A signal f(t) which is known to be periodic signal is defined one of it time period is shown below:-

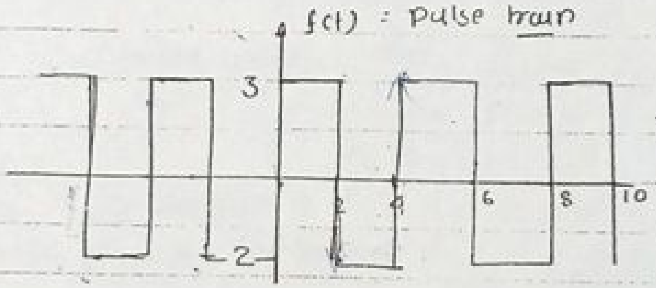
$$f(t) = 3 \quad 0 \le t \le 2$$
$$= -2 \quad 2 < t \le 4$$

A signal g(t) is defined as

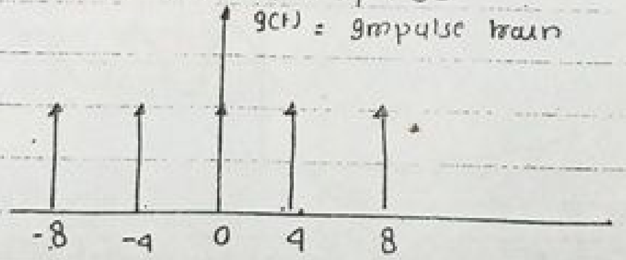
$$g(t) = \sum_{k=-\infty}^{\infty} \delta(t-4k)$$

If $\frac{df}{dt} = A_1 g(t-t_1) + A_2 g(t-t_2)$ Find the value of A_1 & A_2 and t_1 & t_2.

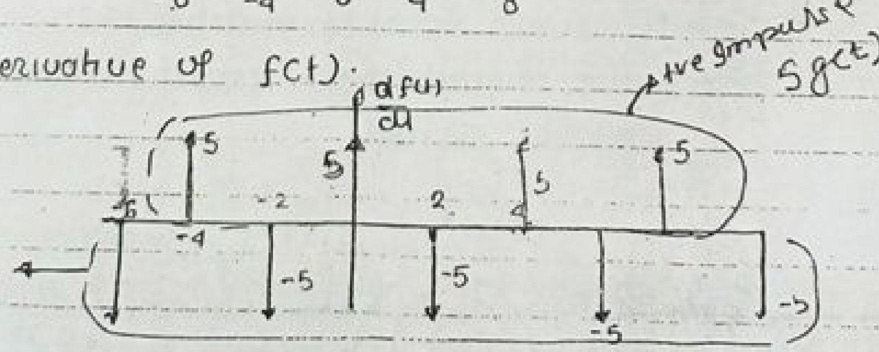
Solution:-



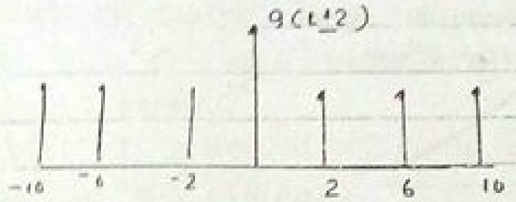
g(t) :- Summation of shifted impulse-



Now derivative of f(t).

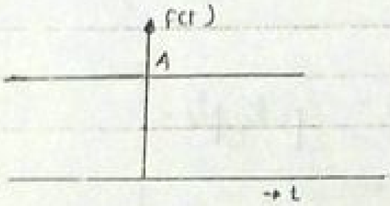


A periodic pulse train when take it derivative then we get two impulse of +ve & -ve



$\therefore \quad \frac{df}{dt} = 5g(t) - 5g(t \pm 2)$.

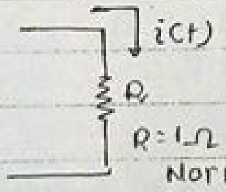
⇒ Identify the signal.



DC signal.
It is periodic but we can not defined fundamental signal.
So it may be periodic & A periodic signal.

Energy and Power Signal

Energy Signal:→



$P(t) = i^2(t)R$. $\quad f^2(t)$

Normalize Resistance

If voltage are applied

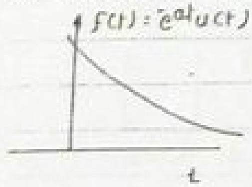
$$\frac{v^2(t)}{R} = v^2(t)$$

$$\text{Power} = \frac{dE}{dt}$$

$$\text{Energy } E = \int_{-\infty}^{\infty} p\,dt$$

$$= \int_{-\infty}^{\infty} |f^2(t)|\,dt$$

Find the energy of signal.

