Unit 2: Signal Distortion in optical Fibers

Attenuation

Signal attenuation within optical fibers, as with metallic conductors, is usually expressed in the logarithmic unit of the decibel. The decibel, which is used for comparing two power levels, may be defined for a particular optical wavelength as the ratio of the input (transmitted) optical power P_i into a fiber to the output (received) optical power P_o from the fiber as:

Number of decibels (dB) = $10 \log_{10} P_i/P_o$

This logarithmic unit has the advantage that the operations of multiplication and division reduce to addition and subtraction, while powers and roots reduce to multiplication and division. However, addition and subtraction require a conversion to numerical values which may be obtained using the relationship:

$P_i/P_o = 10^{(dB/10)}$

In optical fiber communications the attenuation is usually expressed in decibels per unit length (i.e. dB km^{-1}) following:

$\alpha_{dB} L = 10 \log_{10} P_i/P_o$

where α_{dB} is the signal attenuation per unit length in decibels which is also referred to as the fiber loss parameter and L is the fiber length.

Linear scattering losses

Linear scattering mechanisms cause the transfer of some or all of the optical power contained within onepropagating mode to be transferred linearly (proportionally to the mode power) into a different mode. This process tends to result in attenuation of the transmitted light as the transfer may be to a leaky or radiation mode which does not continue to propagate within the fiber core, but is radiated from the fiber. It must be noted that as with all linear processes, there is no change of frequency on scattering. Linear scattering may be categorized into two major types: Rayleigh and Mie scattering.

Rayleigh scattering

Rayleigh scattering is the dominant intrinsic loss mechanism in the low-absorption window between the ultraviolet and infrared absorption tails. It results from in homogeneities of a random nature occurring on a small scale compared with the wavelength of the light. These in homogeneities manifest themselves as refractive index fluctuations and arise from density and compositional variations which are frozen into the glass lattice on cooling.

The compositional variations may be reduced by improved fabrication, but the index fluctuations caused by the freezing-in of density in homogeneities are fundamental and cannot be avoided. The subsequent scattering due to the density fluctuations, which is in almost all directions, produces an attenuation proportional to $1/\alpha^4$ following the Rayleigh scattering formula. For a single-component glass this is given by:

$$\gamma_R = \frac{8\pi^3}{3\lambda^4} n^8 p^2 \beta_c K T_F$$

where γ_R is the Rayleigh scattering coefficient, λ is the optical wavelength, n is the refractive index of the medium, p is the average photo elastic coefficient, β_c is the isothermal compressibility at a fictive temperature T_F, and K is Boltzmann's constant. The fictive temperature is defined as the temperature at which the glass can reach a state of thermal equilibrium and is closely related to the anneal temperature. Furthermore, the Rayleigh scattering coefficient is related to the transmission loss factor of the fiber following the relation.

$$\mathscr{L} = \exp(-\gamma_R L)$$

Mie scattering

Linear scattering may also occur at inhomogeneities which are comparable in size with the guided wavelength. These result from the non perfect cylindrical structure of the waveguide and may be caused by fiber imperfections such as irregularities in the core–cladding interface, core–cladding refractive index differences along the fiber length, diameter fluctuations, strains and bubbles. When the scattering inhomogeneity size is greater than
λ /10, the scattered intensity which has an angular dependence can be very large.
The scattering created by such inhomogeneities is mainly in the forward direction and is called Mie scattering. Depending upon the fiber material, design and manufacture, Mie scattering can cause significant losses. The inhomogeneities may be reduced by:

- removing imperfections due to the glass manufacturing process;
- carefully controlled extrusion and coating of the fiber;
- increasing the fiber guidance by increasing the relative refractive index difference.

