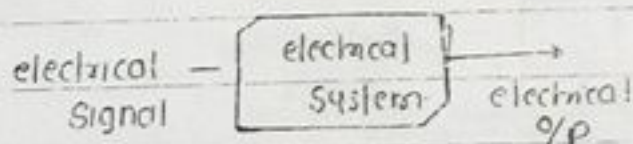
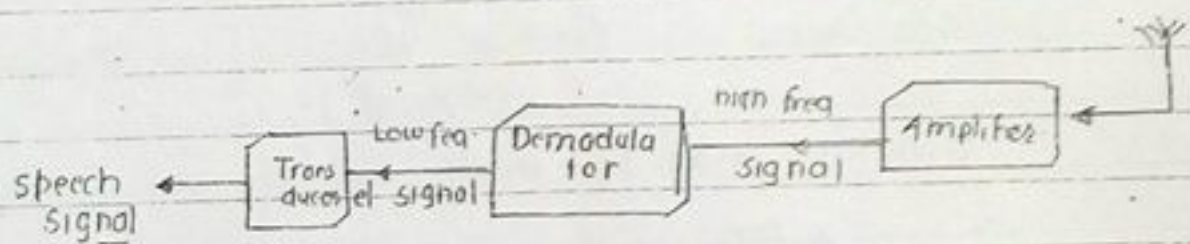
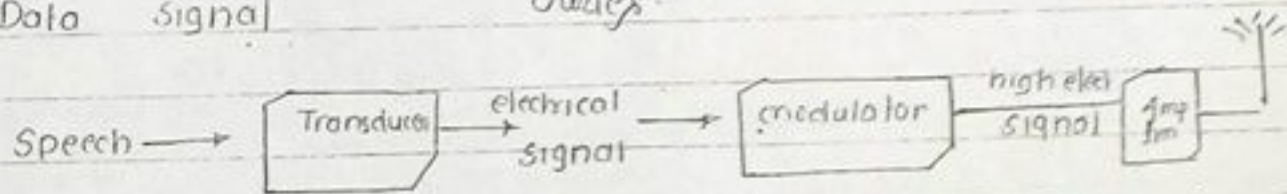


Speech signal	300 Hz to 3400 Hz
Audio signal	20 Hz to 20 kHz
Video signal	0 to 45 MHz [combination of Audio]
Data signal	varies



Fourier series

Fourier Transform

Laplace Transform

Z-Transform

Sampling Theorem

Book

Signal & system - Oppenheim

Signal & system - Simon Hykins

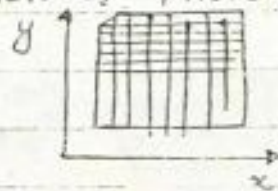
Signal & system - Schaum & series

(2)

Any Quantity having associated information called as signal electrical signal or either voltages or current which are both function of time

Signal is not always function of time.

Ex:- Information of photo.

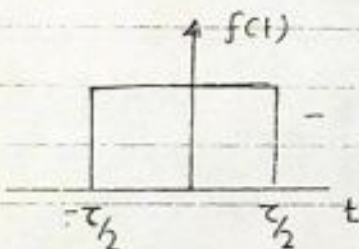


$f(x, y)$   
↳ function of photo is function of  $x, y$

Video signal is function of

$$f(x, y, t)$$

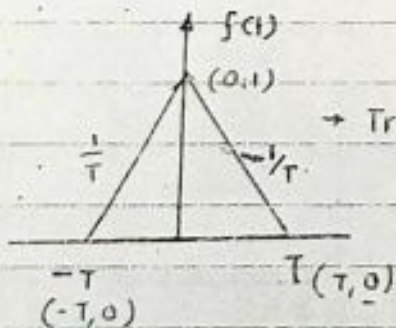
Signal is may be multi variable.



Rectangular pulse

In mathematical form

$$f(t) = \begin{cases} 1, & -\tau/2 < t < \tau/2 \\ 0 & \text{otherwise} \end{cases}$$



Triangular pulse

$$f(t) = mt + c$$

where  $m = \text{slope}$

$$f(t) = \frac{1}{T}t + c$$

$$0 = \frac{1}{T}(-T) + c$$

$$c = 1$$

$$\therefore \boxed{f(t) = -\frac{1}{T}t + 1}, -T \leq t \leq 0$$

eqn of sl. line

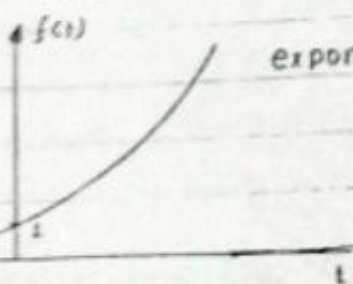
$$f(t) = -\frac{1}{T}t + c$$

$$0 = -\frac{1}{T}T + c$$

$$c = 1$$

$$\boxed{f(t) = -\frac{1}{T}t + 1}$$

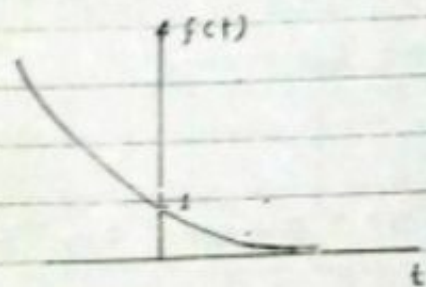
3



exponentially increasing signal.

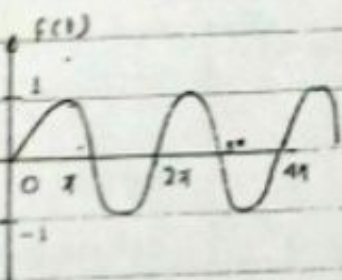
$$f(t) = e^{at}$$

$a$ : time constant



exponentially decreasing signal.

$$f(t) = e^{-at}$$



Sine signal.