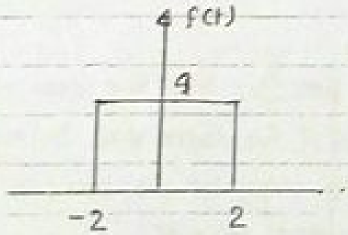


$$\text{Energy } E = \int_{-\infty}^{\infty} |f(t)|^2 dt$$

$$= \int_{-\infty}^{\infty} [e^{-at}u(t)]^2 dt$$

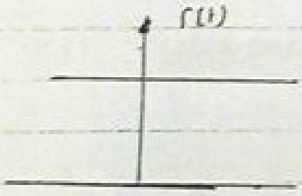
$$= \int_{0}^{\infty} e^{-2at} dt$$

$$= \left[\frac{e^{-2at}}{2a}\right]_0^{\infty} = \frac{1}{2a} \text{ Joule}$$



$$\text{Energy} = \int_{-\infty}^{\infty} |f(t)|^2 dt$$

$$= \int_{-2}^{2} 16\, dt = \underline{\underline{64}} \text{ J.}$$



Energy: ∞

$\because$ Energy is area under the curve.

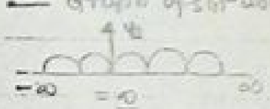
Energy of $4\sin^2 \omega_0 t$.

$$\text{Energy}(\cancel{4\sin^2\omega_0 t}) = \underline{\infty}$$

$$= \int_{-\infty}^{\infty} (4\sin\omega_0 t)^2 d\omega_0 t = \infty$$

$\because$ Graph of $\sin\omega t$

Now Graph of $\sin^2 \omega_0 t$



$$\boxed{\text{Average Power} = \frac{\text{Total energy}}{\text{Total time}}}$$

$$P = \frac{\text{finite}}{\text{time}(\infty)}$$

$$= 0$$

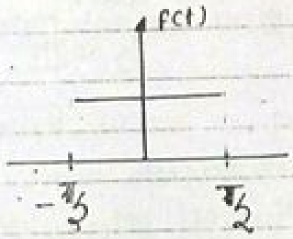
$$\boxed{P = 0}$$

Average power of signal which has infinite energy is

$$P = \frac{\infty}{\infty} = \text{Indeterminat}$$

then we take a interval of finite duration.

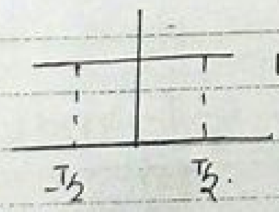
average power of D.C. signal.



$$P = \underset{T\to\infty}{Lt}\ \frac{1}{T}\int_{-\infty}^{\infty} |f^2(t)|^2 dt$$

$$\boxed{P = \underset{T\to\infty}{Lt}\ \frac{1}{T}\int_{-T/2}^{T/2} |f^2(t)|^2 dt}$$ when energy is infinite

Calculation of power.



$E = \infty$

$$P = \underset{T\to\infty}{Lt}\ \frac{1}{T}\int_{-T/2}^{T/2} f^2(t)\,dt$$

$$= A^2\ \underset{T\to\infty}{Lt}\ \frac{1}{T}\left[\int_{-T/2}^{T/2} 1\,dt\right]$$

$$= A^2\ \underset{T\to\infty}{Lt}\ \frac{1}{T}\cdot T$$

$$\boxed{P = A^2} \text{ watt}$$

P = mean square value of signal.

= $(R.M.S)^2$ value of signal.

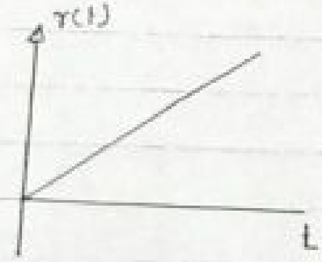
$f(t) = A\sin\omega_0 t$

Power = m.s.v

$= (\text{Rms})^2$

$\text{Rms} = \frac{A}{\sqrt{2}}$

$$P = \left(\frac{A}{\sqrt{2}}\right)^2 = A^2/2 \text{ watt}$$



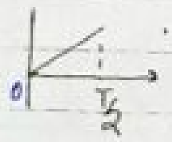
→ Energy $= \int_{-\infty}^{\infty} f^2(t)\,dt$