Nonlinear scattering losses

Optical waveguides do not always behave as completely linear channels whose increase in output optical power is directly proportional to the input optical power. Several nonlinear effects occur, which in the case of scattering cause disproportionate attenuation, usually at high optical power levels. This nonlinear scattering causes the optical power from one mode to be transferred in either the forward or backward direction to the same, or other modes, at a different frequency. It depends critically upon the optical power density within the fiber and hence only becomes significant above threshold power levels.
The most important types of nonlinear scattering within optical fibers are stimulated Brillouin and Raman scattering, both of which are usually only observed at high optical power densities in long single-mode fibers. These scattering mechanisms in fact give optical gain but with a shift in frequency, thus contributing to attenuation for light transmission at a specific wavelength.

Stimulated Brillouin scattering

Stimulated Brillouin scattering (SBS) may be regarded as the modulation of light through thermal molecular vibrations within the fiber. The scattered light appears as upper and lower sidebands which are separated from the incident light by the modulation frequency.
The incident photon in this scattering process produces a phonon of acoustic frequency as well as a scattered photon. This produces an optical frequency shift which varies with the scattering angle because

the frequency of the sound wave varies with acoustic wavelength. The frequency shift is a maximum in the backward direction, reducing to zero in the forward direction, making SBS a mainly backward process.

Brillouin scattering is only significant above a threshold power density. Threshold power P_B is given by:

$$P_B = 4.4 \times 10^{-3} d^2 \lambda^2 \alpha_{dB} \nu \text{ watts}$$

Where d and λ are the fiber core diameter and the operating wavelength, respectively, both measured in micrometers, α is the fiber attenuation in decibels per kilometer and ν is the source bandwidth (i.e. injection laser) in gigahertz.

Stimulated Raman scattering

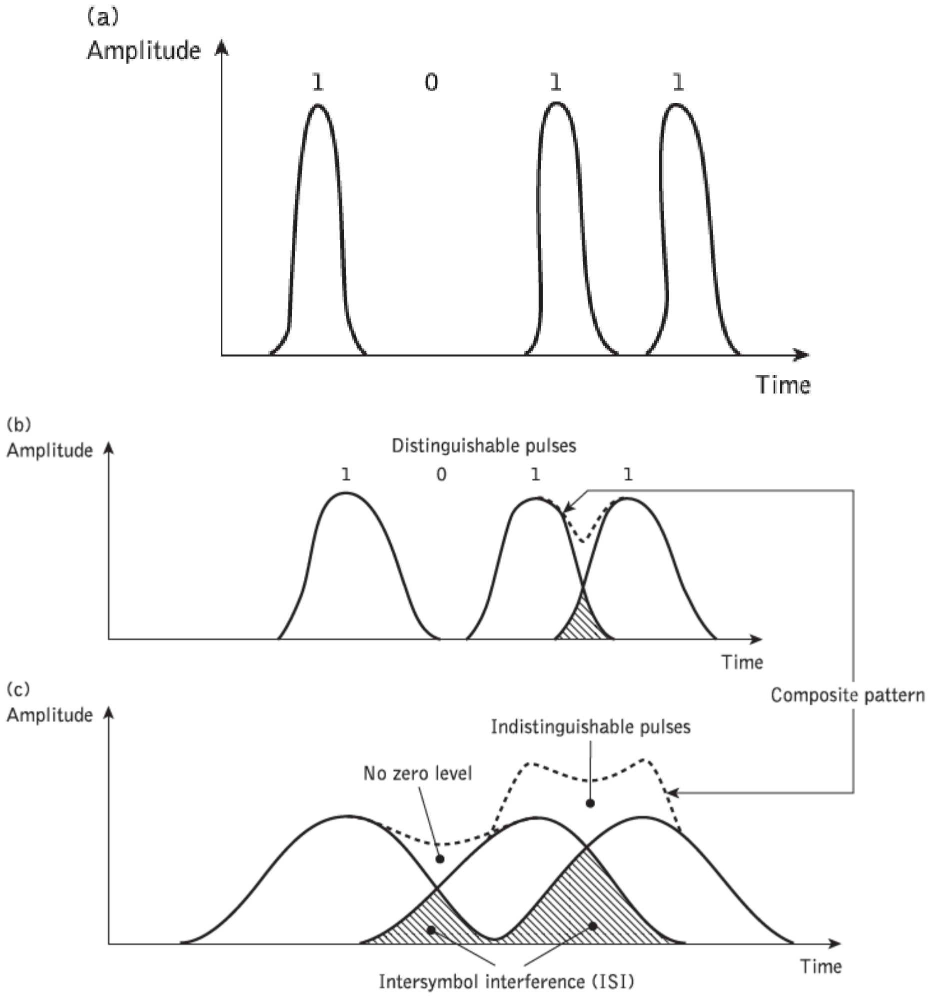
Stimulated Raman scattering (SRS) is similar to SBS except that a high-frequency optical phonon rather than an acoustic phonon is generated in the scattering process. Also, SRS can occur in both the forward and backward directions in an optical fiber, and may have an optical power threshold of up to three orders of magnitude higher than the Brillouin threshold in a particular fiber.

