being exception) due to decrease in density with increase in temperature.

→ General Heat Conduction Equation In Cartesian Co-ordinates

* If the temperature is independent of time and heat flow rate is constant, the system is said to be under steady state. Else it called unsteady.

* Considering an infinitesimal rectangular parallelopiped of sides dx, dy and dz along x, y and z axes respectively in a medium in which temperature is varying with location & time.



→ Special cases of general heat conduction eqⁿ :—

1. For Isotropic Materials :—

$K_x = K_y = K_z = K$ constant

General eqⁿ reduces to

$$\frac{d^2T}{dx^2} + \frac{d^2T}{dy^2} + \frac{d^2T}{dz^2} + \frac{q}{K} = \frac{\rho C_p}{K}\frac{dT}{dt} \quad \left\{\because \frac{1}{\alpha} = \frac{\rho C_p}{K}\right\}$$

* where, α represents the thermal diffusivity of the material

$$\frac{d^2T}{dx^2} + \frac{d^2T}{dy^2} + \frac{d^2T}{dz^2} + \frac{q}{K} = \frac{1}{\alpha}\frac{dT}{dt} \quad \left\{\alpha = \frac{K}{\rho C_p}\right\}$$

2. Steady State Conduction :—

* The system is said to be in steady state if the Temperature of material at any point does not change with time i.e. $\frac{dT}{dt} = 0$ hence general eqⁿ reduces to.

$$\frac{d^2T}{dx^2} + \frac{d^2T}{dy^2} + \frac{d^2T}{dz^2} + \frac{q}{K} = 0 \quad [\text{Poission's eq}^n]$$

3. No. Heat Sources :—

* In absence of any heat generation of energy within the body i.e. $q = 0$, the eqⁿ reduces to

$$\frac{d^2T}{dx^2} + \frac{d^2T}{dy^2} + \frac{d^2T}{dz^2} = \frac{1}{\alpha}\frac{dT}{dt} \quad [\text{Fourier's eq}^n]$$

4. No Heat source and Steady state conditions :—

Since $q = 0$ and $\frac{dT}{dt} = 0$, eqⁿ reduces to

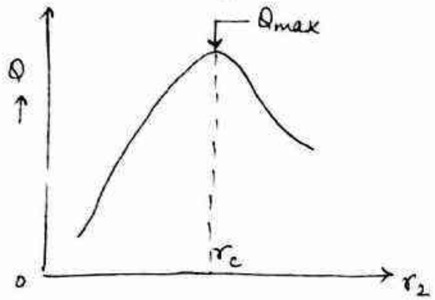
$$\frac{d^2T}{dx^2} + \frac{d^2T}{dy^2} + \frac{d^2T}{dz^2} = 0 \quad [\text{Laplace eq}^n]$$

or $\nabla^2 T = 0$

5. One-dimensional Heat Conduction eqⁿ Without Heat Generation Under Steady State :—

$$\frac{d^2T}{dx^2} = 0$$

* Heat transfer rate (Q) as a function of (r_2) may be seen with the help of a plot in which Q firstly increases and then decreases passing through a maximum value.



* To find the value of r_2 for which Q is maximum, $\left(\frac{dQ}{dr_2}\right)$ should be equated to zero or denominator of above equation should be minimum, hence differentiating denominator with respect to r_2 and equating it to zero, we have.

$$\therefore \frac{d}{dr_2}\left[\frac{\ln(r_2/r_1)}{2\pi K_i L} + \frac{1}{h_o \cdot 2\pi r_2 L}\right] = 0$$

$$\text{or} \quad \frac{d}{dr_2}\left[\frac{\ln(r_2/r_1)}{K_i} + \frac{1}{h_o r_2}\right] = 0$$

$$\frac{d}{dr_2}\left[\frac{\ln r_2}{K_i} - \frac{\ln r_1}{K_i} + \frac{1}{h_o r_2}\right] = 0$$

$$\left[\frac{1}{r_2 K_i} - 0 - \frac{1}{h_o r_2^2}\right] = 0$$

$$\frac{1}{K_i} - \frac{1}{h_o r_2} = 0$$

$$1/K_i = \frac{1}{h_o r_2} \quad \text{or} \quad \boxed{r_2 = \frac{K_i}{h_o} = r_c}$$

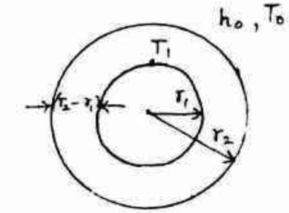
* Hence r_c is called critical radius of insulation after which increase in insulation will further decrease the rate of heat transfer.

→ Important aspects of critical radius of insulation

* With the increase in thickness of insulation (r_2), conductive resistance increases logrithmically and convective resistance decreases linearly, hence total resistance first decreases, attains minimum value (corresponding to maximum Q) and then increases.

3. Insulation in case of spheres :—

Consider a hollow sphere of outer radius (r_1) at temperature T_1 which is covered with an insulation of thickness ($r_2 - r_1$) so that it's outer radius is (r_2). It is also subjected to convective heat transfer (h_o) having fluid temperature T_o. Let K_i be the thermal conductivity of insulation.



* Heat transfer rate across the sphere can be given as

$$Q = \frac{(T_1 - T_o)}{\frac{(1/r_1 - 1/r_2)}{4\pi K_i} + \frac{1}{4\pi r_2^2 h_o}}$$

$$Q = \frac{(T_1 - T_o)}{\frac{(r_2 - r_1)}{4\pi K_i r_1 r_2} + \frac{1}{4\pi r_2^2 h_o}}$$

* To find the value of (r_2), for which Q is maximum, $\frac{dQ}{dr_2}$ should be equated to zero. denominator of above equation should be minimum, hence differentiating denominator with respect to r_2 and equating it to zero.

$$\therefore \frac{d}{dr_2}\left[\frac{(r_2 - r_1)}{4\pi K_i r_1 r_2} + \frac{1}{4\pi r_2^2 h_o}\right] = 0$$

$$\text{or} \quad \frac{d}{dr_2}\left[\frac{1}{K_i r_1} - \frac{1}{K_i r_2} + \frac{1}{r_2^2 h_o}\right] = 0$$

$$\left[0 + \frac{1}{K_i r_2^2} - \frac{2}{h_o r_2^3}\right] = 0$$

$$\boxed{r_2 = \frac{2K_i}{h_o} = r_c}$$

* For reducing heat loss due to insulation $r_{insulation} >> r_c$

from Fourier's Law of heat conduction

$$\dot{Q} = -KA\frac{dT}{dx} = -K_0(1+\alpha T)A\frac{dT}{dx}$$

$$\therefore \frac{\dot{Q}}{A}\cdot dx = -K_0(1+\alpha T)\,dT$$

Integrating above eqⁿ with boundary conditions as at $x=0$, $T=T_1$ and at $x=x$; $T=T_2$

$$\rightarrow \frac{\dot{Q}}{A}\int_0^x dx = -K_0\int_{T_1}^{T_2}(1+\alpha T)\cdot dT$$

$$\rightarrow \frac{\dot{Q}}{A}[x]_0^x = -K_0\left[T+\frac{\alpha}{2}T^2\right]_{T_1}^{T_2}$$

$$\rightarrow \frac{\dot{Q}}{A}(x-0) = -K_0\left[(T_2-T_1)+\frac{\alpha}{2}(T_2^2-T_1^2)\right]$$

$$\frac{\dot{Q}}{A}x = +K_0\left[(T_1-T_2)+\frac{\alpha}{2}(T_1^2-T_2^2)\right]$$

$$\frac{\dot{Q}}{A}x = K_0\left[(T_1-T_2)+\frac{\alpha}{2}(T_1-T_2)(T_1+T_2)\right]$$

$$\frac{\dot{Q}}{A}x = K_0(T_1-T_2)\left[1+\frac{\alpha}{2}(T_1+T_2)\right]$$

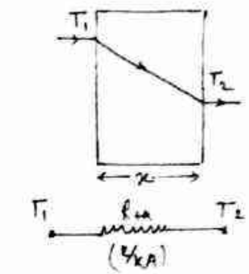
$$\boxed{\dot{Q} = \frac{K_m A(T_1-T_2)}{x}}$$; where $K_m = K_0\left[1+\frac{\alpha}{2}(T_1+T_2)\right]$ represents the mean value of thermal conductivity at mean Temperature of $T = \frac{(T_1+T_2)}{2}$

→ One Dimensional Steady State Heat Conduction Without Heat Generation :—

* Heat transfer in plane wall by conduction :—

from Fourier's Law of conduction

$$\dot{Q} = \frac{KA(T_1-T_2)}{x}$$



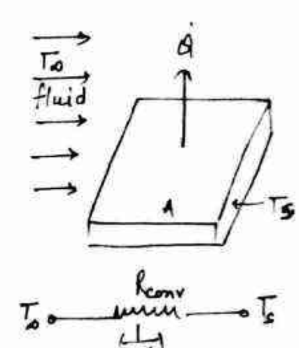
$$\rightarrow \dot{Q} = \frac{(T_1-T_2)}{(x/KA)}$$

$$\rightarrow \dot{Q} = \frac{(T_1-T_2)}{R_{th}}$$

$$\rightarrow R_{th} = \left(\frac{x}{KA}\right)$$

* Heat transfer by convection :—

from Newton's law of cooling

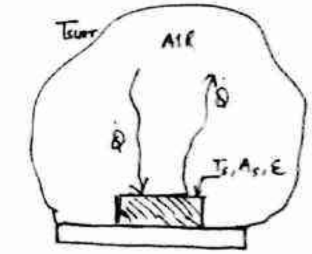


$$\dot{Q} = h\cdot A\cdot(T_s-T_\infty)$$

$$\rightarrow \dot{Q} = \frac{T_s-T_\infty}{(1/hA)} = \frac{T_s-T_\infty}{R_{conv}}$$

$$\rightarrow R_{conv} = \left(\frac{1}{hA}\right)$$

* Heat transfer by radiation



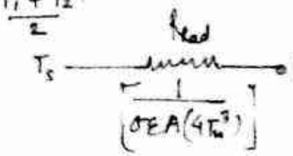
$$\dot{Q} = \sigma\varepsilon A_s(T_s^4 - T_{surr}^4)$$

$$\dot{Q} = \sigma\varepsilon A_s(T_s^2+T_{surr}^2)(T_s^2-T_{surr}^2)$$

$$\dot{Q} = \sigma\varepsilon A_s(T_s^2+T_{surr}^2)(T_s-T_{surr})(T_s+T_{surr})$$

for $T_s \simeq T_{surr}$

$$\dot{Q} = \sigma\varepsilon A_s(4T_m^3)(T_s-T_{surr}) \text{ where } T_m = \frac{(T_1+T_2)}{2}$$

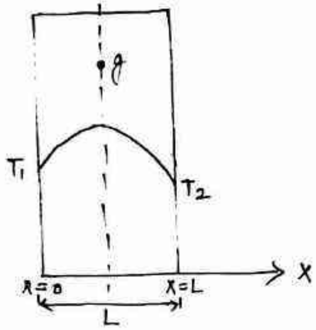


$$\dot{Q} = \frac{(T_s-T_{surr})}{\left[\frac{1}{\sigma\varepsilon A_s(4T_m^3)}\right]} = \frac{(T_s-T_{surr})}{R_{rad}}$$

$$\rightarrow R_{rad} = \left[\frac{1}{\sigma\varepsilon A_s(4T_m^3)}\right]$$

→ Unidirectional Steady State Heat Conduction With Internal Heat Generation In Plane Wall (Poisson's Equation) :—

(A) Specified Temperature On Both Sides :— (6)



* Let, 'q' be the heat generated per unit vol^m by a heat source in the system.

