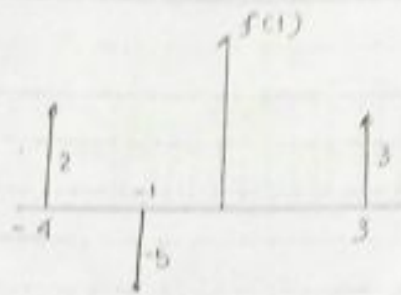


(20)

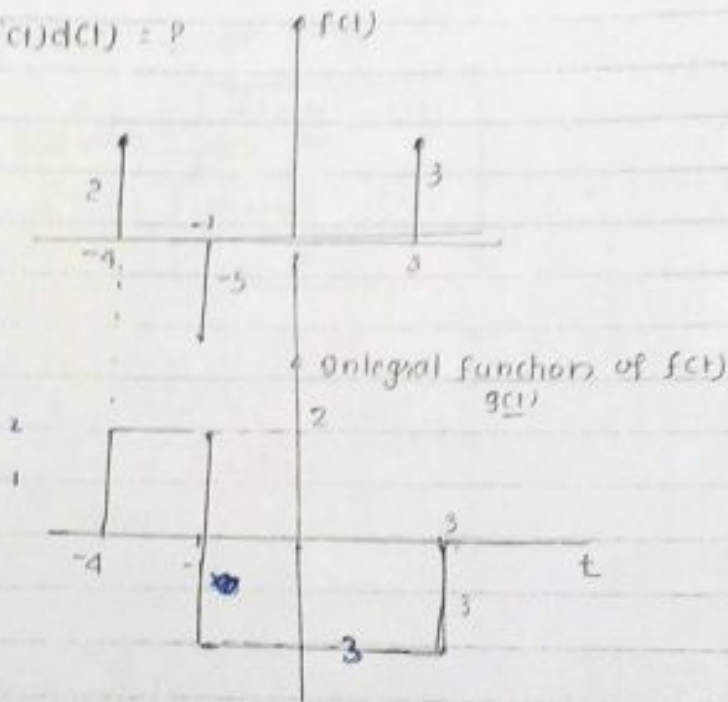
Solution:-



Height of impulse is area under area

Here

$$\int_{-\infty}^{\infty} f(t) dt = P$$



$$f(t) = \frac{d}{dt} g(t)$$

2nd method:-

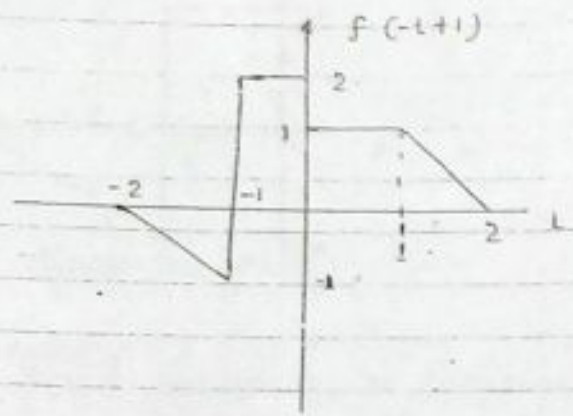
$$f(t) = 2\delta(t+4) - 5\delta(t+1) + 3\delta(t-3)$$

$$g(t) = 2u(t+4) - 5u(t+1) + 3u(t-3)$$

Here step size = area of impulse

(ii) Time Reversal

$$f(L+1) \xrightarrow{t \rightarrow -L} f(-L+1)$$



For above signal $f(t)$ sketch the graph of $f(-2t+3)$

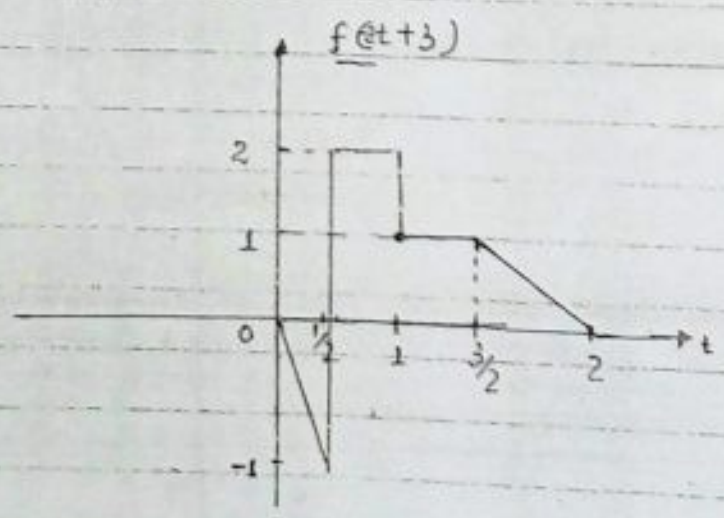
Solution-

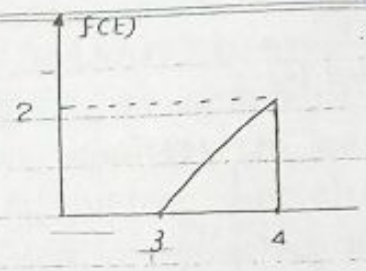
- (i) Shifting: $t \rightarrow t+3$ → Subtract '3'
- (ii) Scaling: $t \rightarrow 2t$ → Divide by 2
- (iii) Reversal: $t \rightarrow -t$ → Add extra -ve sign to t

(i) Shifting.