the threshold optical power for SRS P_R in a long single-mode fiber is given by:

$$P_R = 5.9 \times 10^{-2} d^2 \lambda \alpha_{dB} \text{ watts}$$

Dispersion

Dispersion of the transmitted optical signal causes distortion for both digital and analog transmission along optical fibers. When considering the major implementation of optical fiber transmission which involves some form of digital modulation, then dispersion mechanisms within the fiber cause broadening of the transmitted light pulses as they travel along the channel.



The phenomenon is explained in above figure, where it may be observed that each pulse broadens and overlaps with its neighbors, eventually becoming indistinguishable at the receiver input. The effect is known as intersymbol interference (ISI). Thus an increasing number of errors may be encountered on the digital optical channel as the

ISI becomes more pronounced. The error rate is also a function of the signal attenuation on the link and the subsequent signal-to-noise ratio (SNR) at the receiver. However, signal dispersion alone limits the maximum possible bandwidth attainable with a particular optical fiber to the point where individual symbols can no longer be distinguished.

For no overlapping of light pulses down on an optical fiber link the digital bit rate B_T must be less than the reciprocal of the broadened (through dispersion) pulse duration (2τ). It is given by

$$B_T \leq \frac{1}{2\tau}$$

Another more accurate estimate of the maximum bit rate for an optical channel with dispersion may be obtained by considering the light pulses at the output to have a Gaussian shape with an rms width of σ. while avoiding any SNR penalty which occurs when ISI becomes pronounced. The maximum bit rate is given approximately by