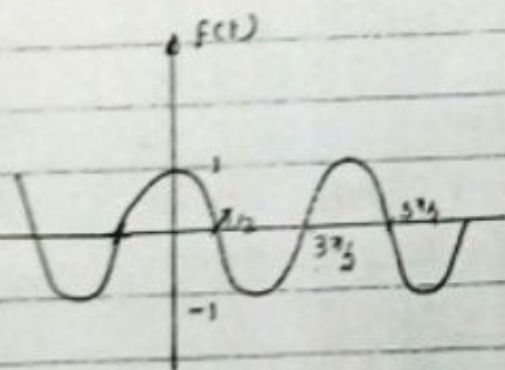
$$f(t) = \sin t$$

$$\omega = 1$$

Zero cross over =  $k\pi$

$$T = \frac{2\pi}{\omega} = 2\pi$$

where  $k$ : Integer



Cosine signal

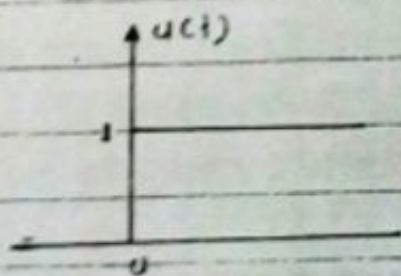
$$f(t) = \cos t$$

$$\text{Zero cross over} = \frac{\pi}{2} (2k+1)$$

where

$$k: 0, 1, \dots$$

Combination of sine and cosine are Sinusoidal signal

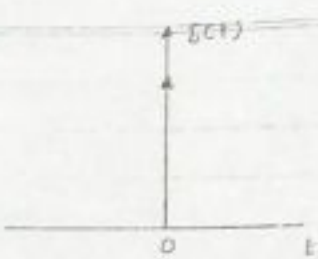


unit step signal.

$$u(t) = 1 \quad t \geq 0$$

$$= 0 \quad t < 0$$

9



Impulse Signal

$$\delta(t) = 0 \quad t \neq 0$$

$$= \infty \quad t = 0$$

$$\int_{-\infty}^{\infty} \delta(t) dt = 1$$

It is also called as unit impulse signal

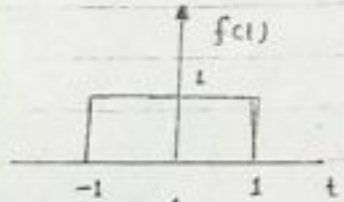
It is also called as Dirac Delta Function signal

Sketch the graph of signal

$f(t) = \frac{\sin t}{t}$  ;  $Sa(t)$  : Sampling Function

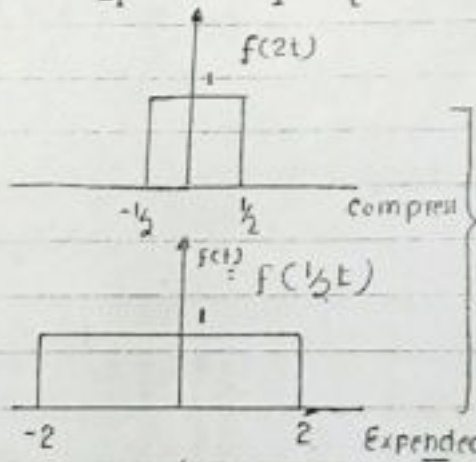
Solution :->

$$f(t) = \frac{\sin t}{t}$$

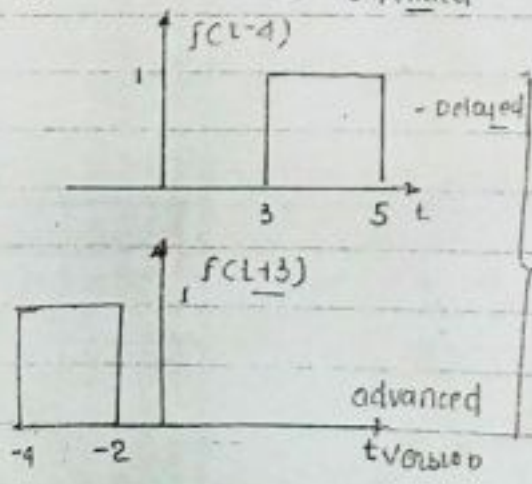


$$f(t) = 1 \quad -1 \leq t \leq 1$$

$$= 0 \quad \text{otherwise}$$



Time scale version  
or/compress version



Time Shifted version of  
Given signal

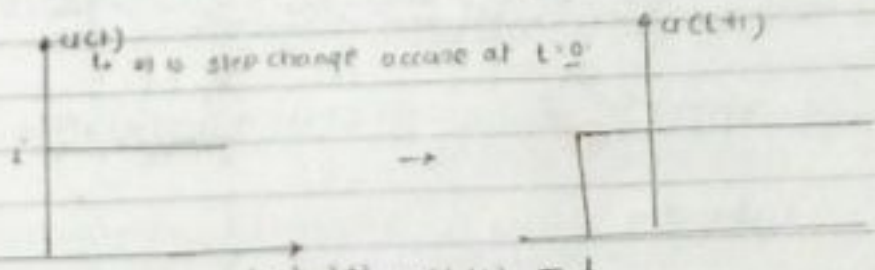
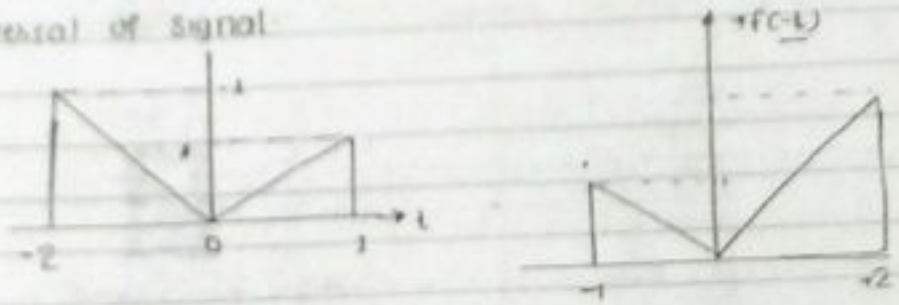
5

Date: \_\_\_\_\_  
Topic: \_\_\_\_\_

Rotation along  $y = ax$  by 180  
or mirror image across  $y = ax$   
Transformation of  $t \rightarrow -t$  is also

