If $f(t)$ is non sysmetric then

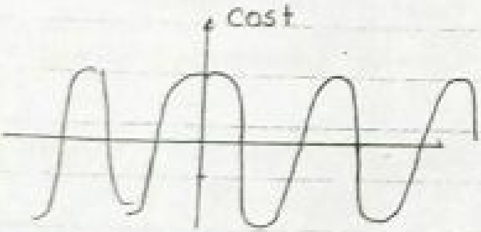
$$\boxed{f(t) = f_e(t) + f_o(t)}$$

where

$$f_e(t) = \frac{f(t) + f(-t)}{2}$$

$$f_o(t) = \frac{f(t) - f(-t)}{2}$$

⇒ <u>Example</u>



even signal

$f(t) = \cos t$

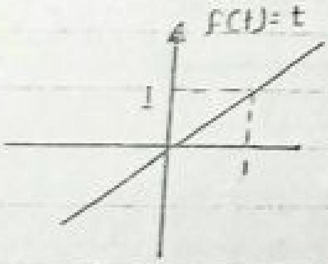
$f(-t) = \cos(-t)$

$\boxed{f(t) = f(-t)}$ even signal

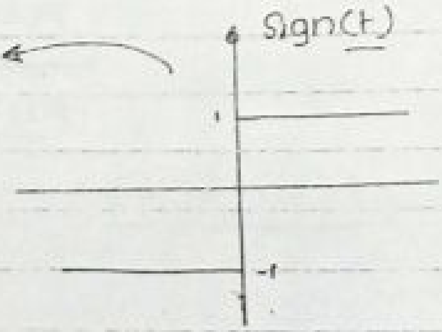
$\boxed{\delta(-t) = \delta(t)}$ even signal

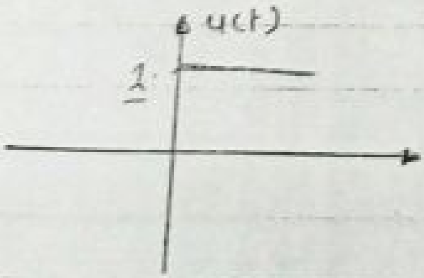
2. sint is odd signal

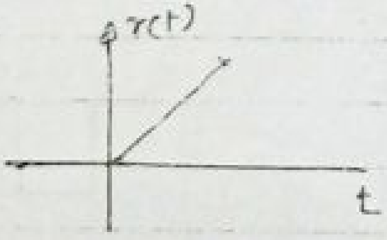
3.



odd signal



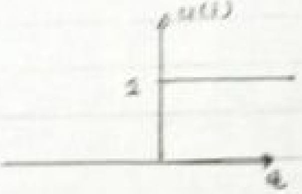


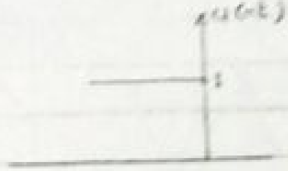


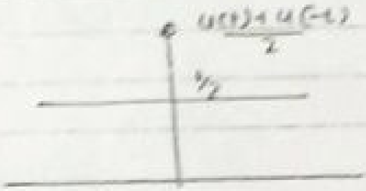
neither even or nor odd

$$f_e(t) = \frac{f(t)+f(-t)}{2}$$

$$f_o(t) = \frac{f(t)-f(-t)}{2}$$

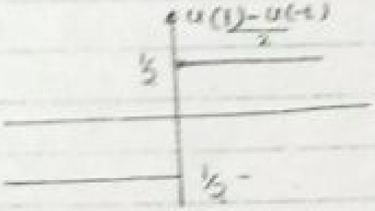






$$u_e(t) = \frac{u(t)+u(-t)}{2} =$$

$$= \frac{1}{2}$$



$$u_o(t) = \frac{u(t)-u(-t)}{2} =$$

$$= \frac{1}{2} x(t)$$

$$= \frac{1}{2} \text{sgn}(t)$$

so -

$$u(t) = u_e(t) + u_o(t)$$

** $\boxed{u(t) = \frac{1}{2} + \frac{1}{2}\text{sgn}(t)}$

$u(t) = 1 \quad t \geq 0$
$\quad = 0 \quad t < 0$

$u(t) = 1 \quad t > 0$
$\quad = 0 \quad t \leq 0$

$u(t) = 1 \quad t > 0$
$\quad = 0 \quad t < 0$
$\quad =$ Indeterminable $\quad t = 0$