$$B_T(\max) \simeq \frac{0.2}{\sigma} \text{ bit s}^{-1}$$

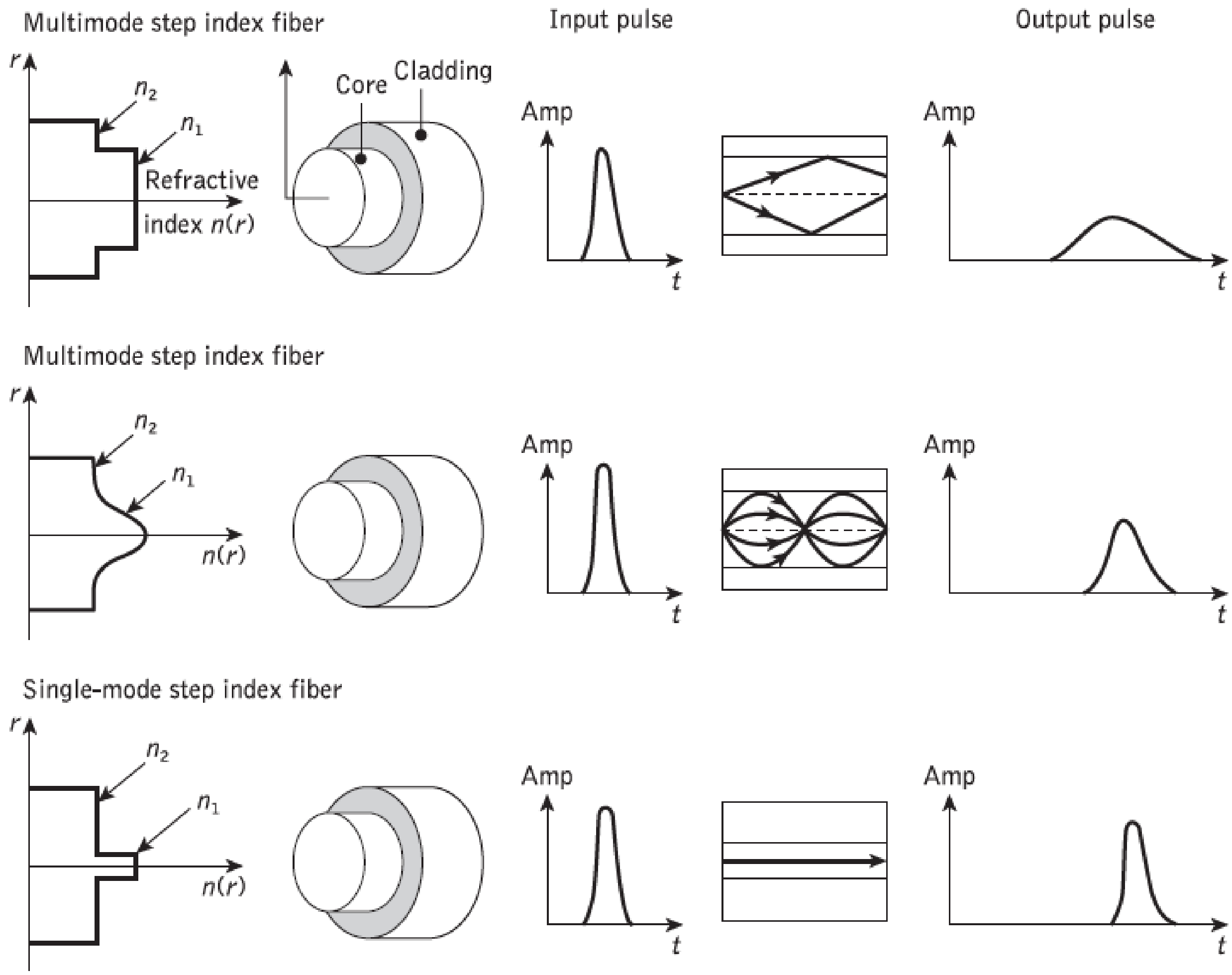


Figure: - diagram showing a multimode step index fiber, multimode graded index fiber and single-mode step index fiber, and illustrating the pulse broadening due to intermodal dispersion in each fiber type

The above figure shows the shows the three common optical fiber structures, namely multimode step index, multimode graded index and single-mode step index, while diagrammatically illustrating the respective pulse broadening associated with each fiber type. It may be observed that the multimode step index fiber exhibits the greatest dispersion of a transmitted light pulse and the multimode graded index fiber gives a considerably improved performance. Finally, the single-mode fiber gives the minimum pulse broadening and thus is capable of the greatest transmission bandwidths which are currently in the gigahertz range, whereas transmission via multimode step index fiber is usually limited to bandwidths of a few tens of megahertz. However, the amount of pulse broadening is dependent upon the distance the pulse travels within the fiber, and hence for a given optical fiber link the restriction on usable bandwidth is dictated by the distance between regenerative repeaters.