$= \int_{-\infty}^{\infty} t^2\,dt$

$$\boxed{E = \infty}$$

Now calculation of power

Consider a finite enterval $-T/2$ to $T/2$



$$\text{Power} = \lim_{T\to\infty} \frac{1}{T}\int_{0}^{T/2} f^2(t)\,dt$$

$$= \lim_{T\to\infty} \cancel{\tfrac{1}{T}}\ \frac{T^2}{24}$$

$$= \infty$$

For a signal of Energy is finite and average power is zero it is called as an energy signal

For a signal of energy is enfinite we go for average power calculation and if the average power happen to be finite it is called as a Power signal

If for a signal both energy and power equal to ∞ it is called as neither energy nor power signal.

Signal having finite time Duration or signal having infinite time duration with their value tending to 0 as ~~treated to~~ $t \to \infty$ are in general energy signal

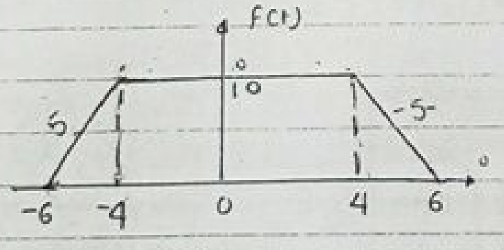
If a signal to be a power signal it should generally infinite time duration signal have infinite time duration with their value tending to non zero constant as $t \to 0$

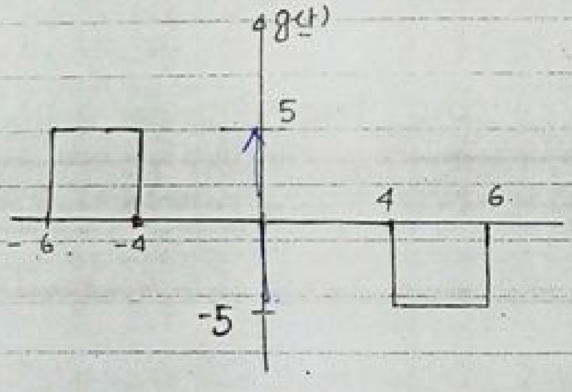
OR signal which are periodic are in General power signal.

Signal having infinite time duration with their value tending to ∞ as ~~treated~~ $t \to \infty$ are in general neither energy nor power signal.

Find the energy in the Derivative of following signal.

(i)





$$\text{Energy} = \int_{-6}^{-4} 5^2 \, dt + \int_{4}^{6} 5^2 \, dt$$

$$= \left[25t\right]_{-6}^{-4} + \left[25t\right]_{4}^{6}$$

$$= 100 \text{ J}$$

A signal $f(t)$ is known to have energy E find the energy of signal $f(-at+b)$

Given that

$$f(t) \rightarrow E = \int_{-\infty}^{\infty} f^2(t)\,dt$$

So, $$E_1 = \int_{-\infty}^{\infty} f^2(-at+b)\,dt$$

Let $-at+b = x$

$-a\,dt = dx$

$dt = \frac{-1}{a}\,dx$

Now

$$E_1 = \int_{\infty}^{-\infty} f^2(x)\cdot\left(-\frac{1}{a}\right)dx$$

$$= \frac{1}{a}\int_{-\infty}^{\infty} f^2(x)\,dx$$

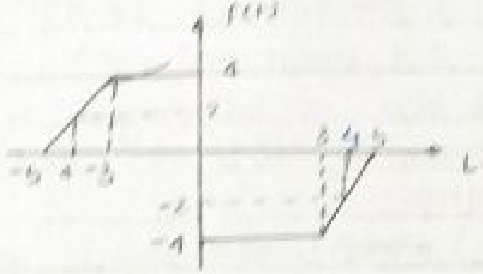
$$= \frac{1}{a}E$$

$$\therefore \boxed{E_1 = \frac{1}{a}E}$$

Note:→

Shifting and time Reversal not effect the Energy. only scaling affect the energy of the signal.

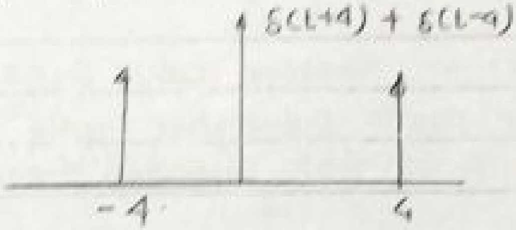
Signal f(t) defined below:→ another signal $g(t)$ is defined by multiply $f(t)$ with $\delta(t+4)+\delta(t-4)$



is the integral of signal $g(t)$, energy signal or power signal hence find energy and power.