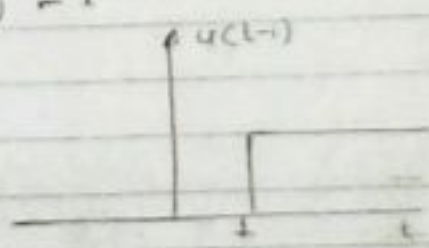
Add extra -ve sign to all  $t$  value

Time Reversal of signal

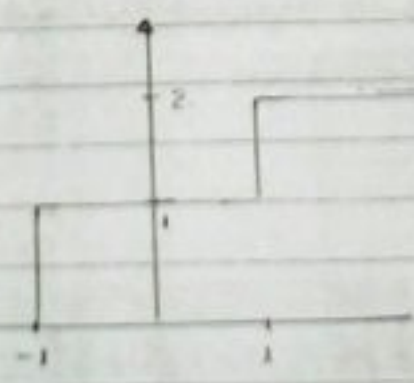


$u(t-1) = ?$

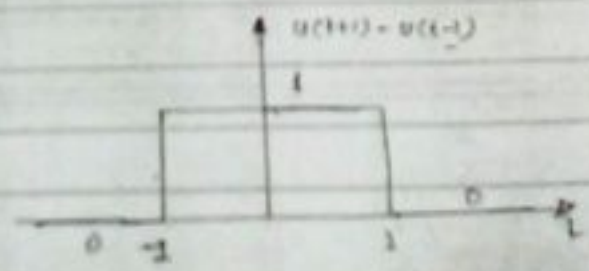
$u(t) \xrightarrow{t \rightarrow t-1}$



$u(t+1) + u(t-1)$



$u(t+1) - u(t-1)$

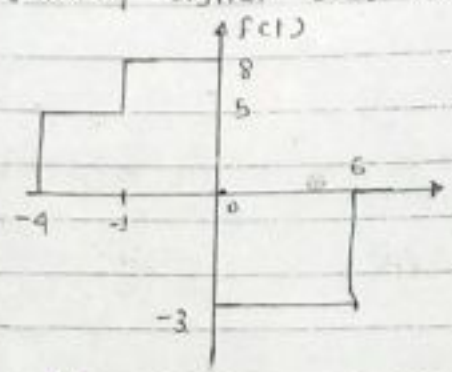


(6)



$3u(t+1) + 4u(t-1)$   
↳ amount of size change.

Represent following signal  $f(t)$  using shifted unit step signal



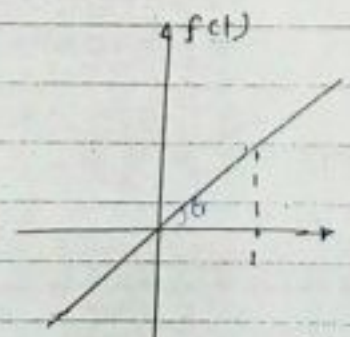
Wrong  $\leftarrow f(t) = 5u(t+4) + 3u(t+1) + (-3u(t-6))$

Right  $\leftarrow f(t) = 5u(t+4) + 3u(t+1) - 11u(t) + 3u(t-6)$

Sum of Co-efficient = 0.  
 $5 + 3 + 3 - 11 = 0$

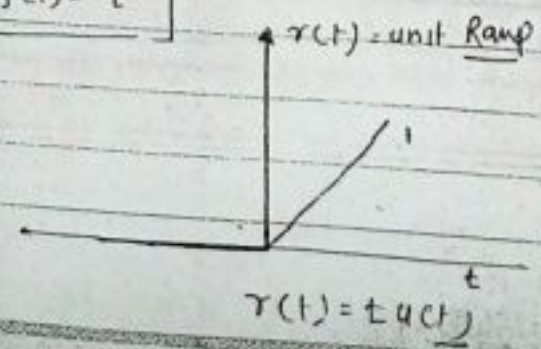
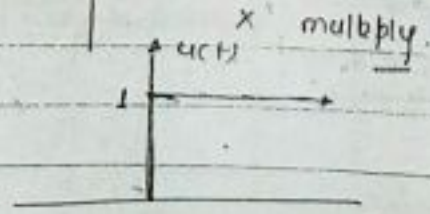
when ever signal have finite duration. then sum of co-efficient is zero.

The above rep. sum of Co-efficient will be always equal to zero as along as the duration of signal is finite



$f(t) = mt$   
 $= 1t$

$f(t) = t$



(7)

$$\begin{aligned} f(t) &= 1 & -1 \leq t \leq 1 \\ f(2t) &= 1 & -1 \leq 2t \leq 1 \\ & & -\frac{1}{2} \leq t \leq \frac{1}{2} \end{aligned}$$

$$\begin{aligned} f\left(\frac{1}{2}t\right) &= 1 & -1 \leq \frac{1}{2}t \leq 1 \\ &= 0 & \text{otherwise} \end{aligned}$$

$$\begin{aligned} f\left(\frac{1}{3}t\right) &= 1 & -2 \leq t \leq 2 \\ &= 0 & \text{otherwise} \end{aligned}$$

Time scaling operation:-

Transformation of  $t$  to  $at$  is called time scaling.

$$\boxed{f(t) \xrightarrow{L+at} f(at)}$$

- $a > 1$  + gt means compression.
- $a < 1$  + gt means expansion.