The poisson's eq^n can be written as

$$\frac{d^2T}{dx^2} + \frac{q}{K} = 0$$

or $$\frac{d^2T}{dx^2} = -\frac{q}{K}$$ — on integration ——— (1)

$$\frac{dT}{dx} = -\frac{q}{K}x + C_1 \quad \text{——— (2)}$$

on second integration

$$T = -\frac{q}{K}\frac{x^2}{2} + C_1 x + C_2 \quad \text{——— (3)}$$

* Boundary conditions are :

(a) At $x = 0$, $T = T_1$

(b) At $x = L$, $T = T_2$

On substituting the above boundary conditions specified at (a) in eq^n (3)

$$T_1 = -\frac{q}{K} \times \frac{(0)^2}{2} + C_1(0) + C_2 \rightarrow C_2 = T_1 \quad \text{——— (4)}$$

∴ Substituting the value of C_2 in eq^n (3)

$$T = -\frac{q}{K}\frac{x^2}{2} + C_1 x + T_1$$

Now at $x = L$; $T = T_2$; above eq^n reduces to

$$T_2 = -\frac{q}{K}\frac{L^2}{2} + C_1 L + T_1$$

or $$C_1 = \frac{(T_2 - T_1)}{L} + \frac{q}{K}\frac{L}{2} \quad \text{——— (5)}$$

On substituting the value of C_1 & C_2 in eq^n (3)

$$\boxed{T = -\frac{q}{K}\frac{x^2}{2} + \left[\frac{(T_1 - T_2)}{L} + \frac{q}{2K}L\right]x + T_1} \quad \text{——— (6)}$$

* Since T is function of x^2, the Temperature distribution is not linear as in case of plane slab without heat generation.

* In case both surfaces of the wall are maintained at equal Temperature i.e. $T_1 = T_2$, eq^n (6) reduces to (7)

$$T = -\frac{q}{K}\cdot\frac{x^2}{2} + \frac{q}{2K}\cdot L\cdot x + T_1$$

or $$T = -\frac{q}{2K}(x^2 - Lx) + T_1$$

or $$T = -\frac{q}{2K}\cdot L^2\left(\frac{x^2}{L^2} - \frac{x}{L}\right) + T_1$$

* above eq^n shows that Temperature variation is parabolic and max^m temperature occurs at the centre of slab.

* On substituting the value of C_1 in eq^n (2) when $T_1 = T_2$

$$\frac{dT}{dx} = -\frac{q}{K}x + \frac{q}{2K}\cdot L = -\frac{q}{2K}(2x - L)$$

at the point of max^m temperature, slope $\frac{dT}{dx} = 0$

$$\therefore -\frac{q}{2K}(2x - L) = 0 \;; \quad x = L/2$$

It shows that the max^m temperature occurs at the centre of slab in case $T_1 = T_2$

CONDUCTION

→ **Thermal Conductivity of Materials :—**

Thermal conductivity indicates the ability of material to conduct heat. Thermal conductivity value varies widely for various engineering materials and it is the function of temperature, density, structure etc.

* Thermal conductivity of metals is mainly due to flow of free electrons while in case of other solids and fluids, it is due to molecular vibrations / collisions.

* Thermal conductivity of materials in decreasing order is as follows.
  Metals → Non-metallic solids → liquids → Gases

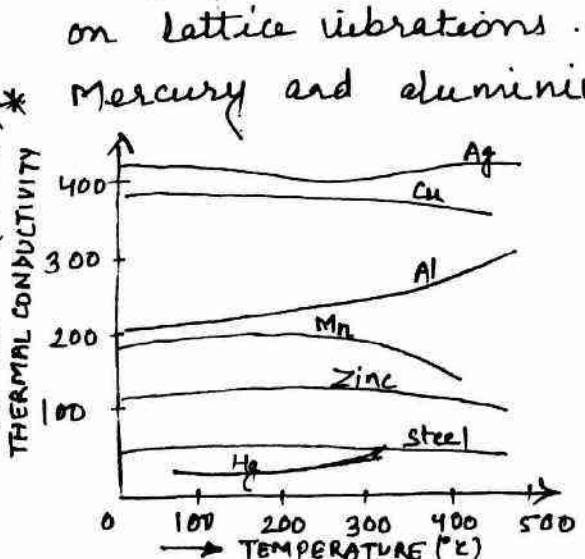
* Mechanism of Heat conduction :—
→ The heat transfer in solids is both by transport by free electrons (70%) and by lattice vibration (30%)
→ The heat transfer in fluids is due to collisions of molecules and diffusion of mass (due to change in densities on temperature variations).

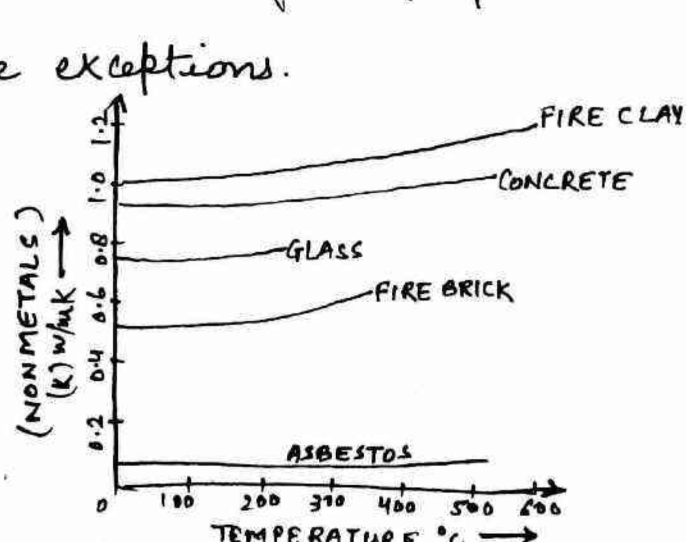
↳ **Effect of variation of temperature on thermal conductivity of solids :—**

* Thermal conductivity of pure metals decreases with increase in temperature because lattice vibrations retards the motion of free electrons.

* Whereas the thermal conductivity of alloys and insulating materials, having few free electrons, increases with increase in temperature because their conductivity largely depends on lattice vibrations.

* Mercury and aluminium are exceptions.





Now substituting the value of C_1 & C_2 in eq^n (3) we have

$$T = \left[\frac{T_2 - T_1}{\ln \frac{r_2}{r_1}}\right] \ln r + \left[T_1 - \frac{(T_2 - T_1)}{\ln(r_2/r_1)} \cdot \ln r_1\right] \quad (3)$$

$$\text{or } T = \left(\frac{T_2 - T_1}{\ln r_2/r_1}\right) \ln r + T_1 - \left(\frac{T_2 - T_1}{\ln r_2/r_1}\right) \ln r_1$$

$$\text{or } T - T_1 = \left[\frac{T_2 - T_1}{\ln\left(\frac{r_2}{r_1}\right)}\right] (\ln r - \ln r_1) = \left(\frac{T_2 - T_1}{\ln r_2/r_1}\right) \ln\left(\frac{r}{r_1}\right)$$

$$\text{or } \left(\frac{T - T_1}{T_2 - T_1}\right) = \frac{\ln r/r_1}{\ln r_2/r_1}$$

$$\text{or } \boxed{\frac{T - T_1}{T_1 - T_2} = \frac{\ln(r/r_1)}{\ln(r_1/r_2)}}$$

This eq^n gives temp. distribution in hollow cylinder in radial dir^n without heat generation.

* From Fourier's law of heat conduction

$Q = -KA \frac{dT}{dx}$ but from eq^n (2) $\frac{dT}{dx} = \frac{C_1}{r}$ and $C_1 = \left(\frac{T_2 - T_1}{\ln r_2/r_1}\right)$

Since $A = 2\pi r L$, above eq^n reduces to

$$Q = -K(2\pi \cancel{r} L) \frac{(T_2 - T_1)}{\cancel{r} \ln(r_2/r_1)} = \frac{-(T_2 - T_1)}{\left[\frac{\ln(r_2/r_1)}{2\pi L K}\right]} \text{ or}$$

$$Q = \frac{(T_1 - T_2)}{\left[\frac{\ln(r_2/r_1)}{2\pi K L}\right]} = \frac{\Delta T}{R_{th}} \Rightarrow \boxed{R_{th} = \left[\frac{\ln(r_2/r_1)}{2\pi K L}\right]}$$

Imp → **Logarithmic Mean Area (LMA) For Hollow Cylinder :—**

* Since area changes with radius, therefore it is convenient to calculate a mean area (A_m) for use in analogous formula of slab, $\left(Q = -KA \frac{dT}{dx}\right)$.

Rewriting the eq^n.

(4)

$$Q = \frac{2\pi K L \cdot \Delta T}{\ln(r_2/r_1)} = K A_m \frac{\Delta T}{(r_2 - r_1)} \quad \text{—— (1)}$$

Multiplying and dividing the eq^n by $(r_2 - r_1)$ we get

$$Q = 2\pi K L \frac{\Delta T}{\ln(r_2/r_1)} \times \frac{(r_2 - r_1)}{(r_2 - r_1)}$$

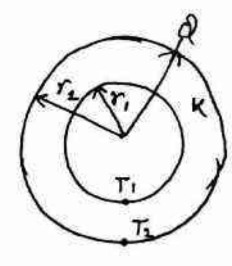
$$Q = \frac{2\pi K L (r_2 - r_1)}{\ln(r_2/r_1)} \times \frac{\Delta T}{(r_2 - r_1)} \quad \text{—— (2)}$$

Comparing eq^n (1) & (2)

$$\boxed{A_m = \frac{2\pi L (r_2 - r_1)}{\ln(r_2/r_1)} = \frac{A_o - A_i}{\ln(A_o/A_i)}}$$

(C) Hollow Sphere :—

* Conduction eq^n for one dimensional (radial) heat flow without heat generation is given as.



$$\frac{1}{r^2} \frac{d}{dr}\left[r^2 \frac{dT}{dr}\right] = 0$$

$$\text{or } \frac{d}{dr}\left[r^2 \frac{dT}{dr}\right] = 0 \quad \text{—— (1)}$$

On integrating

$$r^2 \frac{dT}{dr} = C_1 \text{ or } \frac{dT}{dr} = \frac{C_1}{r^2} \quad \text{—— (2)}$$

On integrating again

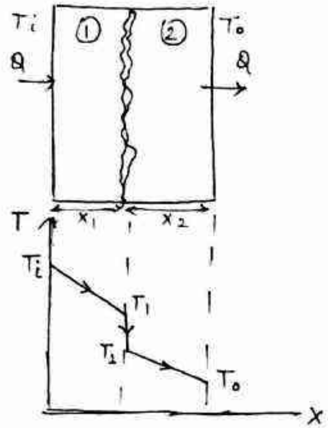
$$T = -\frac{C_1}{r} + C_2 \quad \text{—— (3)}$$

Boundary conditions are

(a) at $r = r_1$; $T = T_1$ and

(b) at $r = r_2$; $T = T_2$

Substituting the boundary conditions in eq^n (3) we get



Contact resistance between two solid surfaces

* Thermal contact resistance at interface develops when two surfaces do not fit tightly and a thin layer of fluid (air or surrounding fluid) is filled between them. This contact resistance is the function of surface roughness, the pressure holding the two surfaces, the property of fluid and the interface temperature.

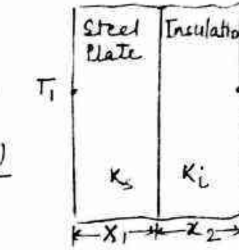
Thermal Insulation :—

* A heat insulating material is one which has low thermal conductivity. It is provided in thermal systems to reduce the heat losses. It retards the heat flow with effectiveness.
* Properties of insulating materials are
  1. It should be able to withstand high or low temperatures
  2. It should have long life and could withstand rough handling
  3. It must be easy to apply and be economical
  4. It also should not have any fire risks.
* Examples of insulating materials are asbestos, glass, rock-wool, cork, man made plastic material like expanded polystyrene etc.
* Application of thermal insulating materials.
  1. Boilers and steam pipes
  2. Airconditioning systems
  3. Food preserving stores and refrigerators.
  4. Insulating bricks
  5. Preservation of liquid gases etc.

Critical Thickness of Insulation :—

* Purpose of insulation is to reduce the heat transfer rate but it is not always true.