$$f(t) \xrightarrow{t \rightarrow t+3} f(t+3)$$

	-1	0	1	2	3
Subtract 3	-4	-3	-2	-1	0
Divided by 2	-2	-3/2	-1	-1/2	0
add extra -ve	2	3/2	1	1/2	0



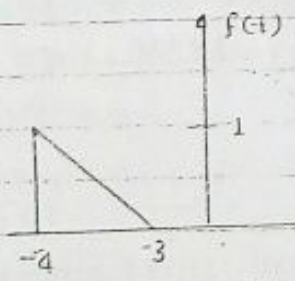


Solution:- (i) Reversal $f(t) \xrightarrow{t \rightarrow -t} f(-t)$
 (ii) Shifting $f(-t) \xrightarrow{t \rightarrow t+7} f(-(t+7)) = f(-t-7)$. This is not our Requirement

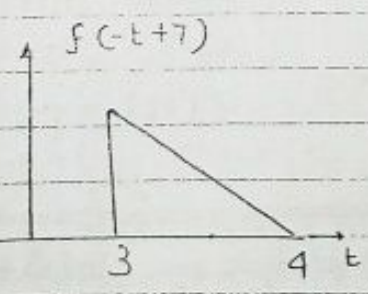
$$f(-t) \xrightarrow{t \rightarrow t-7} f(-(t-7)) = f(-t+7)$$

This is our Requirement

Reversal:

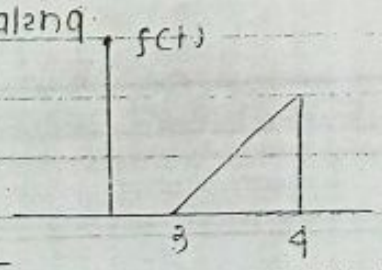


Shifting



$f(t)$ $f(2t+4)$

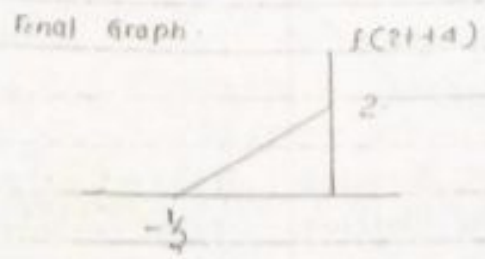
- (i) Shifting
- (ii) Scaling



Shifting $t \rightarrow t+4$

$$f(t) \xrightarrow{\text{subtract 4}} f(t+4)$$

$$f(t+4) \xrightarrow[\text{divided by 2}]{t \rightarrow 2t} f(2t+4)$$



Now 2nd order.

- (i) Scaling
- (ii) Shifting

$$f(t) \xrightarrow{t \rightarrow 2t} f(2t)$$

$$f(2t) \xrightarrow{t \rightarrow t+4} f(2(t+4))$$

$\therefore f(2t+8)$ which is not same as first order.

original shift amount by divided by scaling amount.

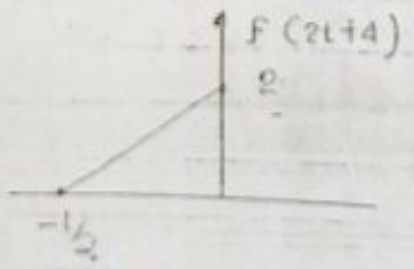
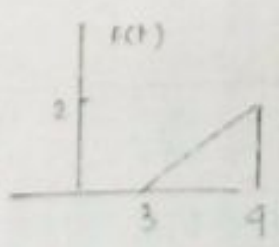
$$\therefore f(2t) \xrightarrow[t \rightarrow t + \frac{4}{2}]{t \rightarrow t + \frac{4}{2}} f(2(t + \frac{4}{2}))$$

$$= f(2t+4)$$

same as first

Now Graph plotting

- (i) Divided by 2.
- (ii) Subtract by 2.



If shifting is performed after Reversal the nature of Shift must be change that is the delay shifting operation changes to advance shifting operation and advance shifting operation change to delay operation if shifting is performed after Reversal.

If shifting is performed after scaling the original Shift amount has to be divided by scaling factor.

Among scaling and Reversal the order of application of operation does not matter.

Relatize the Transformation require to get $f(-at+b)$ if the operations are to perform in the order scaling Reversing and shifting.

Solution:

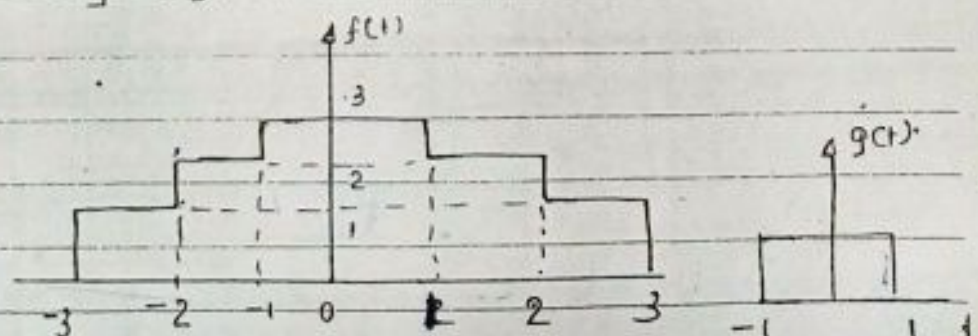
$$f(t) \longrightarrow f(-at+b)$$

$$\text{Scaling } f(t) \xrightarrow{t \rightarrow at} f(at)$$

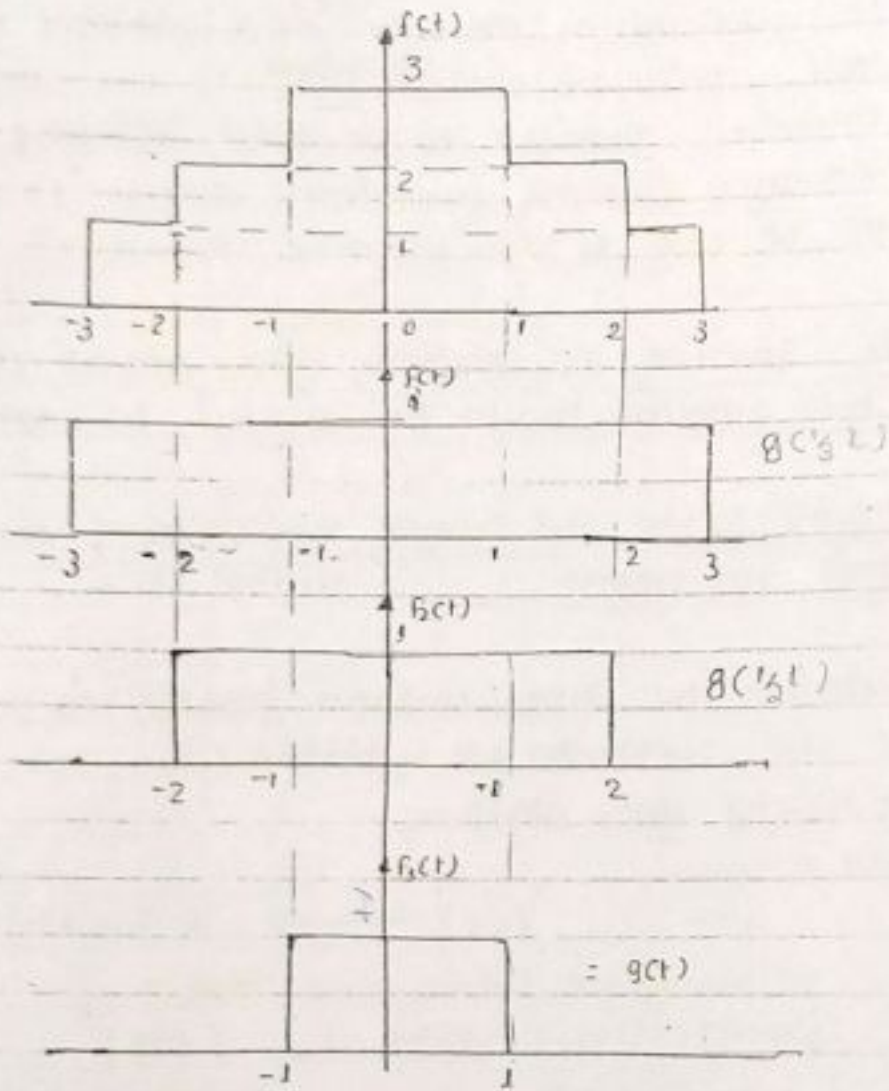
$$f(t) \xrightarrow{t \rightarrow -t} f(-t)$$

$$f(-at) \xrightarrow[\begin{matrix} t \rightarrow t - b/a \end{matrix}]{\begin{matrix} t \rightarrow t + b/a \end{matrix}} f(-a(t - b/a)) \\ = \underline{\underline{f(-at+b)}}$$

⇒ Represent following signal $f(t)$ in term of $g(t)$



Solution →

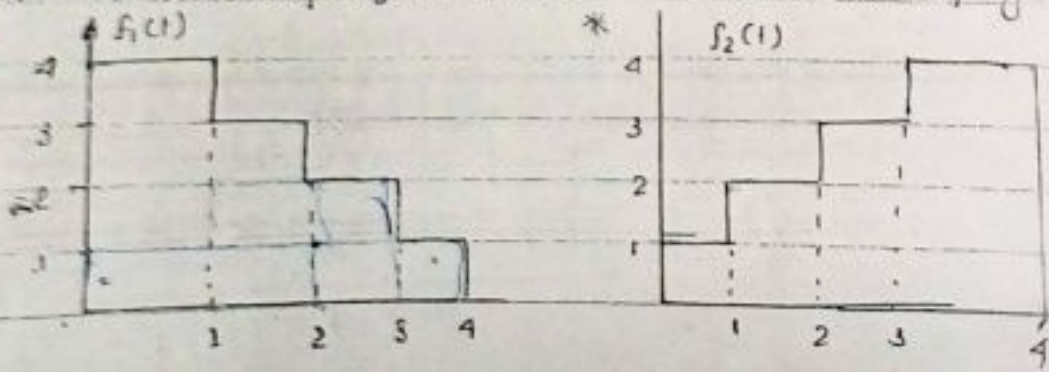


$$f(t) = f_1(t) + f_2(t) + f_3(t)$$

$$= g\left(\frac{1}{3}t\right) + g\left(\frac{1}{2}t\right) + g(t)$$

$$f(t) = g\left(\frac{1}{3}t\right) + g\left(\frac{1}{2}t\right) + g(t)$$

Represent the following signal $f_1(t)$ & $f_2(t)$ in terms of $g(t)$



$g(t)$ is given in above problem.