#  $f(t) = 1 \quad -1 \leq t \leq 1$   
 $= 0 \quad \text{otherwise}$

$t \rightarrow t-4$

$$\begin{aligned} f(t-4) &= 1 & -1 \leq t-4 \leq 1 \\ & & -1+4 \leq t \leq 1+4 \\ & & 3 \leq t \leq 5 \\ &= 0 & \text{otherwise} \end{aligned}$$

#  $f(t+3) = 1 \quad -1 \leq t+3 \leq 1$   
 $\quad \quad \quad -1-3 \leq t \leq 1-3$   
 $\quad \quad \quad -4 \leq t \leq -2$   
 $= 0 \quad \text{otherwise}$

②

Time shifting operation:-

$$\text{change } t \rightarrow t - t_0$$

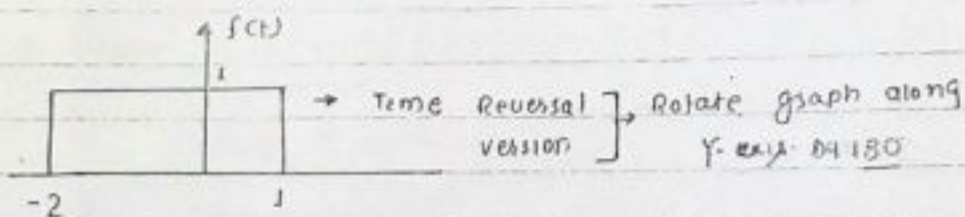
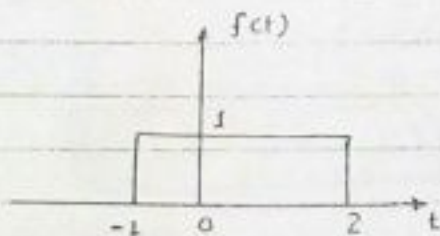
$$L \rightarrow L + t_0$$

$$f(t) \xrightarrow{L \rightarrow L - t_0} f(L - t_0) \rightarrow \text{Delayed shifting} \quad \text{or/ Right shift}$$

$$f(t) \xrightarrow{L \rightarrow L + t_0} f(L + t_0) \rightarrow \text{Advanced shifting} \quad \text{or/ Left shift}$$

→ If also add 'to' transformation → delayed.

→ Subtract 'to' transmission → advance shifting.



$$f(t) = 1, -1 \leq t \leq 2$$

$$= 0 \text{ otherwise}$$

$$f(-t) = 1 \quad -1 \leq -t \leq 2$$

multiply by -1

$$1 \geq t \geq -2$$

$$= 0 \text{ otherwise}$$

### Time Reversal

Transformation of  $t$  to  $-t$

$$f(t) \xrightarrow{t \rightarrow -t} f(-t)$$

$$f(-t) \xrightarrow{t \rightarrow -t} f(-(-t)) = f(t)$$

$$f(t+3) \xrightarrow{t \rightarrow -t} f(-t+3)$$

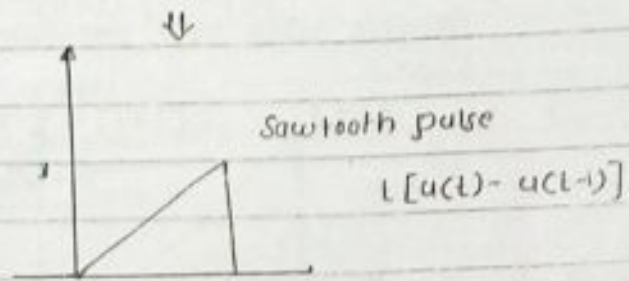
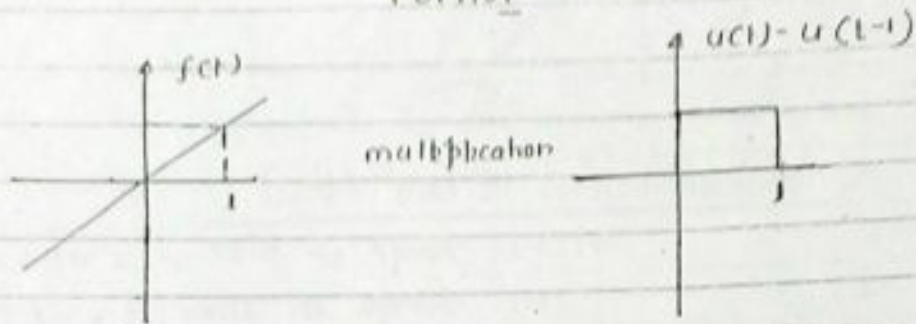


(9)

$$r(t) = t \quad t \geq 0$$

$$= 0 \quad t < 0$$

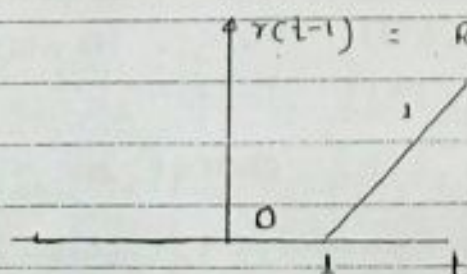
multiplication of  $u(t)$  the signal retain only in  $t > 0$  portion.



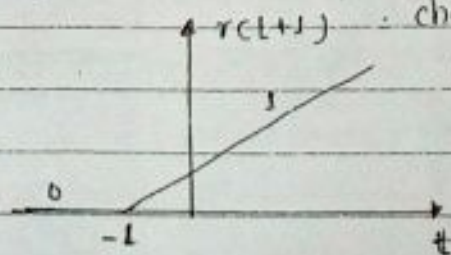
when ever signal is multiply by  $u(t)$  the variation occurring for  $t > 0$  is sustain the variation occurring for  $t < 0$  is completely remove.

A portion of a signal can be considered by multiply by a given signal with a Rectangular pulse having a value 1 in the Required portion of the given signal and a value zero in unrequired portion.