1. Insulation in case of plane walls :—



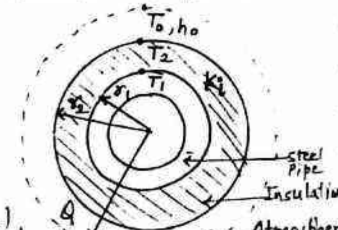
* Considering the case of heat flow across a steel plate with and without insulation

Heat transfer with insulation $Q = \dfrac{(T_1 - T_2)}{\left(\dfrac{X_1}{K_s A}\right)}$

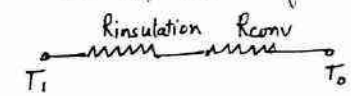
Heat transfer without insulation $Q_1 = \dfrac{(T_1 - T_2)}{\left(\dfrac{X_1}{K_s A} + \dfrac{X_2}{K_i A}\right)}$

* Due to increase in thermal resistance of insulation, the value of Q_1 is always less than Q. It implies that the heat transfer rate will always reduce with insulation in case of plane walls.

2. Insulation in case of cylinder :—



* Considering heat flow from steel pipe of outside radius (r_1), insulated by a layer of insulation having outer radius (r_2).
* Let the temperature of outside surface of steel tube be T_1, conductivity of insulation be K_i (W/mK) and let this insulation be exposed to atmospheric air at temperature T_0 with convective heat transfer coefficient as (h_0) and length of pipe be 'L'.



* Heat transfer rate from insulated steel pipe.

$$Q = \frac{T_1 - T_0}{\left[\dfrac{\ln(r_2/r_1)}{2\pi K_i L} + \dfrac{1}{h_0 2\pi r_2 L}\right]}$$

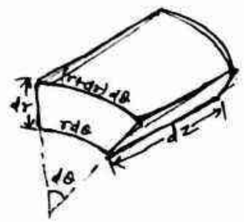
From above eq^n it is clear that on increase of insulation i.e. (r_2) heat flow rate Q may decrease or increase since conductive resistance $\left[\dfrac{\ln r_2/r_1}{2\pi K L}\right]$ increases logarithmically but convective resistance $\left[\dfrac{1}{h_0 2\pi r_2 L}\right]$ decreases linearly.

→ Unsteady state one-dimensional heat conduction equation in cylindrical co-ordinates :—

* Consider an element having polar co-ordinates (r, θ, z). Three sides are dr, dz and $r.d\theta$ are shown in figure.

* As per Fourier's Law of heat conduction

$$Q = -KA\frac{dT}{dx}$$



Hence heat entering in element in radial dir^n is given by

$$dQ_r = -K_r(rd\theta dz)\frac{dT}{dr}$$

and heat leaving the element in radial dir^n is given by.

$$dQ_{r+dr} = dQ_r + \frac{d}{dr}(dQ_r)dr$$

∴ Net heat flow into the element in r-dir^n in certain time 'dt' is given by

$$[dQ_r - dQ_{r+dr}]dt = \left[dQ_r - dQ_r - \frac{d}{dr}(dQ_r)dr\right]dt$$

$$= -\frac{d}{dr}(dQ_r)dr\,dt$$

$$= -\frac{d}{dr}\left[-K_r(rd\theta dz)\frac{dT}{dr}\right]dr\,dt$$

$$= K_r(d\theta dz\,dr)dt\,\frac{d}{dr}\left[r.\frac{dT}{dr}\right]$$

$$= K_r(d\theta dz\,dr)dt\left[r.\frac{d^2T}{dr^2} + \frac{dT}{dr}\right]$$

$$= K_r(rd\theta dz\,dr)\left[\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr}\right]dt$$

* Now consider that there is some heat source within the element which generates heat, given as 'q' (heat generated per unit vol per unit time)

Therefore internal heat generation in time dt is given as

$$= q(rd\theta dz\,dr)dt$$

* Heat gained by the element from internal heat generation will result into energy storage and will increase it's Temperature. Hence net heat storage in the element in time dt will be,

$$mC_p\Delta T = (\rho V)C_p\,dt$$
$$= \rho(rd\theta dz\,dr)C_p\,dt$$

* From energy balance eq^n

$$\begin{pmatrix}\text{Net heat conducted in radial} \\ \text{dir}^n\end{pmatrix} + \begin{pmatrix}\text{Heat generated} \\ \text{within the element}\end{pmatrix} = \begin{pmatrix}\text{Energy stored in} \\ \text{the element}\end{pmatrix}$$

$$\rightarrow K_r(rd\theta dz\,dr)\left[\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr}\right]dt + q(rd\theta dz\,dr)dt = \rho(rd\theta dz\,dr)C_p\,dT$$

$$\rightarrow \left[\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr}\right] + \frac{q}{K_r} = \frac{\rho C_p}{K_r}\frac{dT}{dt} \qquad \left\{\text{Thermal diffusivity } \alpha = \frac{K}{\rho C_p}\right\}$$

$$\rightarrow \boxed{\left[\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr}\right] + \frac{q}{K_r} = \frac{1}{\alpha}\frac{dT}{dt}}$$

* Steady state One dimensional heat conduction eq^n (Poisson's Eq^n)

$$\left[\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr}\right] + \frac{q}{K_r} = 0$$

* In case of no heat generation. i.e. $q = 0$ above eq^n reduces to

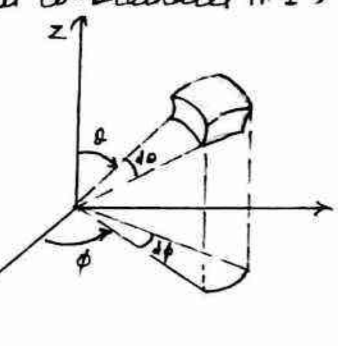
$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} = 0$$

→ General heat conduction eq^n in spherical co-ordinates in 1-D

$$\frac{1}{r^2}\frac{d}{dr}\left[r^2\frac{dT}{dr}\right] + \frac{q}{K_r} = \frac{1}{\alpha}\frac{dT}{dt}$$

or

$$\frac{1}{r}\frac{d}{dr}\left[r.\frac{dT}{dr}\right] + \frac{q}{K_r} = \frac{1}{\alpha}\frac{dT}{dt}$$



* In case of no heat generation & steady state

$$\frac{d}{dr}\left[r.\frac{dT}{dr}\right] = 0$$

5. Radiation :—

* Radiation is the energy emitted by matter in the form of electromagnetic waves (or photons) as a result of the changes in electronic configurations of the atoms or molecules in a wavelength band between 0.1 to 100μ.
* Unlike conduction & convection, the transfer of heat by radiation does not require the presence of an interven-ing medium. In fact, heat transfer by radiation is fastest (at the speed of light). This is how the energy of sun, reaches the earth.
* Thermal radiation is the form of radiation emitted by bodies because of their temperature. All bodies at a a temperature above absolute zero emit thermal radiation.
* Radiation is a volumetric phenomenon, and all solids, liquids, and gases emit, absorb, or transmit radiation to varying degrees.
* The maximum rate of radiation that can be emitted from a surface at a thermodynamic temperature 'T_s' is given by the Stefan-Boltzmann law as.

$$\boxed{\dot{Q}_{emit,max} = \sigma A_s T_s^4 \quad (\text{Watt})}$$

where, $\sigma = 5.670 \times 10^{-8}\ W/m^2K^4$ is boltzmann constant. The idealized surface that emits radiation at this max^m rate is called a blackbody.

* The radiation emitted by all real surfaces is less than the radiation emitted by a blackbody at the same temperature, and is expressed as

$$\boxed{\dot{Q}_{emit} = \varepsilon \sigma A_s T_s^4}$$

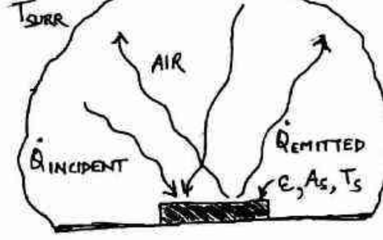
where 'ε' is the emissivity of the surface. The property emissivity, whose value is in the range $0 \leq \varepsilon \leq 1$, is a measure of how closely a surface approximates a blackbody for which $\varepsilon = 1$.

* Another important radiation property of a surface is its absorptivity α, which is the fraction of the radiation energy incident on a surface that is absorbed by the surface. Its value is in the range $0 \leq \alpha \leq 1$.
A blackbody absorbs the entire radiation incident on it. That is, a blackbody is a perfect absorber ($\alpha = 1$) as it is a perfect emitter.
* In general both 'ε' & 'α' of a surface depend on the temperature and the wavelength of the radiation. Kirchoff's law of radiation states that the emissivity and the absorptivity of a surface at a given temp. and wavelength are equal.

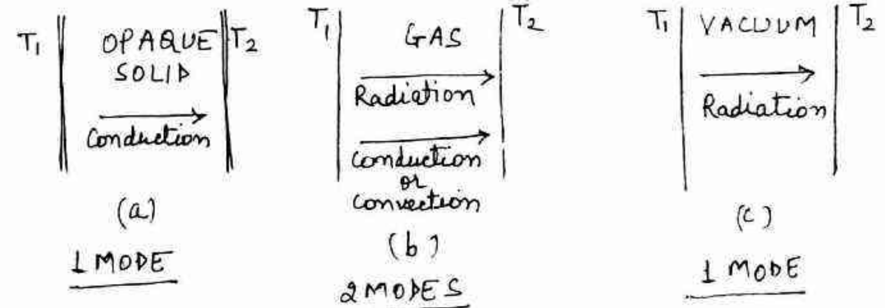
$$\boxed{\dot{Q}_{absorbed} = \alpha \dot{Q}_{incident}} \quad (\text{Watt})$$

* When a surface of emissivity 'ε' and surface area 'A_s' at a thermodynamic temperature 'T_s' is completely enclosed by a much larger (or black) surface at thermodynamic temperature 'T_{surr}' separated by a gas (such as air) that does not intervene with radiation, the net rate of radiation heat transfer between these two surfaces is given by

$$\boxed{\dot{Q}_{rad} = \varepsilon \sigma A_s (T_s^4 - T_{surr}^4)}$$



Note :— Simultaneous heat transfer mechanisms :—



* When the rate of heat transfer ($\dot{Q}$) is available, then the total amount of heat transfer (Q) during time interval (Δt) can be determined from.

$$Q = \int_0^{\Delta t} \dot{Q}\, dt \quad (J)$$

provided that the variation of $\dot{Q}$ with time is known.

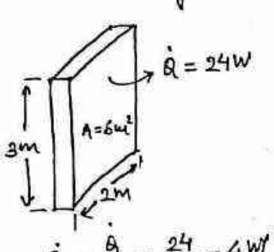
* For special case of $\dot{Q}$ = constant, above eqⁿ reduces to

$$\boxed{Q = \dot{Q}\,\Delta t \quad (J)}$$

* The rate of heat transfer per unit area normal to the dirⁿ of heat transfer is called heat flux, and the avg. heat flux is expressed as.

$$\boxed{\dot{q} = \frac{\dot{Q}}{A_s}} \quad (W/m^2)$$

where 'A_s' is the heat transfer area, normal to dirⁿ of heat flow.

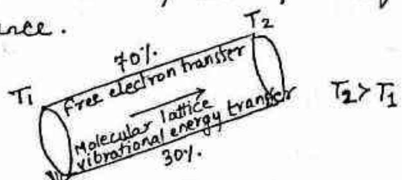


$$\dot{q} = \frac{\dot{Q}}{A} = \frac{24}{6} = 4\, W/m^2$$

→ Heat transfer mechanisms / Modes of heat transfer :—

* The transfer of energy as heat is always from the higher temp. medium to the lower-temp one, and heat transfer stops when two mediums reaches the same temperature.

* Heat can be transferred in three different modes: Conduction, convection, and radiation. All modes of transfer require the existance of a temp. difference.