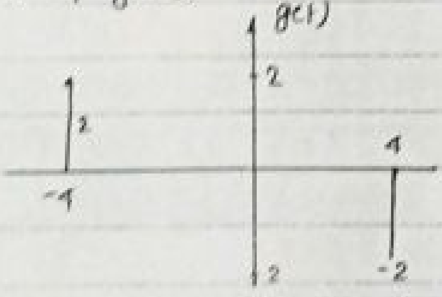
Solution:→

$$g(t) = \{\delta(t+4)+\delta(t-4)\} f(t)$$

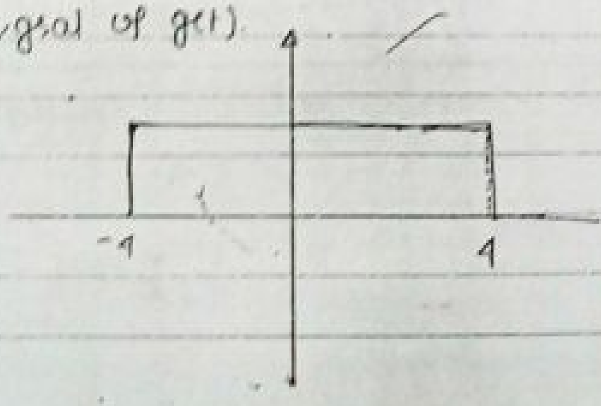


$$f(t)\,\delta(t+4) = f(-4)\,\delta(t+4)$$
$$f(t)\,\delta(t-4) = f(4)\,\delta(t-4)$$

Now graph of $g(t)$



Now integral of $g(t)$



Energy

$$= \int_{-4}^{4} 2^2\,dt$$

$$= 32 \text{ Joule}$$

Repeat above problem if $g(t)$ is defined as

$$g(t) = [\delta(t+4) - \delta(t-4)]\, f(t)$$

Hence find energy and power.

A signal $f(t)$ is defined as $f(t) = 4e^{j3t}$ is this signal is energy or power signal.

→ It is power signal ∵ Periodic signal

Power of signal = ?

$$\begin{array}{lcl} & & P \\ A\sin w_0 t & \longrightarrow & A^2/2 \\ A\sin(w_0 t + \phi) & \longrightarrow & A^2/2, \\ A\sin w_1 t & \longrightarrow & A^2/2. \end{array}$$

$f(t)$ → Complex value signal.

For power calculation only use magnitude part not consider frequency & phase.

$$f_1(t) = |f(t)|\cancel{\angle f(t)}$$

$f(t)$ → Real valued signal

$$E = \int_{-\infty}^{\infty} f^2(t)\,dt$$

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-\infty}^{\infty} f^2(t)\,dt$$

If $f(t)$ → Complex valued signal

$$E = \int_{-\infty}^{\infty} |f(t)|^2 dt$$

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-\infty}^{\infty} |f(t)|^2 dt$$

$|f(t)| = 4$

$P = 16 \text{ watt}$

Find the energy in the odd conjugate part of signal.

$$f[n] = \{-4-j5 \quad \underset{\uparrow}{1+j2} \quad 4\}$$

For Discreate time signal

$f[n]$: Real or complex.

$$\boxed{E = \sum_{n=-\infty}^{\infty} |f[n]|^2}$$

$$*\left\{ P = \sum_{n=-N/2}^{N/2} |f[n]|^2 \right\}$$

OR/

$$P = \lim \frac{1}{2N+1} \sum_{n=-N}^{N} |f[n]|^2$$

Average power

$$\boxed{P = \underset{N \to \infty}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} |f[n]|^2}$$

$$f_{oc} = \{-4-j2.5 \quad 2j, \quad 4-j2.5\}$$

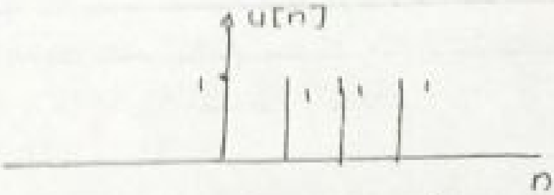
so

$$\text{Energy} = \left[(-4)^2 + (-2.5)^2 + 2^2 + (4^2) + (-2.5)^2 \right]$$

$= 48.5$ Joule

Is signal $f[n] = u[n]$ energy signal or power signal. Hence find energy and power.