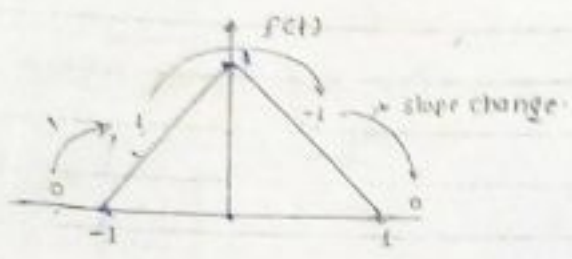
$r(t-1)$  : Repersahve change in slope at  $t=1$



$r(t+1)$  : change in slope at  $t=-1$   
 chang slope : 0 to +1



$r(t-t_0)$  = change of slope of 1 at  $t=t_0$



Representation of this signal:

$r(t+1)$ : change in slope by 1 at  $t = -1$

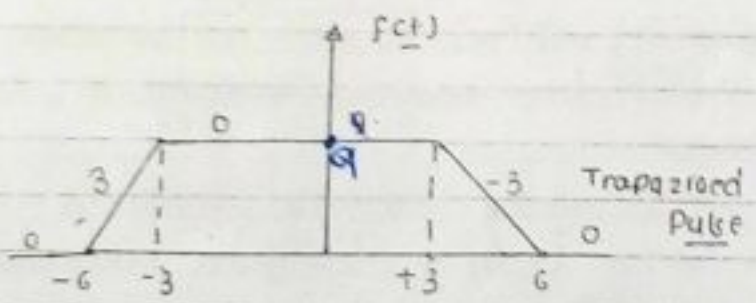
$r(t)$ : change in slope by 2 at  $t = 0$

$r(t-1)$ : change in slope by -1 at  $t = 1$

$$f(t) = r(t+1) - 2r(t) + r(t-1)$$

↓ amount of slope change

Represent the following following signal using shifted Ramp Function.



$r(t+6)$ : change in slope at  $t = -6$

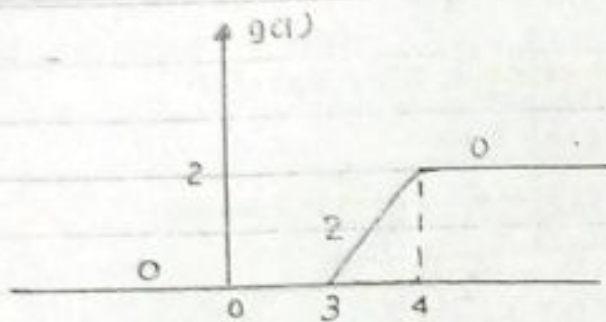
$r(t+3)$ : change in slope at  $t = -3$

$r(t-3)$ : change in slope at  $t = 3$

$r(t-6)$ : change in slope at  $t = 6$

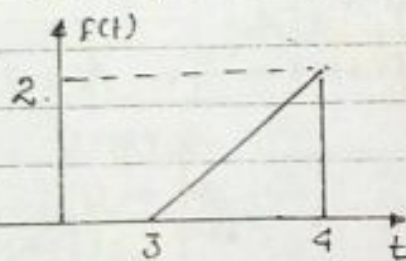
$$f(t) = (3-0)r(t+6) + (0-3)r(t+3) + (0-3)r(t-3) + (0-3)r(t-6)$$

(11)

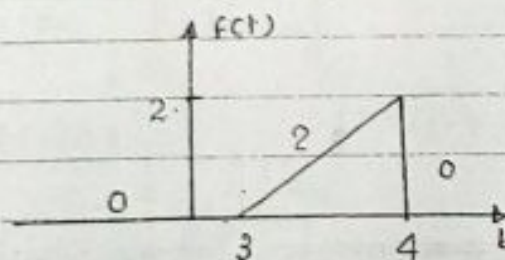


$$g(t) = 2r(t-3) - 2r(t-4)$$

Represent the following sawtooth pulse using shifted Ramp and shifted unit step signal.



For shifted Ramp



0, 2, 0 change in slope

$$f(t) = 2r(t-3) - 2r(t-4) - 2u(t-4)$$

For shifted unit step

For above signal  $f(t) + f(-t+7)$  is equal to

- (a)  ~~$r(t-3) - r(t-4)$~~
- (b)  $2[r(t-3) - r(t-4)]$
- (c)  $u(t-3) - u(t-4)$
- (d)  $2[u(t-3) - u(t-4)]$

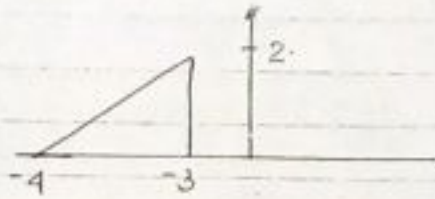
$f(-t+7)$

time shifting + Time reversal

(i) shifting

$t \rightarrow t+7$

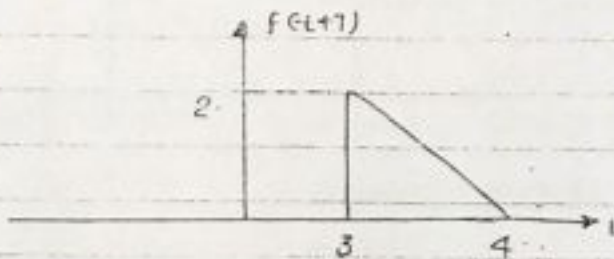
$f(t) \rightarrow f(t+7) \rightarrow$  subtract



(ii) Reversal

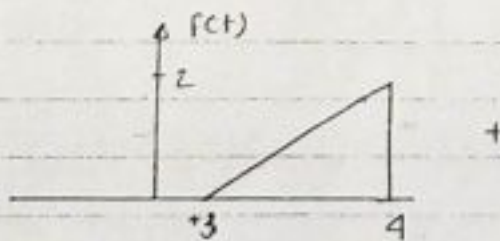
$t \rightarrow -t$

$f(t+7) \xrightarrow{-t} f(-t+7)$

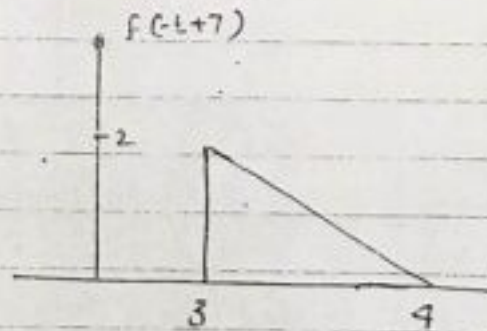


Now

$f(t) + f(-t+7)$

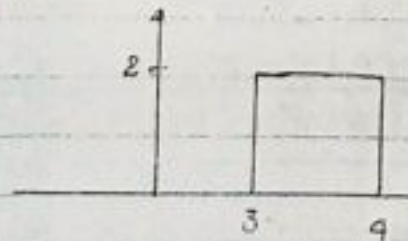


+



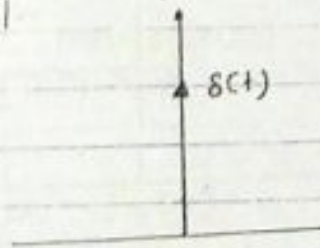
$f(t) = p(t) [u(t+3) - u(t+4)]$   
 $= 2u(t) [u(t+3) - u(t+4)]$