1. Conduction :—

* Conduction is a mechanism of heat propagation from a region of higher temperature to a region of low temperature within a medium (solid, Liquid or gaseous) or between different mediums in direct physical contact. It doesn't involve any movement of macroscopic portions of matter relative to one another.

or

* Conduction is the transfer of energy from more energetic particles of a substance to the adjacent less energetic ones as a result of interactions between the particles. It can take place in solids, liquids, or gases.

* In gases and liquids, conduction is due to the collisions and diffusion of the molecules during their random motion. In solids, it is due to the combination of vibrations of the molecules in a lattice and the energy transported by free electrons.

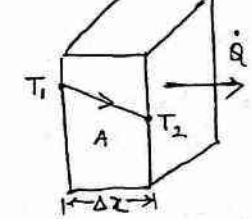
* The rate of heat conduction through a medium depends on the geometry of the medium, it's thickness, and the meterial of the medium, as well as the temperature difference across the medium.

→ Fourier's Law of heat conduction :—

* The rate of heat conduction through a plane layer is proportional to the temp. difference across the layer and the heat transfer area, but is inversely proportional to the thickness of the layer, that is

$$\text{Rate of heat conduction} \propto \frac{\left(\begin{array}{c}\text{Area normal to}\\ \text{heat transfer}\end{array}\right) \times (\text{temp difference})}{(\text{Thickness})}$$

or



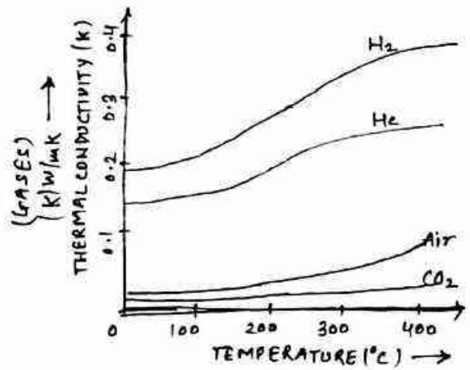
$$\dot{Q}_{conducted} \propto \frac{A_s \times (T_1 - T_2)}{\Delta x}$$

or

$$\boxed{\dot{Q}_{conducted} = k A_s \frac{(T_1 - T_2)}{\Delta x} = -k A_s \frac{\Delta T}{\Delta x}}$$

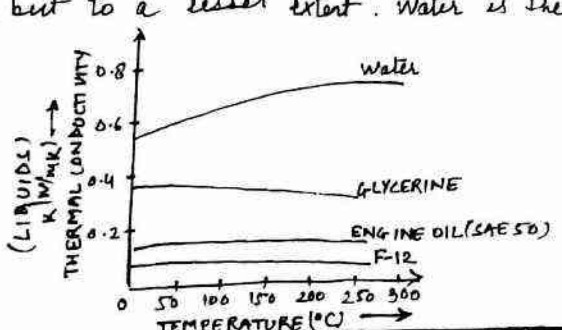
Effect of temperature on thermal Conductivity of gases :—

* The transport of heat energy in gases is due to random motion of molecules exchanging energy by momentum transfer.
* Since kinetic energy of molecules is the function of temperature hence when molecules at higher Temperature region collide with molecules at lower temperature, they loose their K.E. by collision.
* Therefor in case of gases, the thermal conductivity of ideal gases increases with increase in temperature.



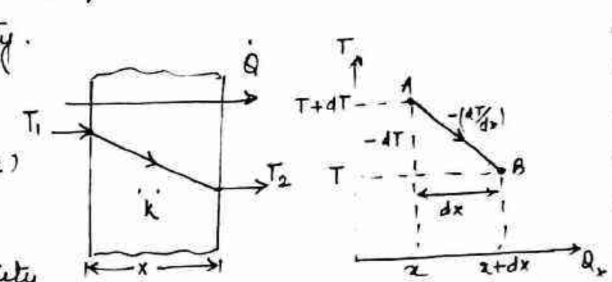
Effect of Temperature on thermal conductivity of liquids

* The mechanism of heat conduction in liquids is quite similar to gases except the fact that the molecules of liquids are more closely spaced compared to gases.
* Since density of liquids decreases on increase in temperature hence it is observed that thermal conductivity of liquids decreases with increase in temperature.
* Though thermal conductivity of liquids also depends on pressure but to a lesser extent. Water is the exception.



→ Heat Conduction Through a Wall / Slab :—

* Consider a wall of surface area 'A' of thickness 'x'. let $\dot{Q}$ be the rate of heat transfer in x-direction and 'K' be the thermal conductivity.



* Assumptions :—
1. 1-D heat transfer i.e $T = f(x)$
2. Steady state heat transfer i.e $T \neq f(\text{time})$
3. Uniform thermal conductivity i.e. $K = \text{constant}$
4. No heat generation within the slab

* Rate of heat transfer along x-direction is given by Fourier's Law as :

$\dot{Q}_x = -KA\frac{dT}{dx}$ (watts) ; Integrating between boundary conditions

At $x = 0$; $T = T_1$ and at $x = x$; $T = T_2$.

$$\dot{Q}_x\int_0^x dx = -KA\int_{T_1}^{T_2} dT$$

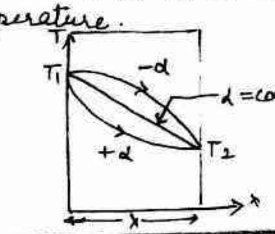
or $\dot{Q}_x(x) = -KA\,(T_2 - T_1)$

or $$\boxed{\dot{Q}_x = \frac{(T_1 - T_2)}{\left(\frac{x}{KA}\right)} = \frac{(T_1 - T_2)}{R_{th.}}}$$

* Heat Conduction In a Thick Wall With Variable Thermal Conductivity :—

The dependence of thermal conductivity on temperature can be expressed as : $K = K_0(1 + \alpha \cdot T)$ where K_0 is the thermal conductivity at reference or zero temperature, α is constant for a given material and 'T' is the Temperature.

* When $\alpha = 0 \rightarrow K = K_0 \rightarrow$ constant
* When α is negative $K\downarrow$ with $x\uparrow$
* When α is positive $K\uparrow$ with $x\downarrow$



In case of gases, K value increases with temperature.

* Diamond / Quartz are exception having very high thermal conductivity and low electric conductivity because of highly ordered lattice structure

→ Effect of temperature :

* Thermal conductivity of pure metals decreases with increase in temperature because the lattice vibrations impede the mation of free electrons. (Mercury (Hg) is an exception)

* Thermal conductivity of alloys and insulating materials, increases with increase in temperature since they have very few free electrons and the heat transfer in them mainly depends on lattice vibrations.

* Thermal conductivity of gases increases with increase in temperature since the number of collisions increase with increase in temperature (higher K.E.)

* Thermal conductivity for liquids depends pressure and temperature. It tends to decrease with increase in temperature due to decrease in density.

→ Integration Form of Fourier's Law :—

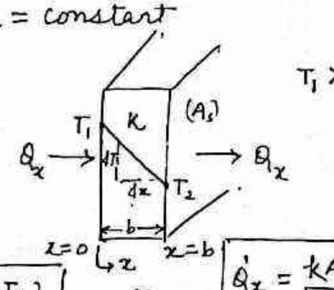
Imp

* Assumptions made in Fourier's Law are :

1. One dimensional heat transfer i.e. $T = f(x)$ only.
2. Steady state heat transfer i.e. $T \neq f(\text{time})$
3. Uniform thermal conductivity i.e. $K = \text{constant}$
4. No heat generation in the slab

Rate of heat transfer along x-dir^n

$$\boxed{\dot{Q}_x = -KA_s\left(\frac{dT}{dx}\right) \text{ watts}}$$



At $x=0$; $T = T_1$ & at $x = b$; $T = T_2$

$$\int_{x=0}^{b} \dot{Q}_x \, dx = -\int_{T_1}^{T_2} KA_s \, dT \rightarrow \boxed{\dot{Q}_x b = KA_s(T_1 - T_2)} \text{ or } \boxed{\dot{Q}_x = \frac{KA_s(T_1 - T_2)}{b}}$$

→ Electrical Analogy :—

* When two physical systems are described by similar equations and have similar boundary conditions, these are said to be analogous.

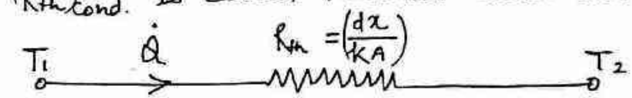
* Heat transfer process may be compared by analogy with flow of electric current.

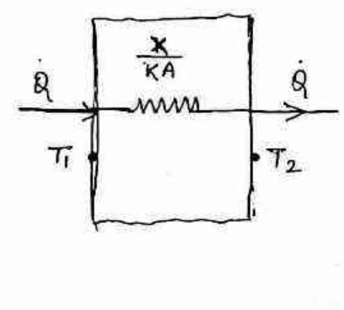
Thermal System	Electrical system
* Heat flow rate is directly proportional to temperature difference ($\dot{Q} \propto dT$)	* Current flow is directly proportional to potential difference ($I \propto dV$)
* As per Fourier's Law by analogy. $\dot{Q} = \dfrac{dT(\text{temp. difference})}{(dx/KA)}$	* As per Ohm's Law $I = \dfrac{dV(\text{potential difference})}{R(\text{resistance})}$

* By comparing both above equations, I is analogous to $\dot{Q}$; dV is analogous to dT and R is analogous to quantity $\left(\frac{dx}{KA}\right)$. The quantity $\left(\frac{dx}{KA}\right)$ is called thermal conduction resistance (R_{th})cond. i.e.

$$\boxed{(R_{th})_{cond} = \left(\frac{dx}{KA}\right)}$$

* The reciprocal of $(R_{th})_{cond.}$ is called thermal conductance





Unit - 1

→ Introduction to heat transfer :—

* We can determine the amount of heat transfer for any system undergoing any process using a thermodynamic analysis alone. The reason is that thermodynamics is concerned with the amount of heat transfer as a system undergoes a process from one equilibrium state to another, and it gives no indication about how long the process will take. It only tells us how much heat is transferred.

* Thermodynamics deals with equilibrium states and changes from one equilibrium state to another. Heat transfer, on the other hand, deals with systems that lack thermal equilibrium, and thus it is a nonequilibrium phenomenon.

→ Note : The basic requirement for heat transfer is the presence of temperature difference. The temperature difference is the driving force for heat transfer. The rate of heat transfer in a certain direction depends on the magnitude of the temperature gradient (the temperature difference per unit length or the rate of change of temperature) in that direction.
The larger the temp. gradient, the higher the ~~heat~~ rate of heat transfer.

→ Energy transfer :—

* Energy can be transferred to or from a given mass by two mechanisms : heat transfer 'Q' and work 'W'.
* An energy interaction is heat transfer if it's driving force is a temperature difference. Otherwise, it is work.
* The amount of heat transferred during the process is denoted by 'Q'. The amount of heat transferred per unit time is called rate of heat transfer and is denoted by '$\dot{Q}$'. The heat transfer rate ($\dot{Q}$) has the unit (J/s), which is equivalent to Watts (W).

→ Overall Heat Transfer Coefficient :—

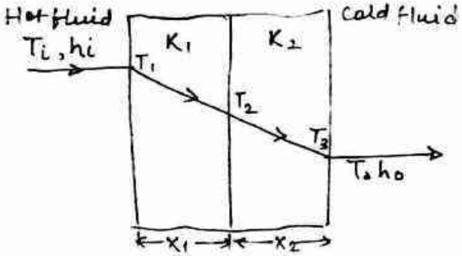
* Heat flow rate through composite wall considering all modes of heat transfer (i.e. by conduction, convection & radiation) can be expressed as.

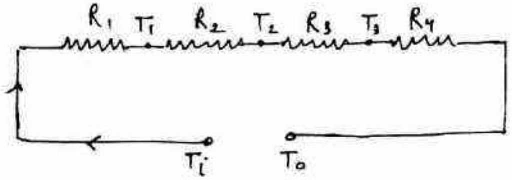
$$\dot{Q} = U \cdot A(T_i - T_o) = \frac{T_i - T_o}{(1/UA)} = \frac{T_i - T_o}{\Sigma R}$$

where T_i & T_o are temperatures at inner & outer sides of composite wall respectively and $(T_i > T_o)$. U is overall heat transfer coefficient and $\Sigma R = (1/UA)$ represents overall / combined resistance of composite wall.