Hence, the number of optical signal pulses which may be transmitted in a given period, and therefore the information-carrying capacity of the fiber, is restricted by the amount of pulse dispersion per unit length. In the absence of mode coupling or filtering, the pulse broadening increases linearly with fiber length and

thus the bandwidth is inversely proportional to distance. This leads to the adoption of a more useful parameter for the
Information-carrying capacity of an optical fiber which is known as the bandwidth–length product (i.e. $B_{opt} \cdot L$).

Chromatic dispersion

Chromatic or intramodal dispersion may occur in all types of optical fiber and results from the finite spectral linewidth of the optical source. Since optical sources do not emit just a single frequency but a band of frequencies (in the case of the injection laser corresponding to only a fraction of a percent of the center frequency, whereas for the LED it is likely to be a significant percentage), then there may be propagation delay differences between the different spectral components of the transmitted signal. This causes broadening of each transmitted mode and hence intramodal dispersion. The delay differences may be caused by the dispersive properties of the waveguide material (material dispersion) and also guidance effects within the fiber structure (waveguide dispersion).

It is of two types

- Material Dispersion
- Waveguide Dispersion

Material Dispersion

Pulse broadening due to material dispersion results from the different group velocities of the various spectral components launched into the fiber from the optical source. It occurs when the phase velocity of a plane wave propagating in the dielectric medium varies nonlinearly with wavelength, and a material is said to exhibit material dispersion when the second differential of the refractive index with respect to wavelength is not zero ie $d^2n/d\lambda^2 \neq 0)$.
The pulse spread due to material dispersion may be obtained by considering the group delay τ_g in the optical fiber which is the reciprocal of the group velocity ν_g. Hence the group delay is given by:

$$\tau_g = \frac{d\beta}{d\omega} = \frac{1}{c}\left(n_1 - \lambda\frac{dn_1}{d\lambda}\right)$$

where n_1 is the refractive index of the core material. The pulse delay τ_m due to material dispersion in a fiber of length L is therefore:

$$\tau_m = \frac{L}{c}\left(n_1 - \lambda\frac{dn_1}{d\lambda}\right)$$

For a source with rms spectral width σ_λ and a mean wavelength λ, the rms pulse broadening due to material dispersion σ_m may be obtained from the above eqn.

$$\sigma_m = \sigma_\lambda \frac{d\tau_m}{d\lambda} + \sigma_\lambda \frac{2d^2\tau_m}{d\lambda^2} + \ldots$$

However, it may be given in terms of a material dispersion parameter M which is defined as:

$$M = \frac{1}{L}\frac{d\tau_m}{d\lambda} = \frac{\lambda}{c}\left|\frac{d^2 n_1}{d\lambda^2}\right|$$

and which is often expressed in units of ps nm^{-1} km^{-1}.

Example

A multimode graded index fiber exhibits total pulse broadening of 0.1 μs over a distance of 15 km. Estimate:

(a) the maximum possible bandwidth on the link assuming no intersymbol interference;

(b) the pulse dispersion per unit length;

(c) the bandwidth–length product for the fiber.

Solution: (a) The maximum possible optical bandwidth which is equivalent to the maximum possible bit rate (for return to zero pulses) assuming no ISI may be obtained as:

$$B_{opt} = B_T = \frac{1}{2\tau} = \frac{1}{0.2 \times 10^{-6}} = 5 \text{ MHz}$$

(b) The dispersion per unit length may be acquired simply by dividing the total dispersion by the total length of the fiber:

$$\text{Dispersion} = \frac{0.1 \times 10^{-6}}{15} = 6.67 \text{ ns km}^{-1}$$

(c) The bandwidth–length product may be obtained in two ways. Firstly by simply multiplying the maximum bandwidth for the fiber link by its length. Hence:

$$B_{opt}L = 5 \text{ MHz} \times 15 \text{ km} = 75 \text{ MHz km}$$

Alternatively, it may be obtained from the dispersion per unit length using Eq. $B_T \leq \frac{1}{2\tau}$