$\Downarrow$



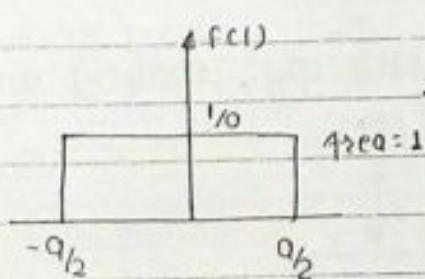
$f(t) + p(-t+7) = 2u(t-3) - 2u(t-4)$

# Unit Impulse signal

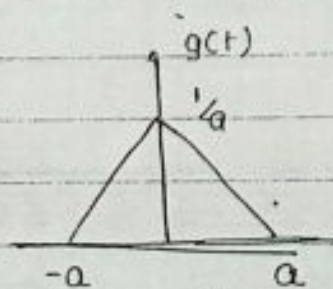
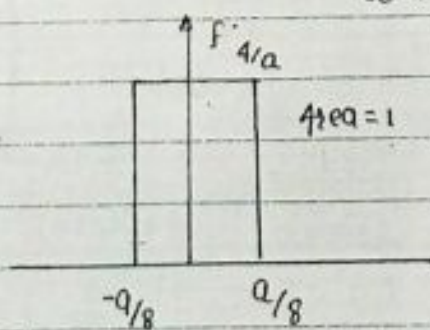
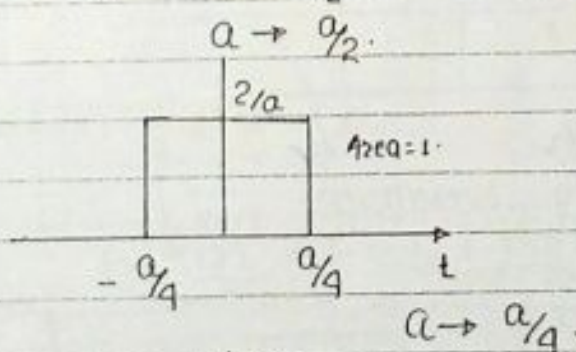


$$\delta(t) = \begin{cases} 0 & t \neq 0 \\ \infty & t = 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(t) dt = 1$$



$$\lim_{a \rightarrow 0} f(t) = \delta(t)$$



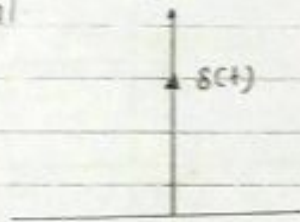
$$\text{area} = \frac{1}{2} \times 2a \times \frac{1}{a} = 1$$

$$\lim_{a \rightarrow 0} g(t) = \delta(t)$$

$\text{area} = 1$

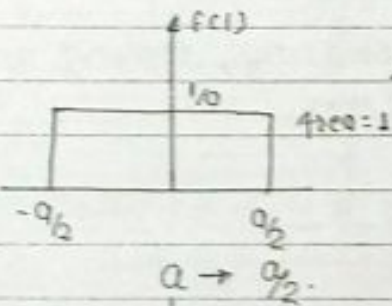
(3)

Unit Impulse signal

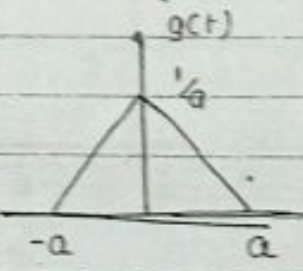
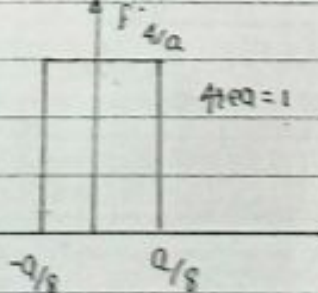
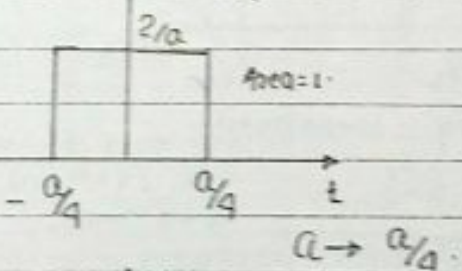


$$\delta(t) = \begin{cases} 0 & t \neq 0 \\ \neq 0 & t = 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(t) dt = 1$$



$$\lim_{a \rightarrow 0} f(t) = \delta(t)$$

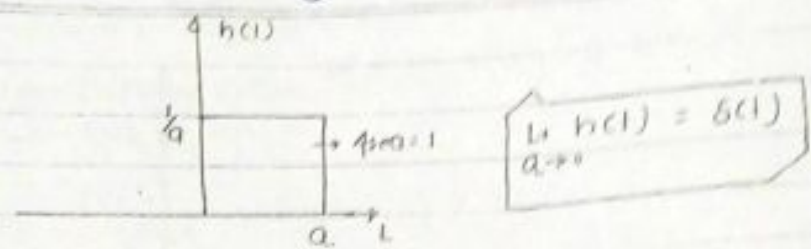


$$\text{Area} = \frac{1}{2} \times 2a \times \frac{1}{a} = 1$$

$$\lim_{a \rightarrow 0} g(t) = \delta(t)$$

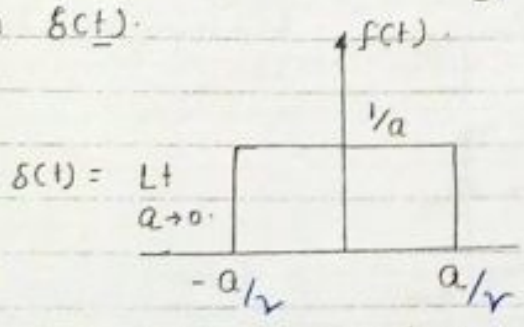
Area = 1

(14)