↳ Heat transfer through composite wall having resistances in series :—

* Consider a composite wall of thickness x_1 and x_2 of surface area 'A' in the perpendicular direction of heat flow having hot fluid at temperature T_i on one side and T_o on other side $(T_i > T_o)$.





Let h_i & h_o be coefficient of heat transfer of films for hot & cold fluid respectively. where

$$R_1 = \frac{1}{h_i A} \;;\; R_2 = \frac{x_1}{K_1 A} \;;\; R_3 = \frac{x_2}{K_2 A} \;;\; R_4 = \frac{1}{h_o A}$$

Total resistance in series; $\boxed{\Sigma R = R_1 + R_2 + R_3 + R_4}$

$$\therefore \quad Q = \frac{T_i - T_o}{\Sigma R} = \frac{T_i - T_o}{R_1 + R_2 + R_3 + R_4}$$

$$Q = \frac{T_i - T_o}{\left(\frac{1}{h_i A} + \frac{x_1}{K_1 A} + \frac{x_2}{K_2 A} + \frac{1}{h_o A}\right)} = \frac{T_i - T_o}{\left(\frac{1}{UA}\right)}$$

Overall heat transfer coefficient $\frac{1}{U} = \left(\frac{1}{h_i} + \frac{x_1}{K_1} + \frac{x_2}{K_2} + \frac{1}{h_o}\right)$

* Estimation of intermediate temperature in the system

Since heat transfer rate remains same across the section under steady state, it follows that

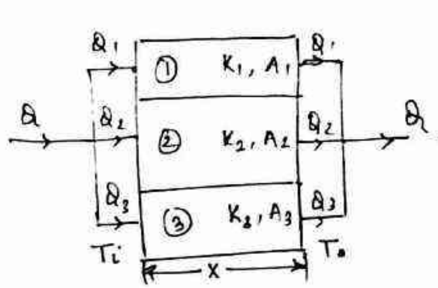
$$Q = h_i A(T_i - T_1) = \frac{K_1 A(T_1 - T_2)}{x_1} = \frac{K_2 A(T_2 - T_3)}{x_2} = h_o A(T_3 - T_o)$$

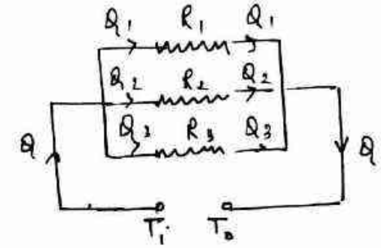
* Using above equation, the intermediate temperatures T_1, T_2 & T_3 can be calculated.

↳ Heat transfer through a composite wall having resistances in parallel :—

→ Neglecting convective heat transfer

* Consider a heat transfer system as shown in figure and it's equivalent electric system.





* Various resistances in system are

$$R_1 = \frac{x}{K_1 A_1} \;;\; R_2 = \frac{x}{K_2 A_2} \;;\; R_3 = \frac{x}{K_3 A_3}$$

Let Q_1, Q_2 & Q_3 are heat transfer rates in slab ①, ② & ③ respectively, then

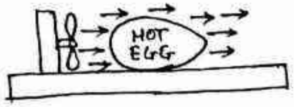
$$Q_1 = \frac{T_i - T_o}{R_1} \;;\; Q_2 = \frac{T_i - T_o}{R_2} \;;\; Q_3 = \frac{T_i - T_o}{R_3}$$

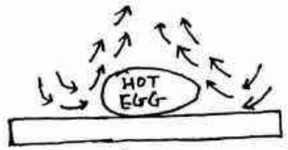
But total heat transfer rate,

$$Q = Q_1 + Q_2 + Q_3 = \frac{T_i - T_o}{\Sigma R}$$

$$\therefore \quad Q = \frac{T_i - T_o}{R_1} + \frac{T_i - T_o}{R_2} + \frac{T_i - T_o}{R_3} = \frac{T_i - T_o}{\Sigma R}$$

$$\therefore \quad \boxed{\frac{1}{\Sigma R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$





* The rate of convection heat transfer is observed to be proportional to the temperature difference, and is expressed by Newton's law of cooling. It is also directly proportional to the area of exposure between the plate & fluid.
* The rate equation for convection heat transfer between a surface and an adjacent fluid is given as.

$$\boxed{\dot{Q}_{convection} = hA_s(T_s - T_\infty)} \quad \text{(watts)}$$

where, $\dot{Q}_{conv.}$ → Rate of heat convection
A_s → Convective heat transfer surface area
h → Convective heat transfer coefficient ($W/m^2{}^\circ C$)
T_s → Surface temp.
T_∞/T_f → Temperature of fluid sufficiently far from surface or free stream fluid temp.

Note: The coefficient of convective heat transfer 'h' (also known as film heat transfer coefficient) may be defined as "the amount of heat transmitted for a unit temperature difference between the fluid and unit area of surface in unit time."

$$\boxed{h = \frac{\dot{Q}_{conv.}}{A_s(T_s - T_\infty)} = \frac{W}{m^2\,{}^\circ C} \text{ or } \frac{W}{m^2\,K}}$$

It is not a property of fluid but a experimentally determined parameter whose value depends on following factors.

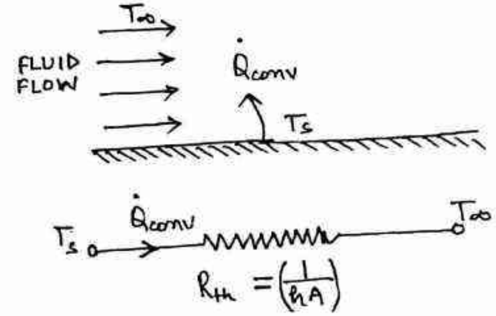
a) Viscosity, density, specific heat etc.
b) Nature of fluid flow
c) Geometry of the surface
d) Prevailing thermal conditions.

→ Electrical analogy :—

$I = dV/R$

$\dot{Q}_{conv} = hA_s(T_s - T_\infty)$

$\dot{Q}_{conv} = \frac{(T_s - T_\infty)}{(1/hA)}$



$$Q\left[-\frac{1}{r}\right]_{r_1}^{r_2} = -K4\pi\left[T\right]_{T_1}^{T_2}$$

$$Q\left[\frac{1}{r_1}-\frac{1}{r_2}\right] = 4\pi K(T_1-T_2)$$

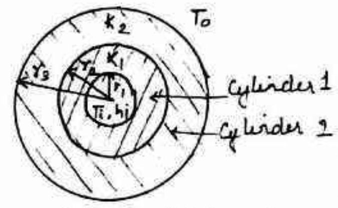
$$Q = \frac{(T_1-T_2)}{\left[\frac{(1/r_1 - 1/r_2)}{4\pi K}\right]} = \frac{(T_1-T_2)}{R} \quad ; \quad R = \frac{(1/r_1 - 1/r_2)}{4\pi K}$$

R represents the thermal resistance.

Heat transfer through composite cylinder with conduction & Convection :—

$$T_i \xrightarrow{R_1} T_1 \xrightarrow{R_2} T_2 \xrightarrow{R_3} T_3 \xrightarrow{R_4} T_o$$

$$\frac{1}{h_i A_i} \quad \left(\frac{\ln r_2/r_1}{2\pi K_1 L}\right) \quad \left(\frac{\ln r_3/r_2}{2\pi K_2 L}\right) \quad \frac{1}{h_o A_o}$$



* Heat transfer can be determined for a composite cylinder by considering the thermal resistance concept.

Resistances due to convection R_1 & R_4 and due to conduction R_2 and R_3 can be given as

$$R_1 = \frac{1}{h_i(2\pi r_1 L)} \; ; \; R_2 = \frac{\ln^{r_2/r_1}}{2\pi K_1 L} \; ; \; R_3 = \frac{\ln^{r_3/r_2}}{2\pi K_2 L} \; ; \; R_4 = \frac{1}{h_o(2\pi r_3 L)}$$

Heat transfer rate through various layers is given as

$$Q = h_i(2\pi r_1 L)(T_i - T_1) = \frac{(T_1-T_2)}{\left[\frac{\ln(r_2/r_1)}{2\pi K_1 L}\right]} = \frac{(T_2-T_3)}{\left[\frac{\ln(r_3/r_2)}{2\pi K_2 L}\right]} = h_o(2\pi r_3 L)(T_3 - T_o)$$

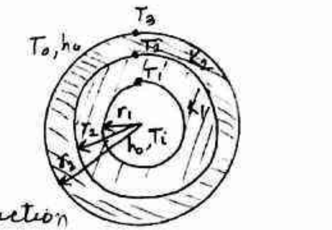
or

$$Q = \frac{(T_i - T_o)}{R_1+R_2+R_3+R_4} = \frac{(T_i - T_o)}{\frac{1}{(h_i 2\pi r_1 L)} + \frac{\ln(r_2/r_1)}{2\pi K_1 L} + \frac{\ln(r_3/r_2)}{2\pi K_2 L} + \frac{1}{h_o 2\pi r_3 L}}$$

Combined Heat transfer by Conduction and Convection in compound sphere :—

* Considering a hollow sphere having inside and outside temperatures T_i and T_o respectively. Let T_1, T_2 and T_3 be interface temperature of two spheres.

$$T_i \xrightarrow{R_1} T_1 \xrightarrow{R_2} T_2 \xrightarrow{R_3} T_3 \xrightarrow{R_4} T_o$$



* Resistances due to convection R_1 & R_4 and conduction R_2 and R_3 can be written as

$$R_1 = \frac{1}{h_i A_i} \; ; \; R_2 = \frac{(1/r_1 - 1/r_2)}{4\pi K_1} \; ; \; R_3 = \frac{(1/r_2 - 1/r_3)}{4\pi K_2} \; ; \; R_4 = \frac{1}{h_o A_o}$$

$$A_i = 4\pi r_1^2 \; ; \; A_o = 4\pi r_3^2$$

* Heat transfer rate through various layers is given as

$$Q = \frac{(T_i - T_1)}{\left[\frac{1}{h_i(4\pi r_1^2)}\right]} = \frac{(T_1-T_2)}{\left[\frac{\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}{4\pi K_1}\right]} = \frac{(T_2-T_3)}{\left[\frac{\left(\frac{1}{r_2}-\frac{1}{r_3}\right)}{4\pi K_2}\right]} = \frac{(T_3-T_o)}{\left[\frac{1}{h_o(4\pi r_3^2)}\right]}$$

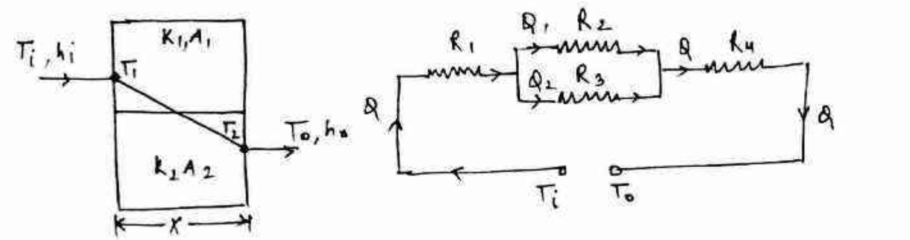
$$Q = \frac{(T_i - T_o)}{R_1+R_2+R_3+R_4} = \frac{(T_i - T_o)}{\left[\frac{1}{(h_i 4\pi r_1^2)} + \frac{(1/r_1 - 1/r_2)}{4\pi K_1} + \frac{(1/r_2 - 1/r_3)}{4\pi K_2} + \frac{1}{(h_o 4\pi r_3^2)}\right]}$$

Thermal Contact Resistance :—

* Consider one dimensional heat flow through a composite slab having two solid surfaces. Since the direct contact between solid surfaces takes place at a limited no. of spots and the void is filled by the surrounding fluid. The heat flow through the filling the voids is mainly by conduction, since there is no convection in such a thin layer of fluid and the radiation effects are negligible. In case the thermal conductivity of fluid filled in voids less than the thermal conductivity of the solids, the interface acts as a resistance to heat flow, called as thermal resistance.