$$B_{opt}L = \frac{1}{2 \times 6.67 \times 10^{-6}} = 75 \text{ MHz km}$$

Waveguide dispersion

The wave guiding of the fiber may also create chromatic dispersion. This results from the variation in group velocity with wavelength for a particular mode. Considering the ray theory approach, it is equivalent to the angle between the ray and the fiber axis varying with wavelength which subsequently leads to a variation in the transmission times for the rays, and hence dispersion. For a single mode whose propagation constant is β, the fiber exhibits waveguide dispersion when $d^2\beta / d\lambda^2 \neq 0$. Multimode fibers, where the majority of modes propagate far from cutoff, are almost free of waveguide dispersion and it is generally negligible compared with material dispersion

Intermodal dispersion

Pulse broadening due to intermodal dispersion (sometimes referred to simply as modal or mode dispersion) results from the propagation delay differences between modes within a multimode fiber. As the different modes which constitute a pulse in a multimode fiber travel along the channel at different group velocities, the pulse width at the output is dependent upon the transmission times of the slowest and fastest modes. This dispersion mechanism creates the fundamental difference in the overall dispersion for the three types of fibers. Thus multimode step index fibers exhibit a large amount of intermodal dispersion which gives the greatest pulse broadening. However, intermodal dispersion in multimode fibers may be reduced by adoption of an optimum refractive index profile which is provided by the near-parabolic profile of most graded index fibers. Hence, the overall pulse broadening in multimode graded index fibers is far less than that obtained in multimode step index fibers (typically by a factor of 100). Thus graded index fibers used with a multimode source give a tremendous bandwidth advantage over multimode step index fibers.

Under purely single-mode operation there is no intermodal dispersion and therefore pulse broadening is solely due to the intramodal dispersion mechanisms. In theory, this is the case with single-mode step index fibers where only a single mode is allowed to propagate. Hence they exhibit the least pulse broadening and have the greatest possible bandwidths, but in general are only usefully operated with single-mode sources.

Multimode step index fiber

Using the ray theory model, the fastest and slowest modes propagating in the step index fiber may be represented by the axial ray and the extreme meridional ray (which is incident at the core–cladding interface at the critical angle ϕ_c) respectively. The paths taken by these two rays in a perfectly structured step index fiber are shown in figure below. The delay difference between these two rays when traveling in the fiber core allows estimation of the pulse broadening resulting from intermodal dispersion within the fiber. As both rays are traveling at the same velocity within the constant refractive index fiber core, then the delay difference is directly related to their respective path lengths within the fiber.

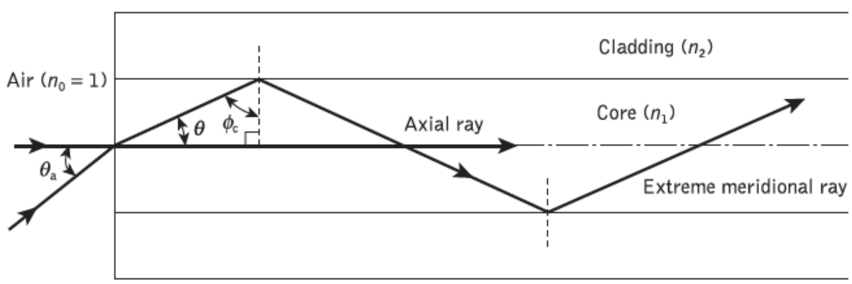


Figure:- The paths taken by the axial and an extreme meridional ray in a perfect multimode step index fiber

Hence the time taken for the axial ray to travel along a fiber of length L gives the minimum delay time T_{Min} and:

$$T_{Min} = \frac{\text{distance}}{\text{velocity}} = \frac{L}{(c/n_1)} = \frac{Ln_1}{c} \qquad \text{....... (1)}$$

Where n_1 is the refractive index of the core and c is the velocity of light in a vacuum. The extreme meridional ray exhibits the maximum delay time T_{Max} where:

$$T_{\text{Max}} = \frac{L/\cos\theta}{c/n_1} = \frac{Ln_1}{c\cos\theta} \qquad \text{.......... (2)}$$