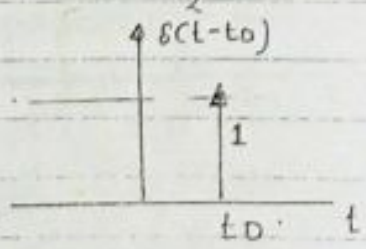
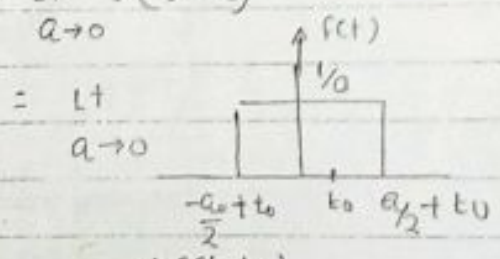
$\delta(t)$  signal can be approached by using a wide variety of signal that is  $\delta(t)$  does not have a unique definition hence it is also called as a Generalised Function.

operation of Time shifting, Scaling and time Reversal on  $\delta(t)$ .



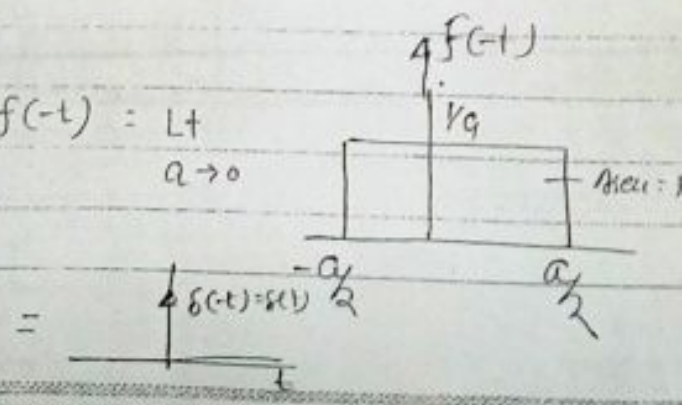
Time shifting operation:-

$$\delta(t-t_0) = \lim_{a \rightarrow 0} f(t-t_0)$$



Time Reversal-

$$\delta(-t) = \lim_{a \rightarrow 0} f(-t) = \lim_{a \rightarrow 0} f(t)$$

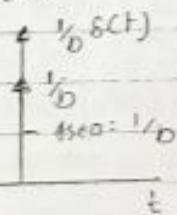
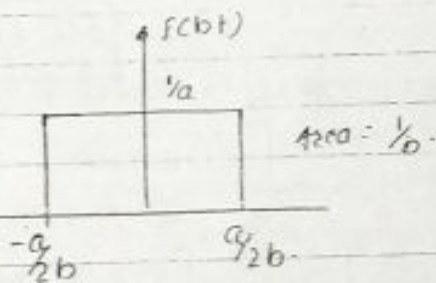


There is no effect of time reversal on the  $\delta(t)$ .

Time scaling:

$$\delta(bt) = \int_{-\infty}^{\infty} f(ct) \delta(bt) dt$$

$$= \int_{-\infty}^{\infty} f\left(\frac{t}{b}\right) \delta(t) dt$$



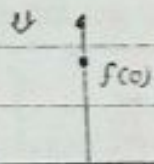
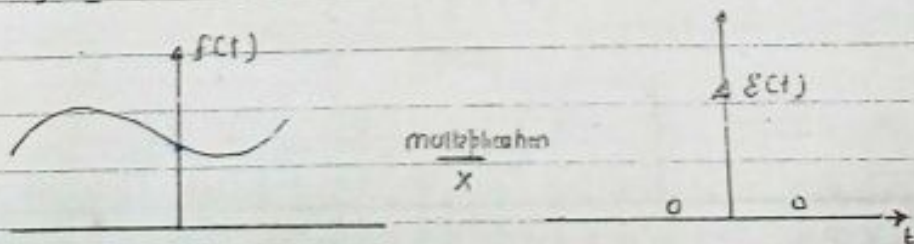
□  $\delta(bt) = \frac{1}{|b|} \delta(t)$

□  $\delta(-t) = \delta(t)$  bcz there is no effect of time reversal

↑ is +ve @ -ve

$$\delta(bt) = \frac{1}{|b|} \delta(t)$$

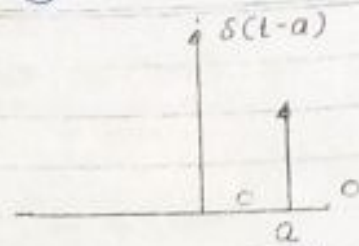
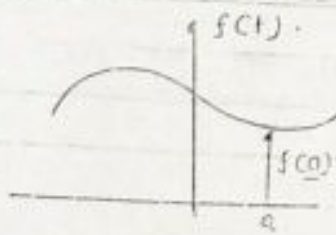
if time scale in impulse it, influence area of corresponding impulse signal.



$$f(t) \delta(t) = f(0) \delta(t)$$



(16)



$$f(t) \delta(t-a) = f(a) \delta(t-a)$$

$$\int_{-\infty}^{\infty} f(t) \delta(t) dt$$

$$= \int_{-\infty}^{\infty} f(0) \delta(t) dt$$

$$= f(0) \int_{-\infty}^{\infty} \delta(t) dt$$

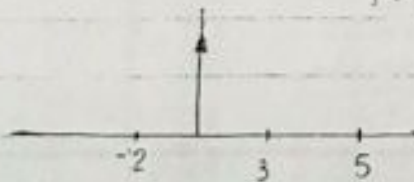
$$\int_{-\infty}^{\infty} f(t) \delta(t) dt = f(0)$$

$$\int_{-\infty}^{\infty} f(t) \delta(t-a) dt$$

$$= \int_{-\infty}^{\infty} f(a) \delta(t-a) dt$$

$$= f(a) \int_{-\infty}^{\infty} \delta(t-a) dt$$

$$= f(a)$$

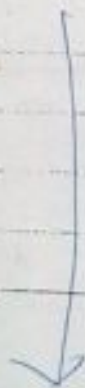


$$\int_{-\infty}^{\infty} \delta(t) dt = 1$$

$$\int_{-2}^3 \delta(t) dt = 1$$

$$\int_3^5 \delta(t) dt = 0$$

$$\int_0^{0^+} \delta(t) dt = \underline{\underline{1}}$$



(17)