Solution



Power signal

$$P = \underset{N\to\infty}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} |f[n]|^2$$

$$= \underset{N\to\infty}{Lt} \frac{1}{2N+1} \sum_{n=0}^{N} 1$$

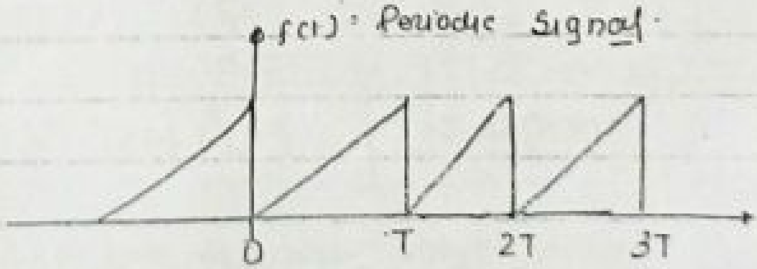
$$= \underset{N\to\infty}{Lt} \frac{1}{2N+1} (N+1)$$

$$= \underset{N\to\infty}{Lt} \frac{N+1}{2N+1}$$

$$= \frac{N(1+1/N)}{N(2+1/N)}$$

$$= 1/2 \text{ watt}$$

*



$$\text{Power}: P_1 = \frac{1}{T}\left(\int_{-T/2}^{T/2} f^2(t)\,dt\right) \text{ of } \underline{A}$$

P_2 = Power in two complete Period.

$$P_2 = \frac{1}{2T}\int_{-T/2}^{T/2} f^2(t)\,dt$$

$$= \frac{2A}{2T} = A/T$$

P_4 : Power in 4 cycle

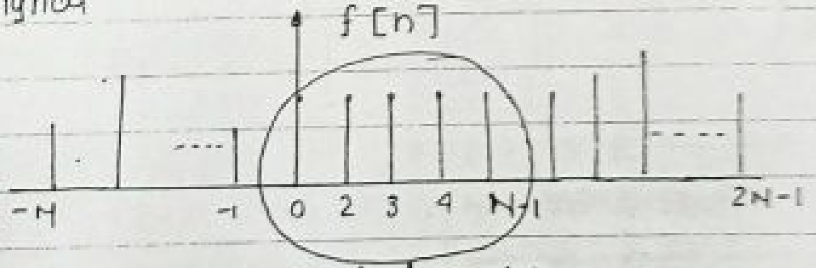
$$= \frac{1}{4T}\int_{-2T}^{2T} f^2(t)\,dt$$

$$= \frac{4A}{4T} = A/T$$

If $f(t)$ = periodic with period T then
For both Real and complex valued

$$\boxed{\text{Power} = \frac{1}{T}\int_{-T/2}^{T/2} |f(t)|^2\,dt}$$

DT Signal



periodic

Power

$$= \frac{1}{N}\sum_{n=0}^{N-1} f^2[n]$$

If $f[n]$ is periodic of period N then

$$P = \frac{1}{N}\sum_{n=0}^{N-1} |f[n]|^2$$

Find the powers of signal

$$f[n] = 4\cos(n\pi/2).$$

Here

$$\omega_0 = \pi/2$$

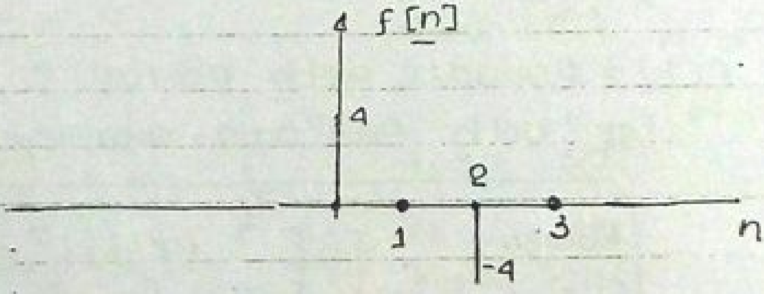
$$\frac{2\pi}{\omega_0} = 2\pi/\pi/2 = 4 \quad \text{Rational}$$

Period

$$N = r \cdot 2\pi/\omega$$

$$= 1 \cdot 4/1 = \underline{4}$$

$$N-1 = 3.$$



$$\text{Power} = \frac{1}{4}\left[4^2 + 4^2\right]$$

$$= \underline{\underline{8 \text{ watt}}}$$

$$f(t) = 4\cos\omega_0 t$$

$$\text{Power} = \frac{4^2}{2}$$

$$= \underline{8 \text{ watt}}$$

$$f(t) = f_e(t) + f_o(t)$$

$$E = \int_{-\infty}^{\infty} f(t)\,dt$$

Random or Deterministic Signal:→

A signal whose feature value can be evaluated exectly staying at present time is called as Deterministic signal. For this to be possible signal must be having well defined mathematical expression.