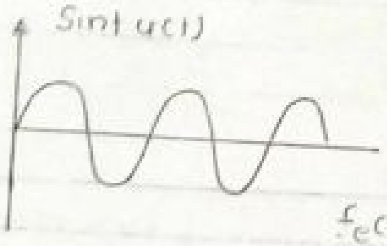
– exact Definition of $u(t)$

$u(t) = 1 \quad t > 0$
$\quad = 0 \quad t < 0$
$\quad = 1/2 \quad t = 0$

Find the even part of signal

$$f(t) = \sin t\, u(t).$$

Solution:→



$$f_e(t) = \frac{f(t) + f(-t)}{2}$$

$$= \frac{\sin t\, u(t) + \sin(-t)\, u(-t)}{2}$$

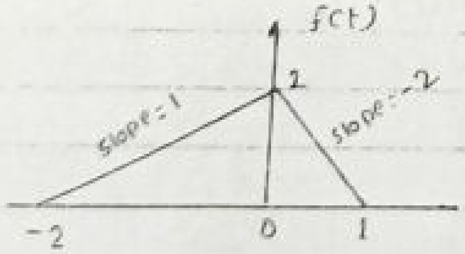
$$= \frac{\sin t\, u(t) - \sin t\, u(-t)}{2}$$

$$= \sin t \left[\frac{u(t) - u(-t)}{2}\right]$$

$$= \frac{1}{2} \sin t\,(\mathrm{sgn}\, t)$$

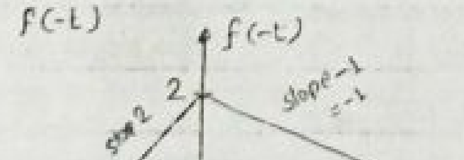
$$\boxed{f_e(t) = \frac{1}{2} \sin \mathrm{sgn} t}$$

⇛ Find the odd part of signal



$$f_o(t) = \frac{f(t) - f(-t)}{2}$$

f(-t)