(27)

Classification of signal :-

1. Real value or Complex valued signal.

If a signal is defined as imaginary along with real part then signal is complex valued signal.

$$f(t) = 3t + j2t$$

$$e^{jt} \rightarrow \cos t + j\sin t$$

↳ Euler's signal.

Real value signal when add with imaginary part become complex valued signal.

Complex valued signal consisting of the non physical imaginary part are used in signal analysis as they reduce amount of analysis require to get the result.

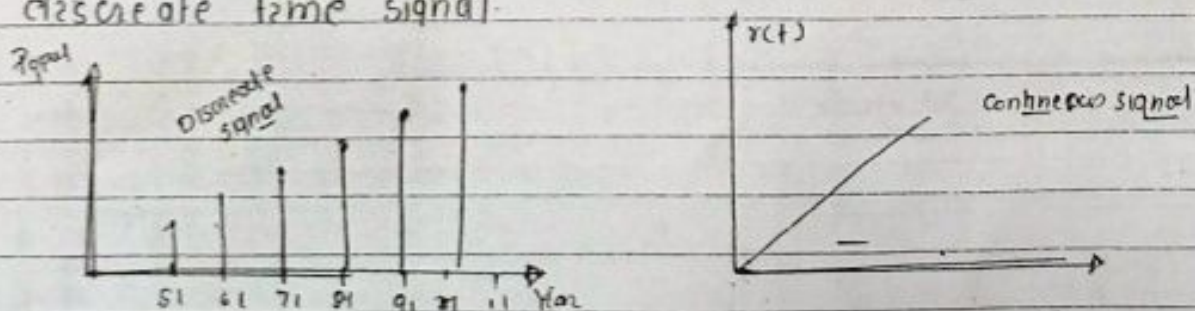
The most common complex valued signal is Euler signal is defined as

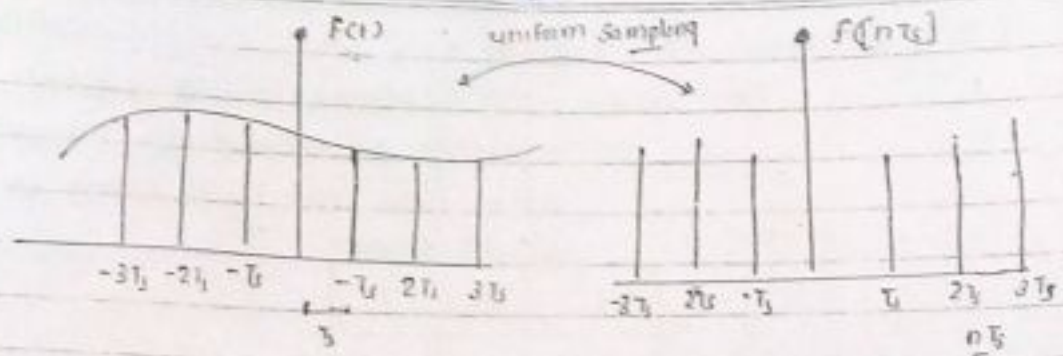
$$e^{jt} = \cos t + j\sin t$$

2. Continuous time or Discrete time signal :-

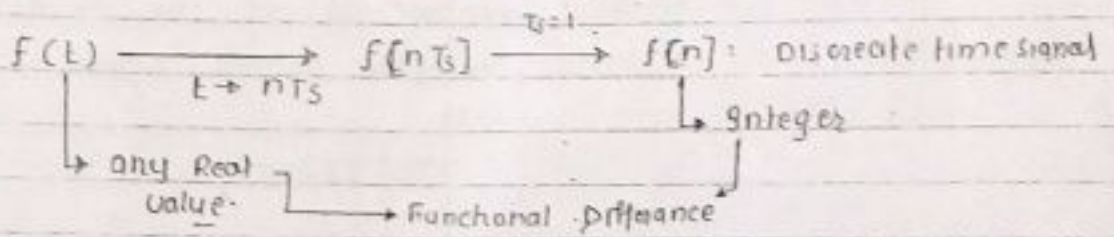
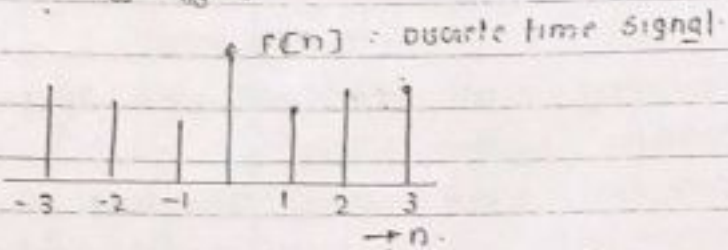
If a signal value is defined for all value of time over a range of time it is called as continuous time signal.

If a signal has a defined value only for some specific instant of time and if the value is not define for remaining time then the signal is discrete time signal.

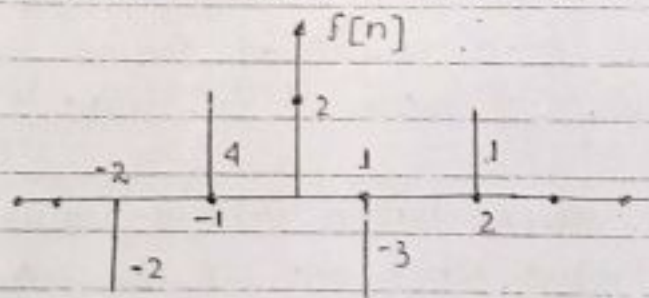




If $T_s = 1$



Graphical model of Discrete time Signal.