For signal which do not have well defined mathem expression feature value can not be evaluated exactly at present point of time and such signal are called as Random signal.

Analog Signal or Digital signal.

If a signal is allow to assumed all possible real value in its dynamic Range it is called as a analog signal where as.

If a signal is allows to assume to some specific value for its amplude in its dynamic range It is called as Digital signal.

Sketch the possible Graph for

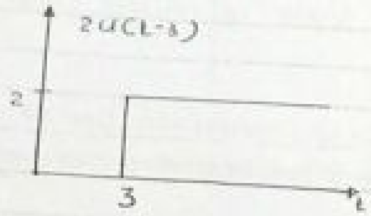
(i) Contineous time analog

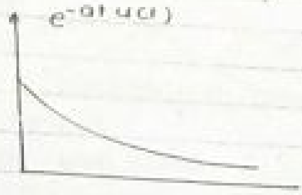
(ii) Contineous time Digital

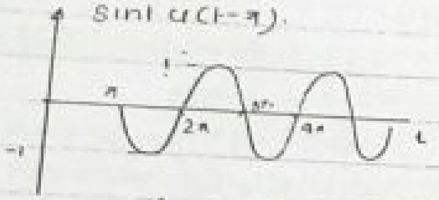
(iii) Discreate time analog

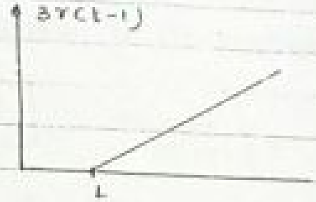
(iv) Discreate time digital signal

Causal or Non Causal Signal.



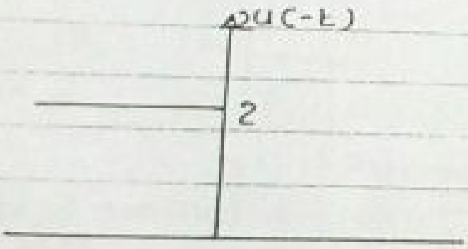


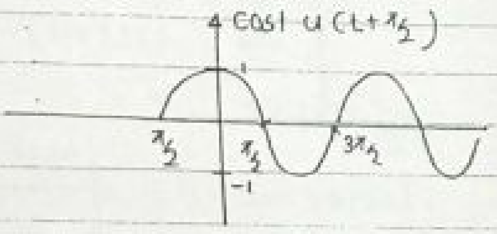


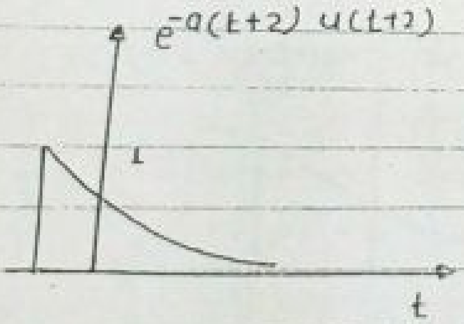


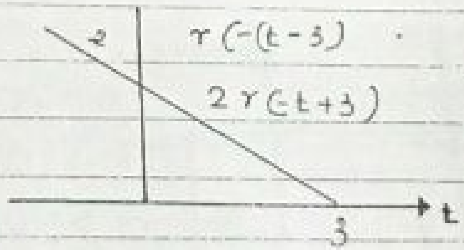
$f(t) = 0 \quad t < 0$

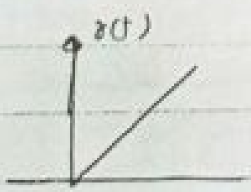
↳ Causal signal.

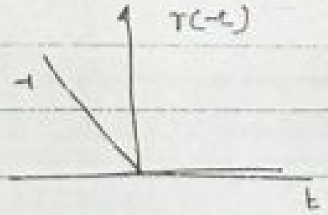












$f(t) \neq 0 \quad t < 0$

Non causal signal.