Evaluate the value of following integral.

$$(i) \int_{-\infty}^{\infty} \sin(t - \pi/2) \delta(t) dt = \sin(0 - \pi/2) = -1$$

$$(ii) \int_{-\infty}^{\infty} \sin(t - \pi/2) \delta(t - \pi) dt = \sin(\pi - \pi/2) = \sin \pi/2 = 1$$

$$(iii) \int_{-\infty}^{\infty} \sin(t - \pi/2) \delta(3t - \pi) dt = \int_{-\infty}^{\infty} \sin(t - \pi/2) \delta(3(t - \pi/3)) dt = \frac{1}{3} \delta(t - \pi/3)$$

$$\rightarrow (iv) \int_{-\pi/6}^{\pi/6} \sin(t - \pi/2) \delta(3t - \pi) dt = \frac{1}{3} \int_{-\infty}^{\infty} \sin(t - \pi/2) \delta(t - \pi/3) dt$$

$$= \frac{1}{3} \sin(\pi/3 - \pi/2)$$

$$= \frac{1}{3} \sin(-\pi/6)$$

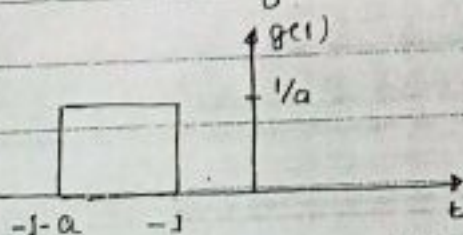
$$= \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$$

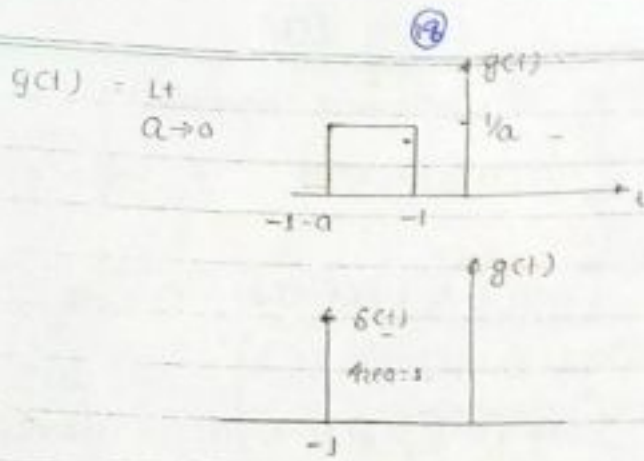
$$(iv) \int_{-\pi/6}^{\pi/6} \sin(t - \pi/2) \delta(3t - \pi) dt = \frac{1}{3} \int_{-\pi/6}^{\pi/6} \sin(t - \pi/2) \delta(t - \pi/3) dt$$

$$= \underline{\underline{0 \text{ Ans}}}$$

Evaluate the value of following integral.

$$(i) \int_{-\infty}^{\infty} \frac{\sin \frac{a}{2} (t-2)}{t^2+4} g(t) dt$$

where  $g(t)$  is following signal as  $a \rightarrow 0$ 



$$\int_{-\infty}^{\infty} f(t) \delta(t-a) dt = f(a)$$

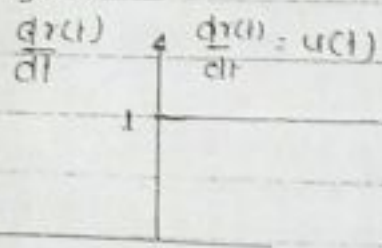
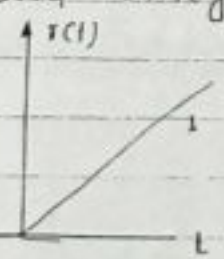
So now given integral:

$$\int_{-\infty}^{\infty} \frac{\sin \frac{\pi}{2}(t-2)}{t^2+1} \cdot \delta(t+1) dt$$

$$= \frac{\sin \left( \frac{\pi}{2}(-1-2) \right)}{(-1)^2+1}$$

$$= \frac{1}{5}$$

### Derivative and Integral of signal.



$$y = mx + c$$

$$\frac{dy}{dx} = m + 0$$

$$u(t) = \frac{d}{dt} r(t)$$

$$\frac{dy}{dx} = m$$

$$\frac{d r(t)}{dt} = 1 \text{ unit step}$$

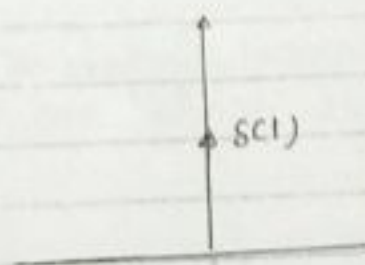
$$r(t) = \text{integral of } u(t)$$



$$\int_{-\infty}^{\infty} f(t) dt = \begin{matrix} \int \rightarrow \text{finite} \\ \int \rightarrow \text{infinite} \end{matrix}$$

$$\int_{-\infty}^L f(t) dt = g(t) \cdot \text{integral of } f(t)$$

$$\begin{aligned} u(t) &= \frac{d}{dt} r(t) \\ r(t) &= \int_{-\infty}^t u(t) dt \end{aligned}$$



$$\int_{-\infty}^t s(t) dt = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

$$u(t) = \int_{-\infty}^t s(t) dt$$

$$u(t) = \int_{-\infty}^t s(t) dt$$

$$s(t) = \frac{d}{dt} u(t)$$

Derivative of discontinuous signal is defined

Problem:- find integral of following signal