Equational Representation-

$f[n] = -2$	$n = -2$	n
$= 4$	$n = -1$	$n+5$
$= 2$	$n = 0$	$n+2$
$= -3$	$n = 1$	$n-4$
$= 1$	$n = 2$	$n-1$
$= 0$	$n = 3$	$n-1$
	other	

other representation: $\{-2, 4, \overset{n=0}{\underset{\uparrow}{2}}, -3, 1\}$

A discrete time signal is derived from a continuous time signal by procedure called as uniform sampling. The functional difference b/w a continuous & discrete time signal is that the argument of continuous time signal can take any real value but argument of discrete time signal can take integer value.

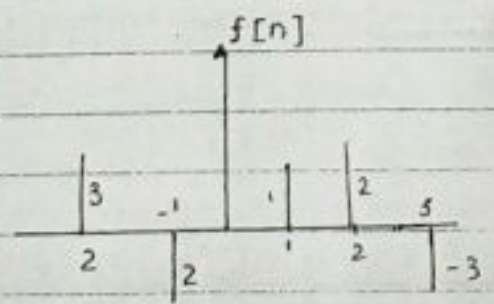
that is discrete time signal value like this $f[3], f[5], f[9], \dots$
 $f[3.6], f[4.9], \dots$ are undefined

For a discrete time signal model proposed above there is only space for integer time value for time axis. There is no space of non integer value.

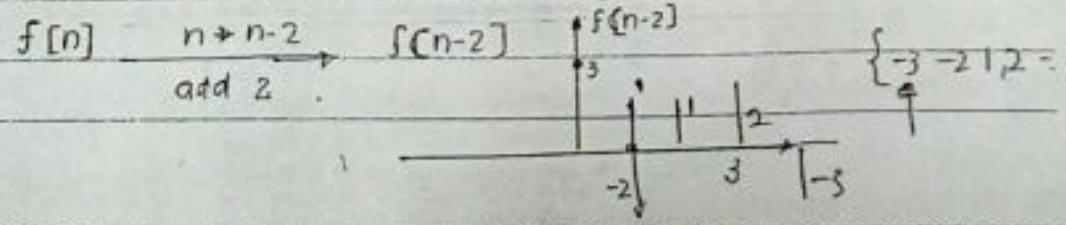
Discrete time signal information also as a sequence of no which indicate the sample value of the signal with proper indexing.

$\{-2, 4, \overset{n=0}{\underset{\uparrow}{2}}, -3, 1\}$

Graphical model of Discrete time signal:

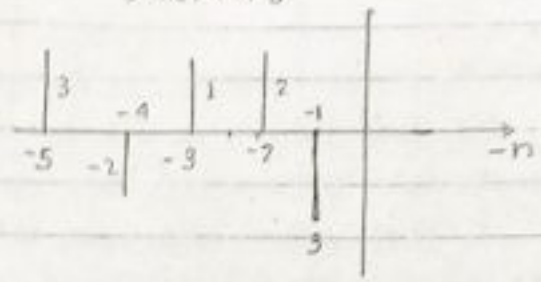


Sequence: $\{3, -2, 1, 2, -3\}$



$f[n+3] : P$

$$f(n) \xrightarrow[n \rightarrow n+3]{\text{Subtract 3}} f(n+3)$$

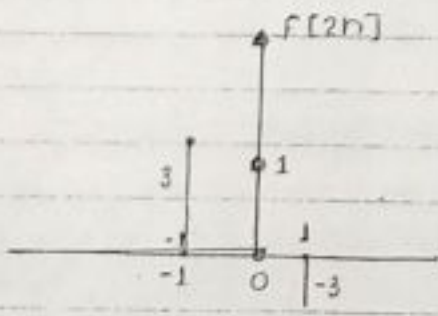


$$\{ 3, -2, 1, 2, -3, 0 \}$$

Shift Right by 3 unit

$f[2n] : P$

$$f(n) \xrightarrow[n \rightarrow 2n]{\text{Divided by 2 transformation}} f[2n]$$



$n = 1$ is not represent on the graph bcz on the axis only represent enteger value but $\frac{1}{2} = 0.5$

Sequence

$$= \{ 3 \quad 1 \quad -3 \}$$

$$f[n] : \{ \overset{-4}{1}, \overset{-3}{2}, \overset{-2}{3}, \overset{-1}{4}, \overset{0}{5}, \overset{1}{6}, \overset{2}{7}, \overset{3}{6}, \overset{4}{5}, \overset{5}{4}, \overset{6}{3}, \overset{7}{2}, \overset{8}{1} \}$$

$$f[2n] : \{ 1, 3, 5, 7, 5, 3, 1 \}$$

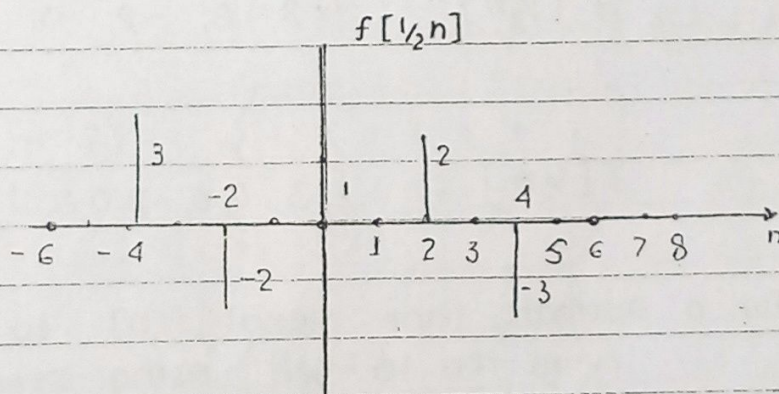
$$f[3n] : \{ 1, 4, 7, 4, 1 \}$$

$$f[4n] = \{3, 7, 3\}$$

To transform $n \rightarrow an$ for a discrete time signal retain the sample value occurring at $n=0$ as it is from there on retain every a th sample towards both side of $n=0$ sample value.