Bounded signal:- Signal having finite value at any instant of time is called Bounded Signal.

unbounded Signal:-

Signal which having infinite value as $t \to \infty$ the signal is called unbounded signal.

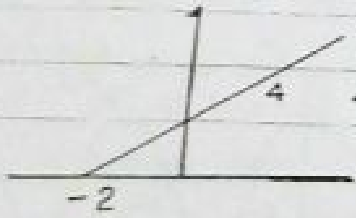
mathematical Representation:→

$$\boxed{|f(t)| \leq M} \rightarrow \text{Bounded Signal}$$

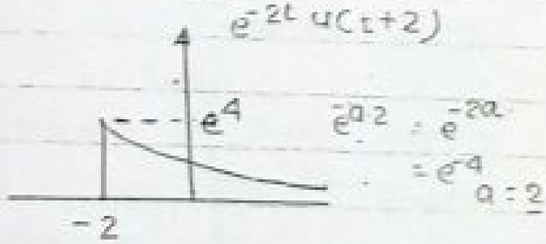
where m: finite value

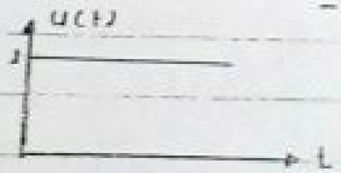
this satisfied for all value of t.

Right Sided Signal or Left Sided Signal.

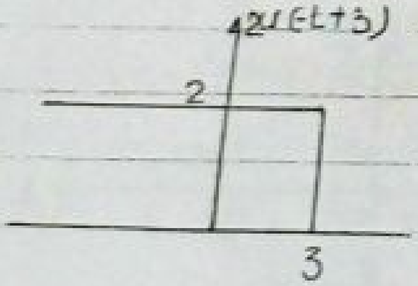


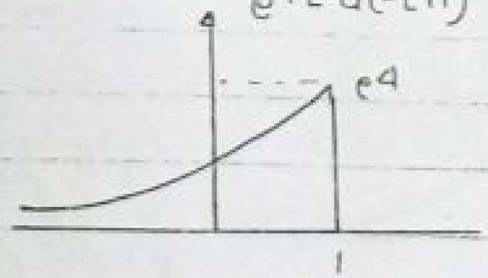
$4r(t+4)$.

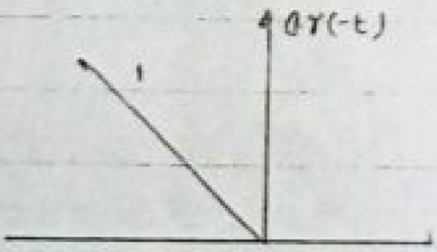




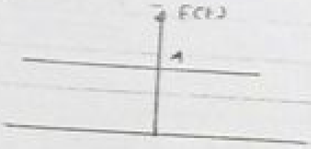
→ All are Right sided signal







All are Left sided sign.



These two signal are two sided OR Double sided signal

match the following

Expression of f(t)		Nature of f(t)
A.	$f(t)[1-u(t)] = 0$	1. Decaying Exponential
B.	$f(t) + k\frac{df}{dt} = 0$, $k \to +ve$	2. Increasing exponential
C.	$f(t) + k\frac{d^2f}{dt^2} = 0$, $k \to +ve$	3. Impulse
d.	$f(t) \cdot [g(t) - g(0)] = 0$, $g(t)$ = Arbitrary	4. Causal
		5. Sinusoidal.

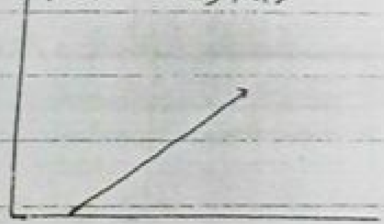
4. $f(t)[1-u(t)] = 0$

$$f(t) - f(t)u(t) = 0$$

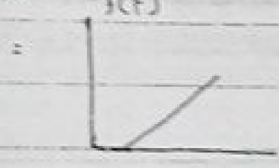
$$f(t) = f(t)u(t)$$

Here $f(t) = 0 \quad t<0$.

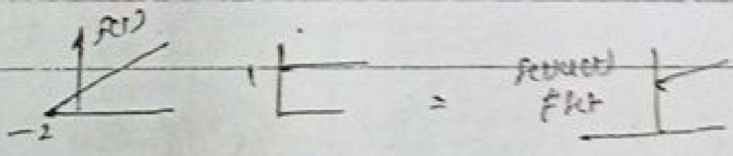
$\therefore$ f(t) is causal signal.