→ With convective heat Transfer.

* Consider the heat transfer system as shown in figure and it's equivalent electrical system.



* Various resistances are, where $A = A_1 + A_2$

$$R_1 = \frac{1}{h_i A} \;;\; R_2 = \frac{X}{K_1 A_1} \;;\; R_3 = \frac{X}{K_2 A_2} \;;\; R_4 = \frac{1}{h_o A}$$

equations for heat flow rate are

$$Q = \frac{T_i - T_1}{R_1} \;;\; Q_1 = \frac{T_1 - T_2}{R_2} \;;\; Q_3 = \frac{T_1 - T_2}{R_3} \;;\; Q = \frac{T_2 - T_o}{R_4}$$

* Combined resistance of the system can be calculated as follows

For resistance in parallel

$$\frac{1}{R_e} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{R_2 + R_3}{R_2 R_3} \;;\quad R_e = \frac{R_2 R_3}{R_2 + R_3}$$

$$\therefore \Sigma R = R_1 + R_e + R_4 = R_1 + \frac{R_2 R_3}{R_2 + R_3} + R_4$$

and $$Q = \frac{T_i - T_o}{\Sigma R}$$

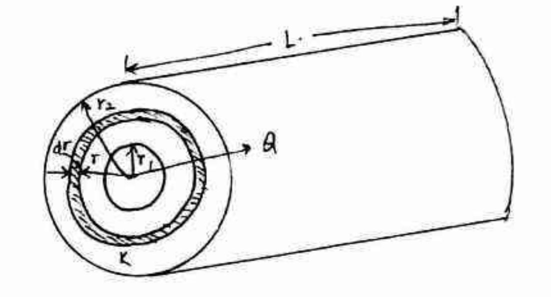
↳ <u>Heat transfer in an infinitely long cylinder</u> :—

* Consider a hollow cylinder of internal radius r_1 & external radius r_2 with respective internal & external temperatures of T_i and T_o as shown in figure.

* Let 'L' be the length and 'K' be the thermal conductivity of cylinder

* Considering heat transfer in radial direction only.

$$Q = -KA \frac{dT}{dx}$$



Since $A = 2\pi r L$

$$Q = -K(2\pi r L)\frac{dT}{dr}$$

Integrating between the limits

$$Q\int_{r_1}^{r_2} \frac{dr}{r} = -2\pi K L \int_{T_1}^{T_2} dT$$

$$Q\left[\ln r\right]_{r_1}^{r_2} = -2\pi K L \left[T\right]_{T_1}^{T_2}$$

$$Q \ln(r_2/r_1) = -2\pi K L (T_2 - T_1) = 2\pi K L (T_1 - T_2)$$

$$Q = \frac{(T_1 - T_2)}{\left[\frac{\ln(r_2/r_1)}{2\pi K L}\right]} = \frac{T_1 - T_2}{R} \;;\quad R = \left[\frac{\ln(\frac{r_2}{r_1})}{2\pi K L}\right]$$

'R' represents the thermal resistance

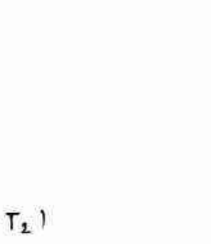
↳ <u>Heat transfer through a hollow sphere</u> :—

* Consider a hollow sphere of internal & external radius as r_1 and r_2 respectively with respective Temperatures T_i & T_o.

* Considering heat transfer in radial direction.

* Let us consider a ring at radius 'r'.

since surface area, $A = 4\pi r^2$



$$Q = -KA\frac{dT}{dx}$$

$$Q = -K(4\pi r^2)\frac{dT}{dr}$$

$$Q\int_{o}^{r} \frac{dr}{r^2} = -K 4\pi \int_{T_1}^{T_2} dT$$

from eq^n ① $\quad -K\left[\frac{dT}{dx}\right]_{x=L} = h\left[T_w - T_\infty\right]$

On substituting the value of $\left(\frac{dT}{dx}\right)$ from eq^n ③ at $x=L$ and the value of T_w from eq^n ⑤

$$-K\left[-\frac{q}{K}\cdot L\right] = h\left[-\frac{q}{2K}L^2 + C_2 - T_\infty\right]$$

$$C_2 = \frac{q\cdot L}{h} + \frac{q}{2K}L^2 + T_\infty \quad \text{—— ⑥}$$

Substituting the value of C_2 in eq^n ④

$$T = -\frac{q}{K}\frac{x^2}{2} + \frac{q\cdot L}{h} + \frac{q}{2K}L^2 + T_\infty \quad \text{—— ⑦}$$

$$\boxed{T = \frac{q}{2K}\left(L^2 - x^2\right) + \frac{q\cdot L}{h} + T_\infty}$$

This eq^n represents the temp. profile at any section at distance x from centre.

* Since maximum temperature occurs at $x=0$,

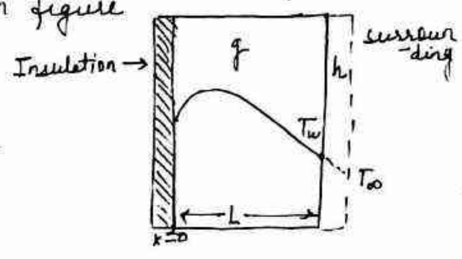
$$T_{max} = \frac{q\cdot L}{h} + \frac{qL^2}{2K} + T_\infty$$

* For surface temperature, T_w at $x=L$; from eq^n ⑦

$$\boxed{T_w = \frac{qL}{h} + T_\infty}$$

(C) Heat Conduction In Plane Wall With One Surface Insulated And Other Exposed To Surrounding Fluid :—

* Considering a slab of thickness 'L' with internal heat generation 'q' (W/m³) whose left surface is thickly insulated while the other surface is exposed to surrounding fluid at temperature T_∞ as shown in figure



* Boundary conditions are :

(a) at $x=0$; $Q=0$ it follows that

$$-KA\left(\frac{dT}{dx}\right)_{x=0} = 0$$

Since neither K no A can be zero, it implies that

$$\left(\frac{dT}{dx}\right)_{x=0} = 0 \quad \text{—— ①}$$

(b) at $x=L$ $\left(\begin{array}{c}\text{Heat conducted}\\ \text{to right face}\end{array}\right) = \left(\begin{array}{c}\text{Heat convected to}\\ \text{surroundings}\end{array}\right)$

$$-KA\left(\frac{dT}{dx}\right)_{x=L} = h\cdot A\cdot\left(T_{x=L} - T_\infty\right)$$

$$-K\left(\frac{dT}{dx}\right)_{x=L} = h\left(T_{x=L} - T_\infty\right) \quad \text{—— ②}$$

* Poisson's eq^n ; $\quad \frac{d^2T}{dx^2} + \frac{q}{K} = 0$

on integration $\quad \frac{dT}{dx} = -\frac{q}{K}x + C_1 \quad \text{—— ③}$

But $\frac{dT}{dx} = 0$ at $x=0$ therefore

$$0 = -\frac{q}{K}(0) + C_1 \rightarrow C_1 = 0 \quad \text{—— ④}$$

$$\therefore \frac{dT}{dx} = -\frac{q}{K}x \quad \text{and} \quad \left(\frac{dT}{dx}\right)_{x=L} = -\frac{q}{K}L \quad \text{—— ⑤}$$

On integrating again

$$T = -\frac{q}{K}\frac{x^2}{2} + C_2 \quad \text{—— ⑥}$$

At $x=L$

$$T_{x=L} = -\frac{q}{K}\frac{L^2}{2} + C_2 \quad \text{—— ⑦}$$

On substituting the value from eq^n ③ and ⑤ in eq^n ②

$$-K\left(-\frac{q}{K}L\right) = h\left(-\frac{q}{K}\frac{L^2}{2} + C_2 - T_\infty\right)$$

$$\therefore C_2 = \frac{qL}{h} + \frac{qL^2}{2K} + T_\infty \quad \text{—— ⑧}$$

Substituting the value of C_2 in eq^n ⑥

$$T = -\frac{q}{K}\frac{x^2}{2} + \frac{qL}{h} + \frac{qL^2}{2K} + T_\infty$$

$$\boxed{T = \frac{q}{2K}\left(L^2 - x^2\right) + \frac{qL}{h} + T_\infty}$$

This eq^n gives the temperature profile at any section 'x' of plane wall.

Now substituting the value of C_1 & C_2 in eq^n (3) we have

$$T = \left[\frac{T_2 - T_1}{\ln \frac{r_2}{r_1}}\right] \ln r + \left[T_1 - \frac{(T_2 - T_1)}{\ln(r_2/r_1)} \cdot \ln r_1\right] \quad (3)$$

$$\text{or } T = \left(\frac{T_2 - T_1}{\ln r_2/r_1}\right) \ln r + T_1 - \left(\frac{T_2 - T_1}{\ln r_2/r_1}\right) \ln r_1$$

$$\text{or } T - T_1 = \left[\frac{T_2 - T_1}{\ln\left(\frac{r_2}{r_1}\right)}\right] (\ln r - \ln r_1) = \left(\frac{T_2 - T_1}{\ln r_2/r_1}\right) \ln\left(\frac{r}{r_1}\right)$$

$$\text{or } \left(\frac{T - T_1}{T_2 - T_1}\right) = \frac{\ln r/r_1}{\ln r_2/r_1}$$

$$\text{or } \boxed{\frac{T - T_1}{T_1 - T_2} = \frac{\ln(r/r_1)}{\ln(r_1/r_2)}}$$

This eq^n gives temp. distribution in hollow cylinder in radial dir^n without heat generation.

* From Fourier's law of heat conduction

$Q = -KA \frac{dT}{dx}$ but from eq^n (2) $\frac{dT}{dx} = \frac{C_1}{r}$ and $C_1 = \left(\frac{T_2 - T_1}{\ln r_2/r_1}\right)$

Since $A = 2\pi r L$, above eq^n reduces to

$$Q = -K(2\pi \cancel{r} L) \frac{(T_2 - T_1)}{\cancel{r} \ln(r_2/r_1)} = \frac{-(T_2 - T_1)}{\left[\frac{\ln(r_2/r_1)}{2\pi L K}\right]} \text{ or}$$

$$Q = \frac{(T_1 - T_2)}{\left[\frac{\ln(r_2/r_1)}{2\pi K L}\right]} = \frac{\Delta T}{R_{th}} \Rightarrow \boxed{R_{th} = \left[\frac{\ln(r_2/r_1)}{2\pi K L}\right]}$$

Imp → Logarithmic Mean Area (LMA) For Hollow Cylinder :—

* Since area changes with radius, therefore it is convenient to calculate a mean area (A_m) for use in analogous formula of slab, $\left(Q = -KA \frac{dT}{dx}\right)$.