Using Snell's law of refraction at the core–cladding interface

$$\sin\phi_c = \frac{n_2}{n_1} = \cos\theta \qquad \text{........... (3)}$$

Where n_2 is the refractive index of the cladding. Furthermore, substituting into Eq. (2) for cosθ gives:

$$T_{\text{Max}} = \frac{Ln_1^2}{cn_2} \qquad \text{......... (4)}$$

The delay difference δT_s between the extreme meridional ray and the axial ray may be obtained by subtracting Eq. (1) from Eq. (4). Hence:

$$\delta T_s = T_{\text{Max}} - T_{\text{Min}} = \frac{Ln_1^2}{cn_2} - \frac{Ln_1}{c}$$

$$= \frac{Ln_1^2}{cn_2}\left(\frac{n_1 - n_2}{n_1}\right) \qquad \text{......... (5)}$$

$$\simeq \frac{Ln_1^2\Delta}{cn_2} \quad \text{when } \Delta \ll 1 \qquad \text{.......... (6)}$$

where Δ is the relative refractive index difference. However, when Δ<<1, the relative refractive index difference is given by:

$$\Delta \simeq \frac{n_1 - n_2}{n_2} \qquad \text{........... (7)}$$

Hence rearranging Eq. (5)

$$\delta T_s = \frac{Ln_1}{c}\left(\frac{n_1 - n_2}{n_2}\right) \simeq \frac{Ln_1\Delta}{c} \qquad \text{........... (8)}$$

Also substituting for Δ gives:

$$\delta T_s \simeq \frac{L(NA)^2}{2n_1 c} \qquad \text{........... (9)}$$

Where *NA* is the numerical aperture for the fiber.

The approximate expressions for the delay difference given in Eqs (8) and (9) are usually employed to estimate the maximum pulse broadening in time due to intermodal dispersion in multimode step index fibers. It must be noted that this simple analysis only considers pulse broadening due to meridional rays and totally ignores skew rays with acceptance angles $\theta_{as} > \theta_a$. Again considering the perfect step index fiber, another useful quantity with regard to intermodal dispersion on an optical fiber link is the rms pulse broadening resulting from this dispersion mechanism along the fiber. When the optical input to the fiber is a pulse $p_i(t)$ of unit area, Then

$$\int_{-\infty}^{\infty} p_i(t)\,\mathrm{d}t = 1 \quad \text{.......... (10)}$$

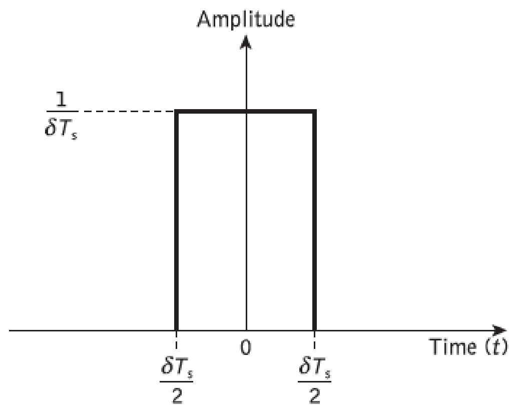


Figure An illustration of the light input to the multimode step index fiber consisting of an ideal pulse or rectangular function with unit area

It may be noted that $p_i(t)$ has a constant amplitude of $1/\delta T_s$ over the range:

$$\frac{-\delta T_s}{2} \le p(t) \le \frac{\delta T_s}{2}$$

The rms pulse broadening at the fiber output due to intermodal dispersion for the multimode step index fiber σ_s (i.e. the standard deviation) may be given in terms of the variance σ_s^2 as

$$\sigma_s^2 = M_2 - M_1^2 \quad \text{.......... (11)}$$

where M_1 is the first temporal moment which is equivalent to the mean value of the pulse and M_2, the second temporal moment, is equivalent to the mean square value of the pulse.
Hence:

$$M_1 = \int_{-\infty}^{\infty} t p_i(t)\,\mathrm{d}t \quad \text{......... (12)}$$

and:

$$M_2 = \int_{-\infty}^{\infty} t^2 p_i(t)\,\mathrm{d}t \quad \text{......... (13)}$$