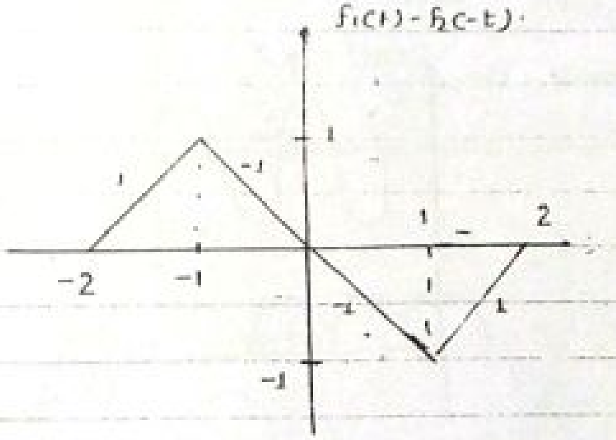


$f_1(t) = m_1 t + c_1$

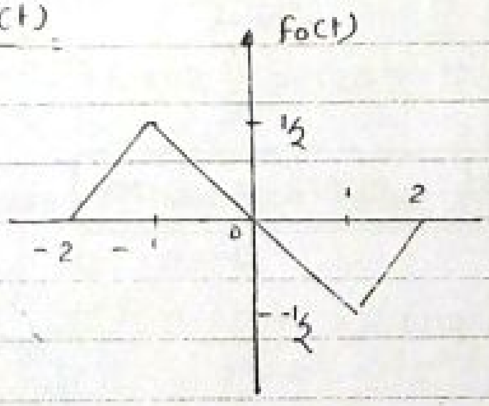
$f_2(t) = m_2 t + c_2$

$f(t) - f_2(t) = (m_1 - m_2)t + c_1$

$f_1(t) - f_2(-t)$

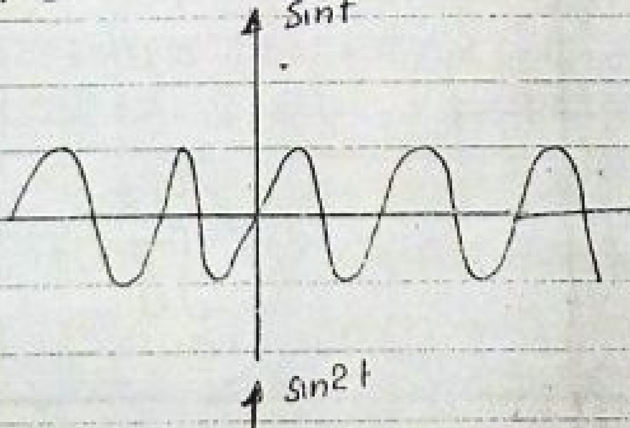


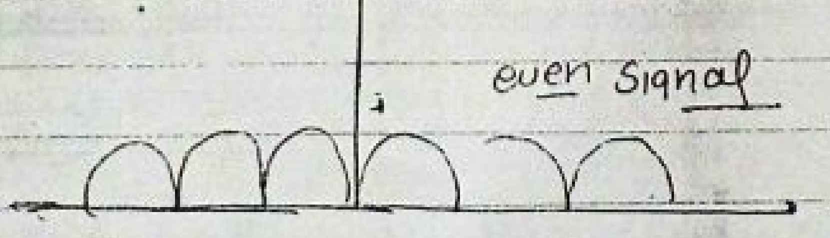
$\therefore \dfrac{f_1(t) - f_2(t)}{2}$



$\boxed{\text{odd} \times \text{odd signal} = \text{even signal}}$

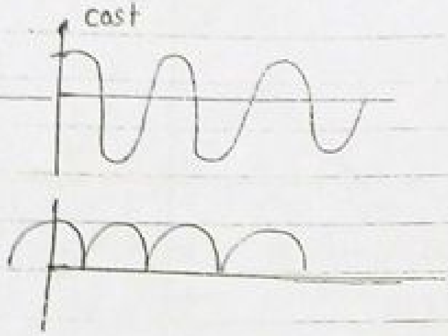
$\sin t \cdot \sin t = \sin^2 t$





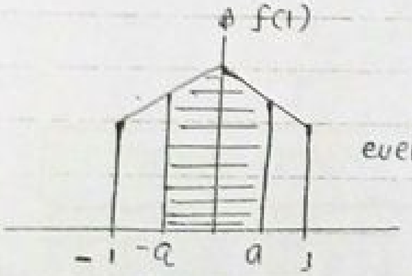
$\cos t \cdot \cos t = \cos^2 t$

even · even → even



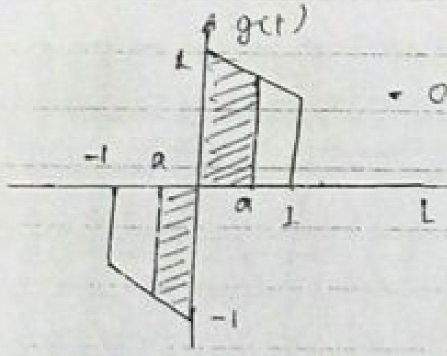
$\sin t \cdot \cos t = \frac{1}{2} \sin 2t$

$\boxed{\text{odd} \cdot \text{even} = \text{odd}}$



even signal

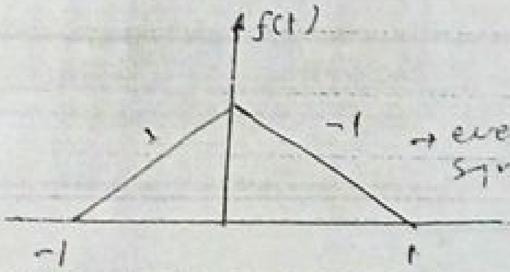
$$\int_{-a}^{a} f(t)\,dt = 2\int_{0}^{a} f(t)\,dt$$