(4) Rewriting the eq^n.

$$Q = \frac{2\pi K L \cdot \Delta T}{\ln(r_2/r_1)} = K A_m \frac{\Delta T}{(r_2 - r_1)} \quad \text{—— (1)}$$

Multiplying and dividing the eq^n by $(r_2 - r_1)$ we get

$$Q = 2\pi K L \frac{\Delta T}{\ln(r_2/r_1)} \times \frac{(r_2 - r_1)}{(r_2 - r_1)}$$

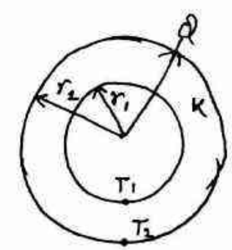
$$Q = \frac{2\pi K L \cdot (r_2 - r_1)}{\ln(r_2/r_1)} \times \frac{\Delta T}{(r_2 - r_1)} \quad \text{—— (2)}$$

Comparing eq^n (1) & (2)

$$\boxed{A_m = \frac{2\pi L (r_2 - r_1)}{\ln(r_2/r_1)} = \frac{A_o - A_i}{\ln(A_o/A_i)}}$$

(C) Hollow Sphere :—

* Conduction eq^n for one dimensional (radial) heat flow without heat generation is given as.



$$\frac{1}{r^2} \frac{d}{dr}\left[r^2 \frac{dT}{dr}\right] = 0$$

$$\text{or } \frac{d}{dr}\left[r^2 \frac{dT}{dr}\right] = 0 \quad \text{—— (1)}$$

On integrating

$$r^2 \frac{dT}{dr} = C_1 \text{ or } \frac{dT}{dr} = \frac{C_1}{r^2} \quad \text{—— (2)}$$

On integrating again

$$T = -\frac{C_1}{r} + C_2 \quad \text{—— (3)}$$

Boundary conditions are

(a) at $r = r_1$; $T = T_1$ and

(b) at $r = r_2$; $T = T_2$

Substituting the boundary conditions in eq^n (3) we get

$$T_2 = -\frac{q}{4K}\cdot r_o^2 + \frac{(T_2-T_1)+\frac{q}{4K}(r_o^2-r_i^2)}{\ln\left(\frac{r_o}{r_i}\right)}\cdot \ln r_o + C_2$$

$$C_2 = T_2 + \frac{q}{4K}\cdot r_o^2 - \left[(T_2-T_1)+\frac{q}{4K}(r_o^2-r_i^2)\right]\frac{\ln r_o}{\ln\left(\frac{r_o}{r_i}\right)}$$

On substituting the value of C_1 and C_2 in eq^n (3) we have

$$T = -\frac{q}{4K}r^2 + \frac{(T_2-T_1)+\frac{q}{4K}(r_o^2-r_i^2)}{\ln\left(\frac{r_o}{r_i}\right)}\ln(r) + T_2 + \frac{q}{4K}r_o^2 - \left[(T_2-T_1)+\frac{q}{4K}(r_o^2-r_i^2)\right]\frac{\ln r_o}{\ln\left(\frac{r_o}{r_i}\right)}$$

→ Heat Conduction In Solid Spheres With Heat Generation

* The one dimensional heat conduction equation in spheres with heat generation is given by poisson's eq^n as

$$\frac{1}{r^2}\frac{d}{dr}\left[r^2\frac{dT}{dr}\right] + \frac{q}{K} = 0$$

or $$\frac{d}{dr}\left[r^2\frac{dT}{dr}\right] = -\frac{q}{K}\cdot r^2$$

On integrating $$r^2\frac{dT}{dr} = -\frac{q}{K}\frac{r^3}{3} + C_1$$

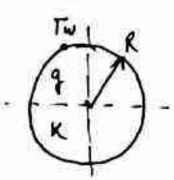
or $$\frac{dT}{dr} = -\frac{q}{K}\frac{r}{3} + \frac{C_1}{r^2} \quad \text{—— (1)}$$

On integrating again,

$$T = -\frac{q}{3K}\frac{r^2}{2} - \frac{C_1}{r} + C_2 \quad \text{—— (2)}$$

— Case 1 : Solid Sphere With Specified Surface Temperature :

Boundary conditions are



(a) At centre $r=0$, $\frac{dT}{dr}=0$ → from eq^n (1) $C_1 = 0$

(b) At centre $r=R$, $T=T_w$

Substituting the above condition in eq^n (2)

$$C_2 = T_w + \frac{qR^2}{6K}$$

Substituting the value C_1 & C_2 in eq^n (2)

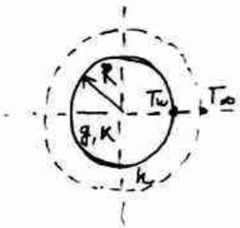
$$T = -\frac{qr^2}{6K} + T_w + \frac{qR^2}{6K}$$

$$\boxed{T = \frac{q}{6K}(R^2-r^2) + T_w}$$

This eq^n gives the temperature distribution through solid sphere.

— Case 2 : Solid Sphere With Heat Convection :

Boundary conditions at the centre are



(a) At $r=0$, $\frac{dT}{dr}=0$ → from eq^n (1) $C_1=0$

Hence eq^n (1) can be written as

$$\frac{dT}{dr} = -\frac{q\cdot r}{3K} \quad \text{—— (3)}$$

(b) Boundary condition at outer surface at $r=R$

(Heat conducted upto surface) = (Heat convected from surface)

$$-KA\left(\frac{dT}{dr}\right)_{r=R} = h\cdot A\cdot\left[T_{r=R} - T_\infty\right]$$

$$-K\left(\frac{-qR}{3K}\right) = h\left[-\frac{qR^2}{6K} + C_2 - T_\infty\right] \quad \left\{\text{from eq}^n \text{ (3) and eq}^n \text{ (2)}\right\}$$

$$C_2 = \frac{qR}{3h} + \frac{qR^2}{6K} + T_\infty$$

On substituting the value of C_2 and C_1 in eq^n (2)

$$T = -\frac{q}{3K}\frac{r^2}{2} + \frac{qR}{3h} + \frac{qR^2}{6K} + T_\infty$$

$$\boxed{T = \frac{q}{6K}(R^2-r^2) + \frac{qR}{3h} + T_\infty}$$

$$\therefore\ Q = -KA\frac{dT}{dr} = -KA\left(-\frac{qr}{3K}\right) = \frac{Aqr}{3}$$

$$\boxed{Q = \frac{Aqr}{3}} \quad \text{and} \quad \boxed{q = \frac{Q}{A} = \frac{qr}{3}}$$

→ Application of steady state, unidirectional heat flow differential equation with heat generation for plane wall

Imp

* Basic eq^ns are

$$\frac{d^2T}{dx^2} + \frac{q}{K} = 0$$

$$\frac{dT}{dx} = -\frac{q}{K} \cdot x + C_1$$

$$T = -\frac{q}{K}\frac{x^2}{2} + C_1 x + C_2$$

* Boundary conditions for various cases are:

① Specified temperature on both sides

(a) At $x = 0$; $T = T_1$

(b) At $x = L$; $T = T_2$

② When heat is transferred on both sides by convection to surroundings at $T = T_\infty$

Note (Length of slab $2L$. Considering centre of slab i.e. $x = 0$)

(a) At $x = 0$; $T = T_{max}$ hence $\frac{dT}{dx} = 0$

(b) At $x = L$; $T = T_w$ and

Heat conducted = Heat convective

$$-KA\left(\frac{dT}{dx}\right)_{x=L} = h \cdot A\,(T_{x=L} - T_\infty)$$

③ One surface is insulated and other exposed to surroundings

(a) At $x = 0$; $Q = 0$ hence $\frac{dT}{dx} = 0$

(b) At $x = L$; $\left[-KA\left(\frac{dT}{dx}\right)_{x=L} = h \cdot A\,(T_{x=L} - T_\infty)\right]$

→ Application of steady state, unidirectional heat flow differential equation with heat generation for cylinder

* Basic eq^ns are

$$\frac{1}{r}\frac{d}{dr}\left[r \cdot \frac{dT}{dr}\right] + \frac{q}{K} = 0$$

$$\frac{d}{dr}\left[r\frac{dT}{dr}\right] = -\frac{q}{K} \cdot r$$

$$\frac{dT}{dr} = -\frac{q}{K} \cdot \frac{r}{2} + \frac{C_1}{r}$$

$$T = -\frac{q}{2K}\frac{r^2}{2} + C_1 \ln(r) + C_2$$

* Boundary conditions for various cases are

① Long solid cylinder with specified surface Temperature

(a) At $r = 0$; $\frac{dT}{dr} = 0$

(b) At $r = R$; $T = T_w$

$$(T - T_w) = \frac{q}{4K}(R^2 - r^2)$$

② Solid cylinder exposed to convection heat transfer from surroundings.

(a) At $r = 0$; $\frac{dT}{dr} = 0$

(b) At $r = R$; $Q_{conducted} = Q_{convective}$

$$T = \frac{q}{4K}(R^2 - r^2) + \frac{qR}{2h} + T_\infty$$

$$(T_{max})_{r=0} = \frac{qR^2}{4K} + \frac{qR}{2h} + T_\infty$$

③ Hollow cylinder with uniform heat generation and specified temperatures

(a) At $r = r_i$; $T = T_1$

(b) At $r = r_o$; $T = T_2$

→ Application of steady state, unidirectional heat flow differential equation with internal heat generation for sphere

* Basic equations are

$$\frac{1}{r^2}\frac{d}{dr}\left[r^2\frac{dT}{dr}\right] + \frac{q}{K} = 0$$

$$\frac{d}{dr}\left[r^2\frac{dT}{dr}\right] = -\frac{q}{K}r^2$$

$$\frac{dT}{dr} = -\frac{qr}{3K} + \frac{C_1}{r^2}$$

$$T = -\frac{qr^2}{6K} - \frac{C_1}{r} + C_2$$

$$T_1 = -\frac{C_1}{r_1} + C_2 \quad \text{—} (4)$$

(5)

and $$T_2 = -\frac{C_1}{r_2} + C_2 \quad \text{—} (5)$$

On solving eq^n (4) & (5)

$$(T_1 - T_2) = C_1\left(\frac{r_1 - r_2}{r_1 r_2}\right)$$

or $$C_1 = \frac{(T_1 - T_2)\, r_1 r_2}{(r_1 - r_2)} \quad \text{—} (6)$$

and $$C_2 = T_1 + \frac{(T_1 - T_2).\, r_2}{(r_1 - r_2)} \quad \text{—} (7)$$

Now substituting the value of C_1 and C_2 in eq^n (3)

$$T = -\frac{1}{r}\,\frac{(T_1 - T_2)\, r_1 r_2}{(r_1 - r_2)} + T_1 + \left(\frac{T_1 - T_2}{r_1 - r_2}\right) r_2$$

$$(T - T_1) = \frac{(T_1 - T_2)}{(r_1 - r_2)}\left[-\frac{r_1 r_2}{r} + r_2\right]$$

$$\left(\frac{T - T_1}{T_1 - T_2}\right) = \frac{1}{(r_1 - r_2)}\left[r_2 - \frac{r_1 r_2}{r}\right] = \frac{1}{(r_1 - r_2)}\left[\frac{r r_2 - r_1 r_2}{r}\right]$$

$$\boxed{\left(\frac{T - T_1}{T_1 - T_2}\right) = \frac{r_2}{r}\left(\frac{r - r_1}{r_1 - r_2}\right)}$$

→ Logarithmic Mean Area (LMA) for Hollow Sphere :—

From Fourier's Law of heat conduction

$$Q = -KA\frac{dT}{dr} = -K(4\pi r^2)\frac{dT}{dr} \quad \text{but from eq}^n \text{ (2) } \frac{dT}{dr} = \frac{C_1}{r^2}$$

$$Q = -KA\frac{C_1}{r_2} = -\frac{K\, 4\pi r^2 C_1}{r^2} = 4\pi K \frac{(T_1 - T_2)}{(r_1 - r_2)}\, r_1 r_2 \quad \left\{C_1 = \left(\frac{T_1 - T_2}{r_1 - r_2}\right) r_1 r_2\right\}$$

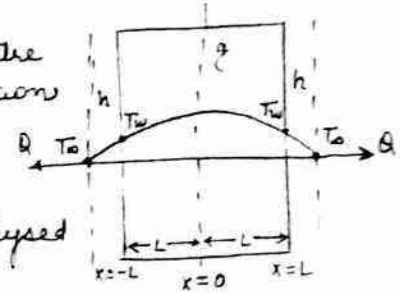
$$Q = \frac{(T_1 - T_2)\, 4\pi r_1 r_2 \times K}{(r_1 - r_2)} = K A_m \frac{(T_1 - T_2)}{(r_1 - r_2)} \rightarrow \boxed{A_m = 4\pi r_1 r_2}$$

and $$Q = \frac{(T_1 - T_2)}{\frac{r_1 - r_2}{4\pi K r_1 r_2}} = \frac{\Delta T}{R_{th}} \Rightarrow \boxed{R_{th} = \left(\frac{r_1 - r_2}{4\pi K r_1 r_2}\right)}$$

(B) When Heat Is Transferred From Both Sides By Convection To Surroundings At T_∞ :— 8

* Considering the case when both surfaces of plate/wall are exposed to surrounding fluid at T_∞ and heat is transferred by convection from the wall surface at T_w to surroundings as shown in fig.