The mean value M_1 for the unit input pulse in above figure is zero, and assuming this is maintained for the output pulse, then from Eqs (11) and (13):

$$\sigma_s^2 = M_2 = \int_{-\infty}^{\infty} t^2 p_i(t)\,\mathrm{d}t \quad \text{......... (14)}$$

Integrating over the limits of the input pulse (previous figure) and substituting for $p_i(t)$ in Eq. (14) over this range gives:

$$\sigma_s^2 = \int_{-\delta T_s/2}^{\delta T_s/2} \frac{1}{\delta T_s} t^2 \, dt$$

$$= \frac{1}{\delta T_s}\left[\frac{t^3}{3}\right]_{-\delta T_s/2}^{\delta T_s/2} = \frac{1}{3}\left(\frac{\delta T_s}{2}\right)^2 \qquad \text{.......... (15)}$$

Hence substituting from Eq. (8) for δT_s gives:

$$\sigma_s \simeq \frac{L n_1 \Delta}{2\sqrt{3}c} \simeq \frac{L(NA)^2}{4\sqrt{3}n_1 c} \qquad \text{.......... (16)}$$

Equation (16) allows estimation of the rms impulse response of a multimode step index fiber if it is assumed that intermodal dispersion dominates and there is a uniform distribution of light rays over the range $0 \le \theta \le \theta_a$ .The pulse broadening is directly proportional to the relative refractive index difference Δ and the length of the fiber L.

Example

A 6 km optical link consists of multimode step index fiber with a core refractive index of 1.5 and a relative refractive index difference of 1%. Estimate:

(a) the delay difference between the slowest and fastest modes at the fiber output;

(b) the rms pulse broadening due to intermodal dispersion on the link;

(c) the maximum bit rate that may be obtained without substantial errors on the link assuming only intermodal dispersion;

(d) the bandwidth–length product corresponding to (c).

Solution: (a) The delay difference is given by Eq. (8) as:

$$\delta T_s \simeq \frac{L n_1 \Delta}{c} = \frac{6 \times 10^3 \times 1.5 \times 0.01}{2.998 \times 10^8}$$

$$= 300 \text{ ns}$$

(b) The rms pulse broadening due to intermodal dispersion may be obtained from Eq. (16) where:

$$\sigma_s = \frac{Ln_1\Delta}{2\sqrt{3}c} = \frac{1}{2\sqrt{3}} \frac{6 \times 10^3 \times 1.5 \times 0.01}{2.998 \times 10^8}$$

$$= 86.7 \text{ ns}$$

(c) The maximum bit rate may be estimated in two ways. Firstly, to get an idea of the maximum bit rate when assuming no pulse overlap where:

$$B_T(\max) = \frac{1}{2\tau} = \frac{1}{2\delta T_s} = \frac{1}{600 \times 10^{-9}}$$

$$= 1.7 \text{ Mbit s}^{-1}$$

Alternatively an improved estimate may be obtained using the calculated rms pulse broadening is given by

$$B_T(\max) = \frac{0.2}{\sigma_s} = \frac{0.2}{86.7 \times 10^{-9}}$$

$$= 2.3 \text{ Mbit s}^{-1}$$

(d) Using the most accurate estimate of the maximum bit rate from (c), and assuming return to zero pulses, the bandwidth–length product is:

$$B_{opt} \times L = 2.3 \text{ MHz} \times 6 \text{ km} = 13.8 \text{ MHz km}$$