→ odd signal

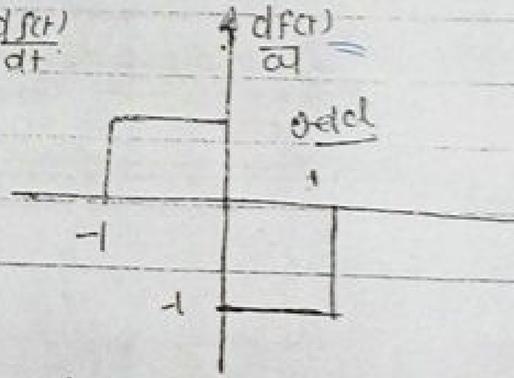
$$\int_{-a}^{a} g(t)\,dt = 0$$

$g(t)$: odd signal



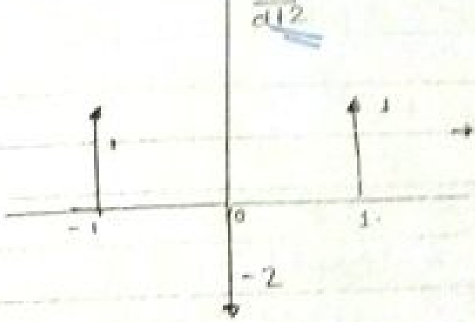
→ even sig

$\frac{df(t)}{dt}$



odd

After differentiation even signal become odd signal and wise versa.



→ even function

$$\frac{d}{dt}(\sin t) = \cos t$$
(odd) (even)

$$\frac{d}{dt}\cos t = -\sin t$$
(even) (odd)

Evenness and oddness of complex valued value signal.

$$c = a + jb$$
$$c^* = a - jb$$
$$|c| = \sqrt{a^2+b^2} = |c^*|$$
$$\angle c = \tan^{-1}\left|\frac{b}{a}\right| = -\angle c^*$$

* $f(t)$ = complex valued or complex even.

$$f(t) = f^*(-t)$$ = even conjugate OR/ conjugate summetric

or $f^*(t) = f(-t)$

$$f(t) = -f^*(-t)$$: Complex odd signal OR/ conjugate antisymmetric signal / odd conjugate signal.

$$f(t) = e^{jt}$$
$$f^*(t) \quad e^{-jt}$$
$$f^*(-t) = e^{-j(-t)}$$
$$= e^{jt}$$
$= f(t)$ → even conjugate signal

$f(t) = te^{jt}$

$f^*(t) = te^{-jt}$

$f^*(-t) = -te^{jt}$

$= f(t)$

↳ odd conjugate

⇒ e^{jt} → even conjugate

$$\boxed{e^{jt} = \underbrace{\cos t}_{\text{even}} + j\underbrace{\sin t}_{\text{odd}}}$$

Ex: $f(t) = te^{jt}$

$= t[\cos t + i t \sin t]$

$$= \underbrace{\underbrace{t}_{\text{odd}}\underbrace{\cos t}_{\text{even}}}_{\text{odd}} + \underbrace{\underbrace{i t}_{\text{odd}}\underbrace{\sin t}_{\text{odd}}}_{\text{even}}$$

For the even conjugate signal Real part is even
Imaginary Part is odd.

For the odd conjugate signal Real part is odd on:

P/10
17.

- $f_{oc}[n]$: odd congugate part
$$: \frac{f[n] - f^*[-n]}{2}$$

Given that $f[n] : [-4-5j, \underset{\uparrow}{1+j2}, 4]$

$$f^*[n] : [-4+5j \quad \underset{\uparrow}{1-2j} \quad 4]$$

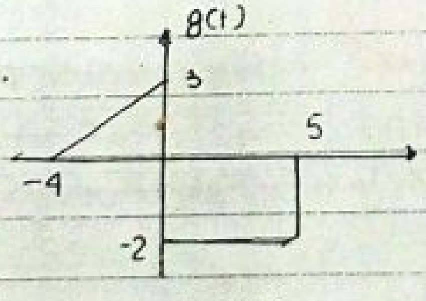
$$f^*[-n] : [4 \quad \underset{\uparrow}{1-2j} \quad -4+5j]$$

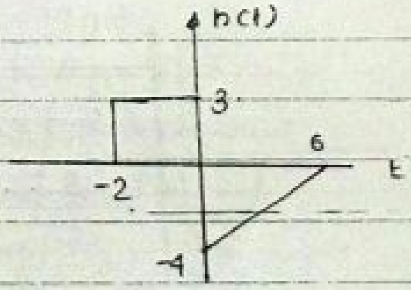
$$f_{oc}[n] : \{-4-j2.5 \quad \underset{\uparrow}{2j} \quad 4-j2.5\}.$$