$$f(n) \longrightarrow f[\frac{1}{2}n]$$

$$f[n] \xrightarrow[\substack{\text{Divided by} \\ \frac{1}{2}}]{n \rightarrow \frac{1}{2}n} f[\frac{1}{2}n]$$



Interpolation

- (i) Zero Interpolation
- (ii) Unity Interpolation
- (iii) Average interpolation.

For a discrete time signal $f(n)$ of n is transfer to an [$a > 1$] the resulting time scale signal will have lesser no. of sample value than that were originally present in given discrete time signal this procedure is called as Desimation of Discrete time signal.

For a discrete time signal of n is transform to $\frac{1}{a}n$ ($a > 1$) there are some unspecified sample value in the Resulty unspaced

which have to be specified by using a suitable interpolation formula and resulting time scale signal will have more no. of sample value than that are originally present on the discrete time signal. This process is interpolation of discrete time signal.

By default zero interpolation is used to specify the unknown sample value.

$$f[\frac{1}{2}n]:$$

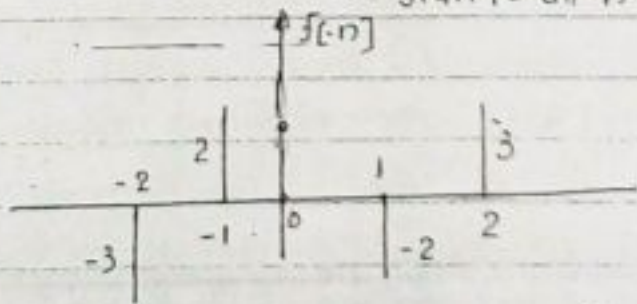
$$f[\frac{1}{2}n]: \{ 3, 0, -2, 0, 1, 0, 2, 0, -3 \}$$

$$f[\frac{1}{3}n] = \{ 3, 0, 0, -2, 0, 0, 1, 0, 0, 2, 0, 0, -3 \}$$

For a discrete time signal $f[n]$ to transform n to $\frac{1}{a}n$ using zero interpolation technique $(a-1)$ zero have to be added b/w every two value.

$$f[-n]: ?$$

$$f[n] \xrightarrow[\text{add extra -ve sign to all 'n'}]{n \rightarrow -n} f[-n]$$



Sequence: $\{ -3, +2, 1, -2, 3 \}$

[P5/19] :-

Given

$$x[n] = \begin{cases} 0 & n < -2, n > 4 \\ 1 & \text{otherwise} \end{cases}$$

Find $x[-n-2] : P$

Solution:-

$$x[n] = \left\{ \begin{array}{cccccc} -2 & -1 & 0 & 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{array} \right\}$$

Shifting-

$$x[n] \xrightarrow[\text{Shift the array left by 2 unit}]{n \rightarrow n-2} x[n-2] = \left\{ \begin{array}{cccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{array} \right\}$$

Reversal

$$x[n-2] \xrightarrow{n \rightarrow -n} x[-n-2] = \left\{ \begin{array}{cccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{array} \right\}$$

$$x[-n-2] = \left\{ \begin{array}{cccccc} n=-6 & & & & & n=0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{array} \right\}$$

$$[c] \quad n < -6, n > 0$$

Question-29

$$y[n] = \begin{cases} x\left(\frac{n}{2}-1\right) & n = \text{even} \\ 0 & n = \text{odd} \end{cases}$$

Solution

$$x[n] \longrightarrow x\left[\frac{n}{2}-1\right]$$

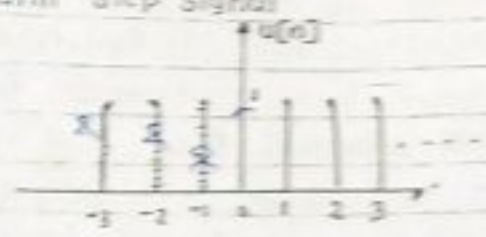
Shifting

$$x[n] \xrightarrow[\text{add 1}]{n \rightarrow n-1} x[n-1] \quad \begin{matrix} x[n] \\ 0 \\ 0 \end{matrix}$$

Scaling

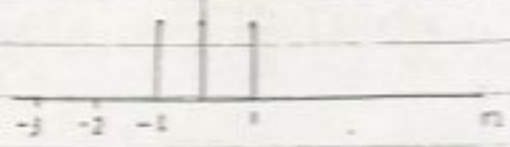
$$x[n-1] \xrightarrow[\text{divided by } \frac{1}{2}]{n \rightarrow \frac{1}{2}n} x\left[\frac{1}{2}n-1\right] \quad \begin{matrix} \frac{1}{2}n = 2 \\ y[n] \end{matrix}$$