यदि f(t)



$f(t) = 0 \quad t < 0$



B. $K\frac{df}{dt} + f(t) = 0$

$KD + 1 = 0$

$D = -1/K$

$f(t) = ae^{-1/K\, t}$

= decaying function

C. $K\frac{d^2f}{dt^2} + f(t) = 0$

$KD^2 + 1 = 0$

$D^2 = -1/K$

$D = \pm j\sqrt{1/K}$. Sinusoidal

D. $f(t)\,[g(t) - g(0)] = 0$

$f(t)g(t) - f(t)g(0) = 0$

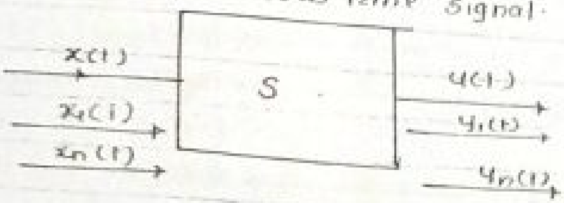
$g(t)f(t) = f(t)g(0)$

$g(t)\delta(t) = g(0)\delta(t)$

so, Impulse signal

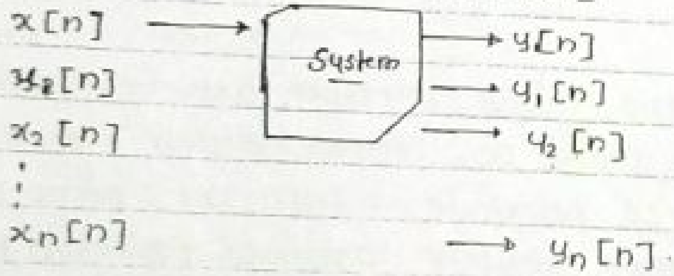
SYSTEM:-

Continuous time signal.



A system is is mathematical identity which map a set of input to set of output.

Discrete time signal



Representation of system:- A system is Represent by the

(i) Relating the response to the input.

(ii) Physical composition.

(iii) Differential equation. [$V = IR$, $V = L\,di/dt$, $I = C\,dV/dt$]

OR/ Difference equation

* (iv) Unit Impulse Response

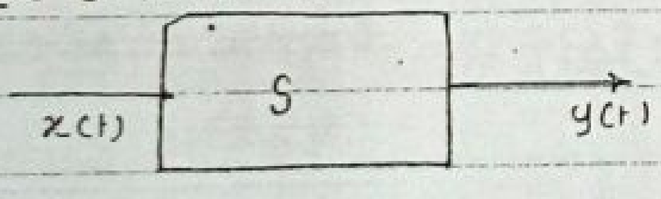
It is Represent by. $h(t)$ $h[n]$

* (v) Transfer Function.

Symbol for this $H(\omega)$, $H(s)$, $H(z)$

(vi) State variable

Symbolic Representation.



Linear or Non Linear System

$$x(t) \xrightarrow{S} y(t)$$

$$a x(t) \xrightarrow{S} a y(t)$$, homogenity principle

$$x_1(t) \xrightarrow{S} y_1(t)$$

$$x_2(t) \xrightarrow{S} y_1(t)$$

$$x_1(t) + x_2(t) \xrightarrow{S} y_1(t) + y_2(t)$$

System has additivity principle

OR/ Superposition principle

If Both the above principle are verify by a system System is called as Linear system. If at least one one of these principle or both the principle are not satisfies then system are called Non linear system

Example :→ $y(t) = 2\,x(t)$.

$$x(t) \xrightarrow{S} 2 \cdot x(t)$$

$$a x(t) \longrightarrow \cancel{a\,x(t)}$$

$$= 2(a\,x(t)]$$

$$= a[2x(t))$$

$$= a\,y(t)$$

homoginity principle

$$x_1(t) \xrightarrow{S} 2x_1(t) = y_1(t)$$

$$x_2(t) \xrightarrow{S} 2x_2(t) = y_2(t)$$

$$x_1(t) + x_2(t) \xrightarrow{S} 2[x_1(t) + x_2(t)]$$

$$= 2x_1(t) + 2x_2(t)$$

$$= y_1(t) + y_2(t)$$

Superposition principle

So system is Linear system