* Max^m temperature occurs at the centre of the slab since the same conditions exist on both sides of the wall.



* Due to symmetry in temperature profile, half the slab can be analysed with boundary conditions as.

(a) At $x = 0$; $\frac{dT}{dx} = 0$

(b) At wall surface i.e. At $x = L$; $T = T_w$

* Since

(Heat conducted to the surface) = (Heat convected from surface To surrounding fluid)

$$-KA\left[\frac{dT}{dx}\right]_{x=L} = h.\, A.\, (T_w - T_\infty)$$

$$-K\left[\frac{dT}{dx}\right]_{x=L} = h(T_w - T_\infty) \quad \text{—} (1)$$

Poisson's eq^n, $\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0$

on integrating $\frac{dT}{dx} = -\frac{\dot{q}}{k}x + C_1$ but at $x = 0$, $\frac{dT}{dx} = 0 \Rightarrow C_1 = 0$

above eq^n reduces to

$$\frac{dT}{dx} = -\frac{\dot{q}}{k}x \quad \text{—} (2)$$

$$\therefore \left[\frac{dT}{dx}\right]_{x=L} = -\frac{\dot{q}}{k}.L \quad \text{—} (3)$$

On integrating eq^n (2)

$$T = -\frac{\dot{q}}{k}\frac{x^2}{2} + C_2 \quad \text{—} (4) \quad \because \text{At } x = L \,;\; T = T_w$$

$$\therefore T_w = -\dot{q}/k\, \frac{L^2}{2} + C_2 \quad \text{—} (5)$$

and $Q = -KA\frac{dT}{dx} = -KA\left(-\frac{q}{K}\cdot x\right) = A\cdot q\cdot x$

$$\boxed{Q = A\cdot q\cdot x}$$

→ Heat Conduction Through Cylinder With Heat Generation :—

* The example of steady state one dimensional heat conduction with internal heat generation is an electrical conductor carrying current can be given by the poission's eq^n as.

$$\frac{1}{r}\frac{d}{dr}\left[r\cdot\frac{dT}{dr}\right] + \frac{q}{K} = 0$$

or $\frac{d}{dr}\left[r\cdot\frac{dT}{dr}\right] = -\frac{q}{K}\cdot r$

On integration

$$r\frac{dT}{dr} = -\frac{q}{K}\frac{r^2}{2} + C_1 \quad \text{—— (1)}$$

or $\frac{dT}{dr} = -\frac{q}{K}\frac{r}{2} + \frac{C_1}{r}$

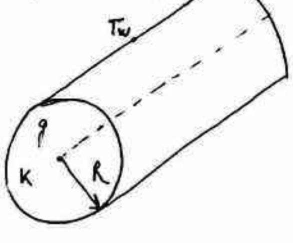
On integrating again

$$T = -\frac{q}{2K}\frac{r^2}{2} + C_1 \ln(r) + C_2 \quad \text{—— (2)}$$

– Case 1: Long Solid Cylinder With Specified Surface Temperature

Let outer surface temperature be (T_w)

Boundary conditions are



(a) The temperature at the centre line must be constant due to symmetry of solid, hence the temperature gradient at $r=0$ must be zero. i.e.

$$\left(\frac{dT}{dr}\right)_{r=0} = 0$$

Substituting above condition in eq^n (1) ; $C_1 = 0$

(b) At $r = R$; $T = T_w$ substituting the condition in eq^n (2)

$$T_w = -\frac{q}{2K}\frac{R^2}{2} + C_2 \quad \text{or} \quad C_2 = T_w + \frac{q}{2K}\frac{R^2}{2} \quad \text{—— (3)}$$

On substituting the value of C_1 & C_2 in eq^n (2), we get

$$T = -\frac{q}{2K}\cdot\frac{r^2}{2} + 0 + T_w + \frac{q}{2K}\cdot\frac{R^2}{2}$$

$$\boxed{(T - T_w) = \frac{q}{4K}(R^2 - r^2)}$$

This eq^n represents the temperature distribution in solid cylinder

* Heat Conduction $Q = -KA\frac{dT}{dr}$

from eq^n (1) since $C_1 = 0 \rightarrow \frac{dT}{dr} = \left(-\frac{q}{K}\frac{r}{2}\right)$

Hence $Q = -K(2\pi rL)\left(-\frac{q}{K}\frac{r}{2}\right)$

$$\boxed{Q = \pi r^2 L q} \quad \text{—— (4)}$$

* If outer surface is exposed to surrounding fluid at T_∞ with convective heat transfer coefficient, h . Then

$$Q = hA(T_w - T_\infty) = h(2\pi RL)(T_w - T_\infty) \quad \text{—— (5)}$$

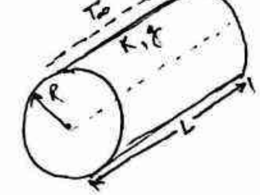
At outer surface $r = R$, in eq^n (4) and equating to eq^n (5)

$$\pi R^2 L q = h(2\pi RL)(T_w - T_\infty)$$

$$(T_w - T_\infty) = \frac{Rq}{2h}$$

or $$\boxed{T_w = \frac{Rq}{2h} + T_\infty}$$

– Case 2: Solid Cylinder Exposed To Convection Of Heat To Surrounding Fluid :



From poissons eq^n

$$\frac{1}{r}\frac{d}{dr}\left[r\cdot\frac{dT}{dr}\right] + \frac{q}{K} = 0$$

$$\frac{d}{dr}\left[r\cdot\frac{dT}{dr}\right] = -\frac{q}{K}\cdot r$$

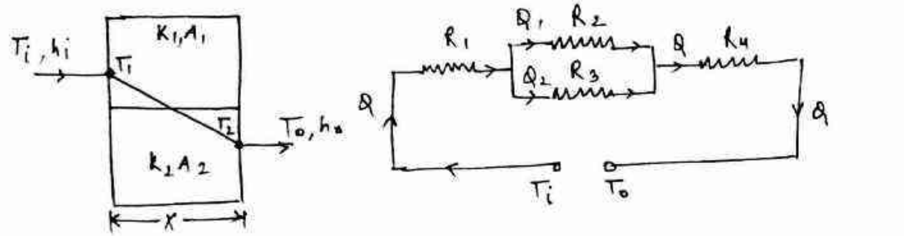
On integration above eq^n

$$\left[r\cdot\frac{dT}{dr}\right] = -\frac{q}{K}\frac{r^2}{2} + C_1 \quad \text{—— (1)}$$

$$\frac{dT}{dr} = -\frac{q}{K}\frac{r}{2} + \frac{C_1}{r} \quad \text{—— (2)}$$

→ With convective heat Transfer.

* Consider the heat transfer system as shown in figure and it's equivalent electrical system.



* Various resistances are, where $A = A_1 + A_2$

$$R_1 = \frac{1}{h_i A} \;;\; R_2 = \frac{X}{K_1 A_1} \;;\; R_3 = \frac{X}{K_2 A_2} \;;\; R_4 = \frac{1}{h_o A}$$

equations for heat flow rate are

$$Q = \frac{T_i - T_1}{R_1} \;;\; Q_1 = \frac{T_1 - T_2}{R_2} \;;\; Q_3 = \frac{T_1 - T_2}{R_3} \;;\; Q = \frac{T_2 - T_o}{R_4}$$

* combined resistance of the system can be calculated as follows
For resistance in parallel

$$\frac{1}{Re} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{R_2 + R_3}{R_2 R_3} \;;\quad Re = \frac{R_2 R_3}{R_2 + R_3}$$

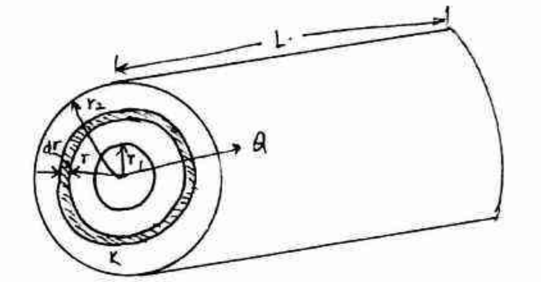
$$\therefore \Sigma R = R_1 + Re + R_4 = R_1 + \frac{R_2 R_3}{R_2 + R_3} + R_4$$

and $$Q = \frac{T_i - T_o}{\Sigma R}$$

↳ Heat transfer in an infinitely long cylinder :—

* Consider a hallow cylinder of internal radius r_1 & external radius r_2 with respective internal & external temperatures of T_i and T_o as shown in figure.
* let 'L' be the length and 'K' be the thermal conductivity of cylinder
* Considering heat transfer in radial direction only.

$$Q = -KA \frac{dT}{dx}$$



Since $A = 2\pi r L$

$$Q = -K(2\pi r L)\frac{dT}{dr}$$

Integrating between the limits

$$Q \int_{r_1}^{r_2} \frac{dr}{r} = -2\pi K L \int_{T_1}^{T_2} dT$$

$$Q \left[\ln r\right]_{r_1}^{r_2} = -2\pi K L \left[T\right]_{T_1}^{T_2}$$

$$Q \ln(r_2/r_1) = -2\pi K L (T_2 - T_1) = 2\pi K L (T_1 - T_2)$$

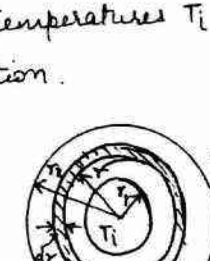
$$Q = \frac{(T_1 - T_2)}{\left[\frac{\ln(r_2/r_1)}{2\pi K L}\right]} = \frac{T_1 - T_2}{R} \;;\quad R = \left[\frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi K L}\right]$$

'R' represents the thermal resistance

↳ Heat transfer through a hallow sphere :—

* Consider a hallow sphere of internal & external radius as r_1 and r_2 respectively with respective Temperatures T_i & T_o.
* considering heat transfer in radial direction.
* let us consider a ring at radius 'r'.

since surface area, $A = 4\pi r^2$



$$Q = -KA\frac{dT}{dx}$$

$$Q = -K(4\pi r^2)\frac{dT}{dr}$$

$$Q \int_{0}^{r} \frac{dr}{r^2} = -K 4\pi \int_{T_1}^{T_2} dT$$

① Solid sphere with specified temperature

(a) At $r = 0$; $\frac{dT}{dr} = 0$

(b) At $r = R$; $T = T_w$

$$T = \frac{q(R^2 - r^2)}{6K} + T_w$$

② Solid sphere with heat convection

(a) At $r = 0$; $\frac{dT}{dr} = 0$

(b) At $r = R$; $\left[Q_{cond}\right]_{r=R} = \left[Q_{conv}\right]_{r=R}$

$$T = \frac{q}{6K}(R^2 - r^2) + \frac{qR}{3h} + T_\infty$$

Note * Heat generated in current carrying conductor

$Q_g = I^2 R$;

$Q_g = VI$

where $R = \frac{\rho \cdot L}{A}$;

$q = \frac{Q_g}{A \cdot L} = \frac{I^2 R}{A L}$

* Current density $i = I/A$ ∴ $R = (i)^2 L$

* Electrical resistivity $\rho = \frac{1}{\text{electrical conductivity}}$

where,

$Q_g \rightarrow$ Heat generated (W)

$q \rightarrow$ Heat generated per unit vol^m (W/m^3)

$R \rightarrow$ Resistance (ohm)

$A \rightarrow$ Area (m^2)

$L \rightarrow$ Length of wire (m)

$\rho \rightarrow$ Electrical resistivity $(\Omega\, m/m^2)$

$i \rightarrow$ Current density (Amp/m^2)