Discrete time unit step signal

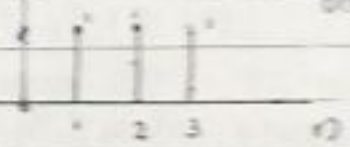


$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

$u[n+1]$: signal sample value change occurring at $n = -1$

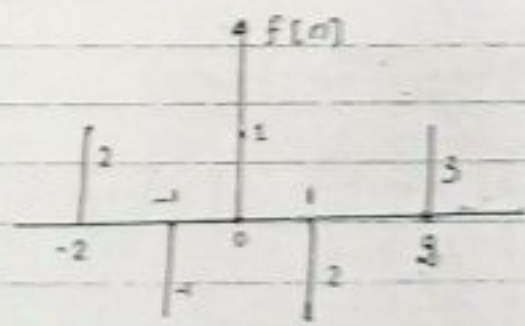


$u[n-1]$: signal sample value change occurring at $n = 1$



$u[n-n_0]$

= Sample value change at $n = n_0$



$$\{2, -1, 1, -2, 3\}$$

$u[n+2]$: sample value change 0 to 2 at $n = -2$

$u[n+1]$: sample value change 2 to -1 at $n = -1$

$u[n]$: sample value change -1 to 1 at $n = 0$

$u[n-1]$

$u[n+2]$

$u[n-3]$

$$f[n] = 2u[n+2] - 3u[n+1] + 2u[n] - 3u[n-1] + 5u[n-2] - 3u[n-3]$$

$f[n]$: In term of shifted unit step signal

Some of co-efficient = 0

or ~~at~~ in $f[n]$

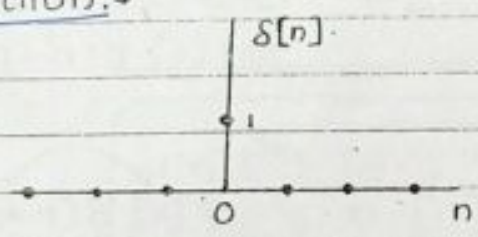
' at $n=1$ $f[n] = 1$ then $f[n]$ becoms.

$$f[n] = 2u[n+2] - 3u[n+1] + 2u[n] + 0 + 2u[n-2] + 3u[n-3]$$

In the above Representation sum of co-efficient will be equal to zero if there are finite no of non-zero sample value in the consides $f[n]$.

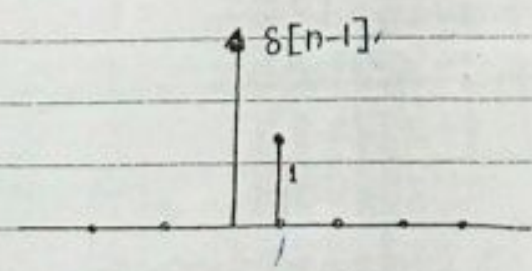
Discrete time:

→ Impulse Function:

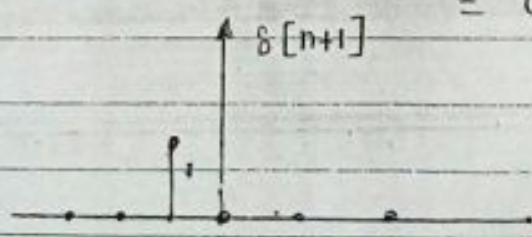


mathematically

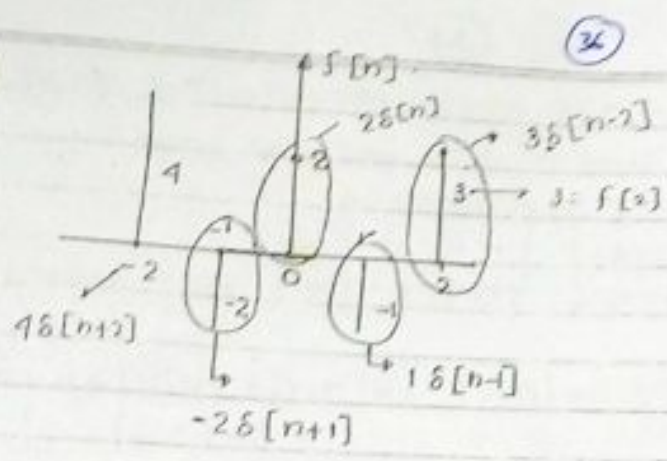
$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$



$$\delta[n-1] = \begin{cases} 1 & n = 1 \\ 0 & n \neq 1 \end{cases}$$



$$\delta[n+1] = \begin{cases} 1 & n = -1 \\ 0 & n \neq -1 \end{cases}$$

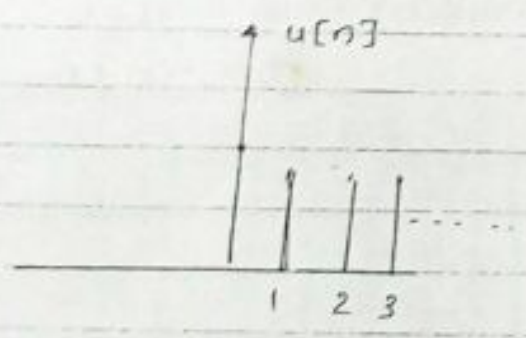


$f[n]$: combination of shifted discrete impulse signal

$$f[n] = 4\delta[n+2] - 2\delta[n+1] + 2\delta[n] - 1\delta[n-1] + 3\delta[n-2]$$

$$= f[-2]\delta[n-(-2)] + f[-1]\delta[n-(-1)] + f[0]\delta[n-0] + f[1]\delta[n-1] + f[2]\delta[n-2]$$

$$f[n] = \sum_{k=-\infty}^{\infty} f[k]\delta[n-k]$$



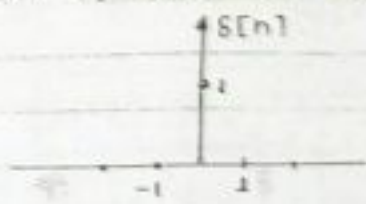
$$u[n] = \sum_{k=0}^{\infty} u[k]\delta[n-k]$$

$$u[n] = \sum_{k=0}^{\infty} \delta[n-k]$$

(37)

Date: _____
Page No: _____

Representation of $\delta[n]$ in term of $u[n]$



$$\delta[n] = u[n] - u[n-1]$$

simple value of from 1 to 0 at n

Simple value change from 0 to 1 hence coefficient is $(1-0)=1$ at $n=0$

$$\sum_{k=-\infty}^n \delta[k] = 0$$

$$\delta[n] + \delta[n-1] + \delta[n-2] + \dots + \delta[-\infty]$$

$$= 0 \quad n < 0$$

$$= 1 \quad n = 0 \quad \left[\delta[0] + \delta[-1] + \delta[-2] + \dots \right]$$

$$= 1 \quad n > 0 \quad \left[\delta[1] + \delta[0] + \delta[-1] + \dots \right]$$

Hence

$$\sum_{k=-\infty}^n \delta[k] = \begin{cases} 0 & n < 0 \\ 1 & n \geq 0 \end{cases}$$

So,

$$u[n] = \sum_{k=-\infty}^n \delta[k]$$

Relation b/w $\delta[n]$ and $u[n]$.

$$\delta[n] = u[n] - u[n-1]$$

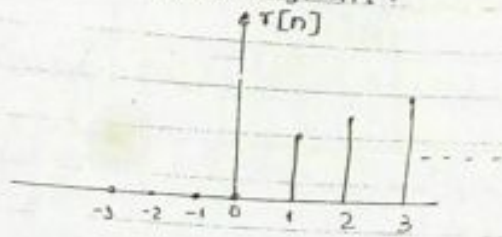
$$u[n] = \sum_{k=-\infty}^n \delta[k]$$

9n continuous time signal:-

$$\delta(t) = \frac{du}{dt}$$

$$u(t) = \int_{-\infty}^t \delta(\tau) d\tau$$

Discrete Time Ramp Signal:→



$$r[n] = \begin{cases} n & n \geq 0 \\ 0 & n < 0 \end{cases}$$

$$r[n] = \sum_{m=-\infty}^{\infty} m u[n-m]$$

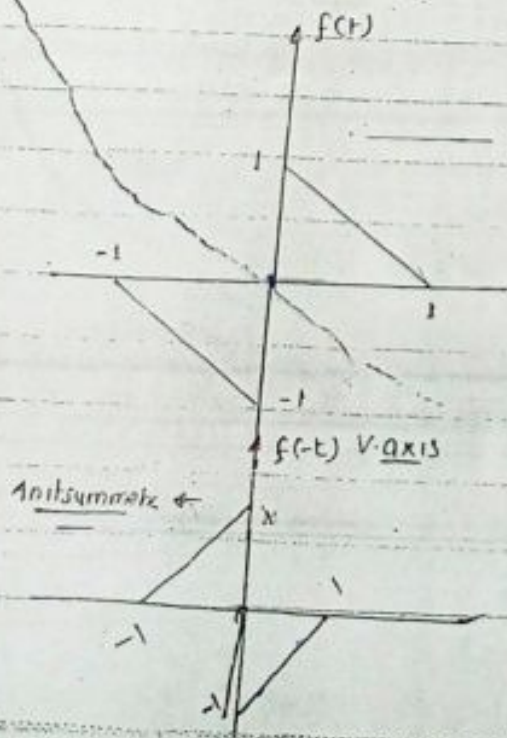
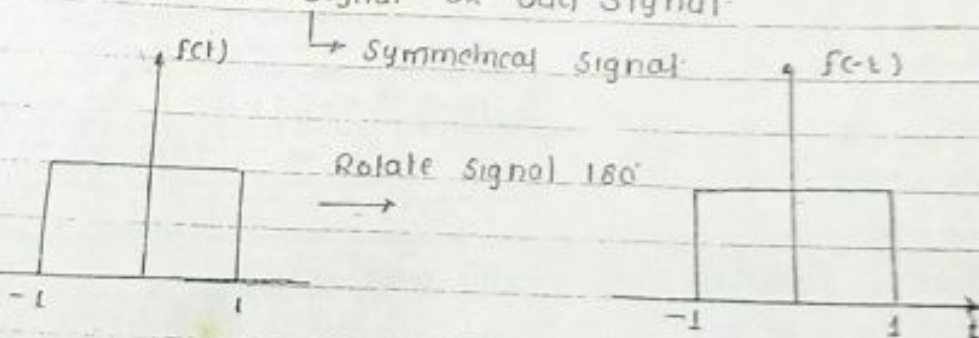
In continuous time signal

$$u(t) = \frac{dr(t)}{dt}$$

$$r(t) = \int_{-\infty}^t u(\tau) d\tau$$

Find out the Relation b/w DT unit Ramp and discrete time unit step signal.

3 Even Signal or odd signal



Rotate about t-axis