$f_{oe}[n]$: even congugate part
$$: \frac{f[n] + f^*[-n]}{2}$$

$$f_{oe} : \{-j2.5 \quad \underset{\uparrow}{1} \quad j2.5\}.$$

A complex valued signal $f(t)$ is defined with a Real part $h(t) + h(-t)$ and an imiginary part $g(t) - g(-t)$ where $g(t)$ and $h(t)$ are defined as shown below:→

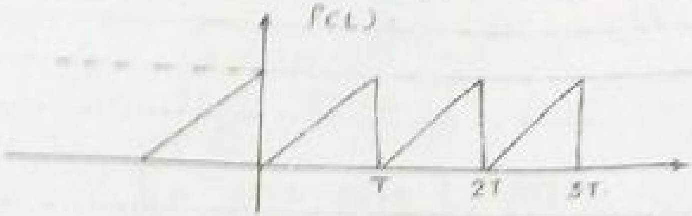




Then signal $f(t)$
(i) even congugate
(ii) odd Conjugate
(iii) Neither even nor odd.
(iv) Can not comment on Symmetry

$$f(t) = \underbrace{\{h(t) + h(-t)\}}_{\text{even}} + j\underbrace{\{g(t) - g(-t)\}}_{\text{odd}}$$

Periodic and Non Periodic Signal



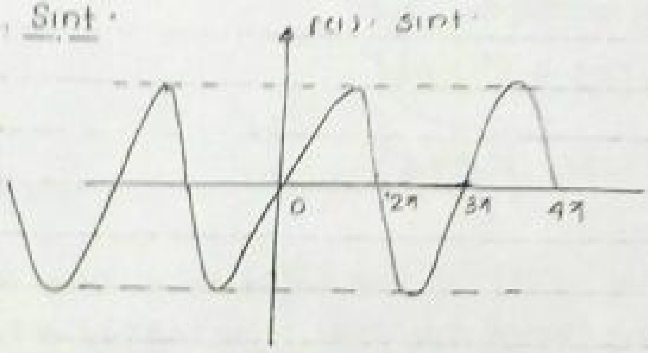
T : Fundamental period.

KT = time period

$\boxed{f(t+KT) = f(t)}$ → condition for a Periodic Signal.

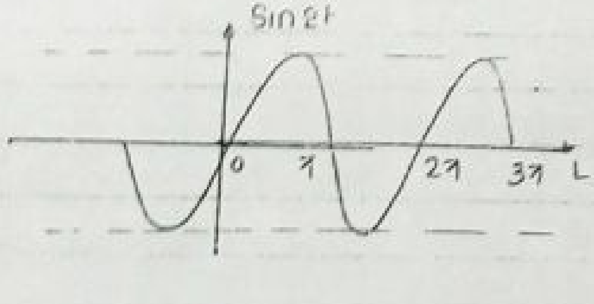
K → Integer

Sint ·



Fundamental period $= 2\pi$

is Periodic signal



Fundamental period $= \pi$.

$\frac{2\pi}{2} = \pi$.

Fundamental time period of Sin KT

where K = Integer

$= \frac{2\pi}{K}$

$\frac{2\pi}{K} \times K = 2\pi$ orignal fundamental time Period.

All have common time Period $= 2\pi$

sint, sin2t, sin3t

Area under a complete cycle of sint is zero

$$\int_0^{2\pi} \sin t = 0$$

$$\int_0^{4\pi} \sin t = 0$$

$\sin t \quad \sin 2t \quad \sin 3t \; \ldots\ldots \rightarrow$ Angular velocity ω_0

So, Repersent as:

$\sin \omega_0 t$

	Time period	
$\sin \omega_0 t$	$\frac{2\pi}{2\omega_0} = \frac{2\pi}{\omega_0}$	All the signal have a Comman time period $= \frac{2\pi}{\omega_0}$
$\sin 2\omega_0 t$	$\frac{2\pi}{2\omega_0} = \frac{\pi}{\omega_0}$	
$\sin 3\omega_0 t$	$\frac{2\pi}{3\omega_0} = \frac{2\pi}{3\omega_0}$	
$\vdots$		
$\sin k\omega_0 t$	$\frac{2\pi}{k\omega_0}$	

$$\int_0^{2\pi/\omega_0} \sin \omega_0 t = 0$$

$$\int_0^{2\pi/\omega_0} \sin 2\omega_0 t = 0$$

$$\int_0^{2\pi/\omega_0} \sin k\omega_0 t = 0$$

where k = Integer.

similary

$$\int_0^{2\pi/\omega_0} \cos k\omega_0 t = 0$$

Evaluate the following Integral

(i) $\int_0^{2\pi/\omega_0} \sin\omega_0 t \, \sin 2\omega_0 t \, dt$

(ii) $\int_0^{2\pi/\omega_0} \cos\omega_0 t \, \cos 2\omega_0 t \, dt$

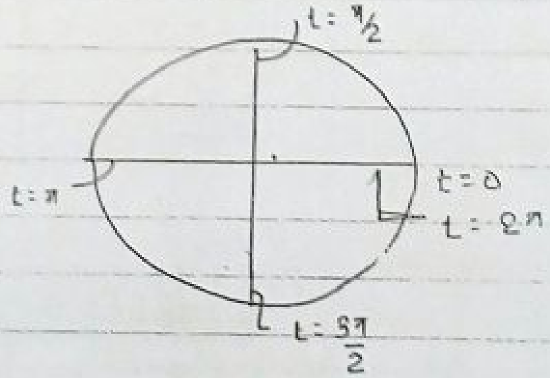
(iii) $\int_0^{2\pi/\omega_0} \sin\omega_0 t \cdot \cos 2\omega_0 t \, dt$.

$\left.\begin{array}{l}\sin\omega_0 t \\ \cos\omega_0 t\end{array}\right\}$ Alway periodic with Fundamental period $= \frac{2\pi}{\omega_0}$.

$e^{jt} = \cos t + j\sin t$.

$= 1\angle t \rightarrow$ phasor Quantity. $e^{j\angle\theta}$ = phasor

Locus of the e^{jt} is Circle.

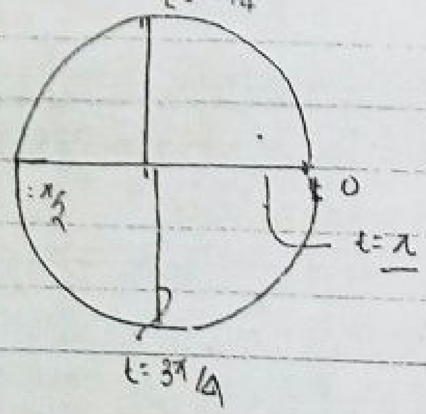


$e^{2jt} = \cos 2t + j\sin 2t$

$= 1\angle 2t$

$=$

Fundamental time Period $= \pi$.

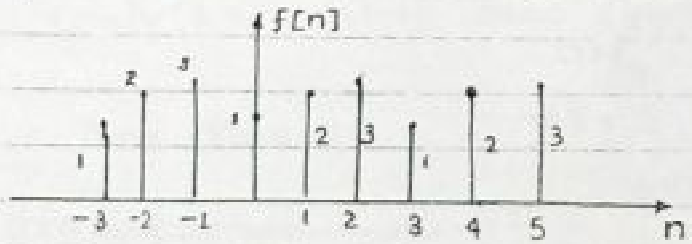


e^{jw_0t} Time Period = $\frac{2\pi}{w_0}$

w_0 = angular velocity

e^{-jw_0t} & e^{jw_0t} are both have same angular velocity an same period but only Difference is compliting of direction

Periodicity of Discreate time signal:→



Fundamental time period of Discreate time signal [N] = 3.

Fundamental time period is defined as the minimum no of sample taken by sample to repeat it self. It is denoted by N and N is always an integer be'z it measure no of sample.

The fundamental of period of a Continuous time periodic signal can be assume any real value but fundamental period of a Discreate time periodic signal is an Integer

e^{jw_0n}

$e^{j(\frac{\pi}{4})n} = 1 \angle \frac{\pi}{4}n$

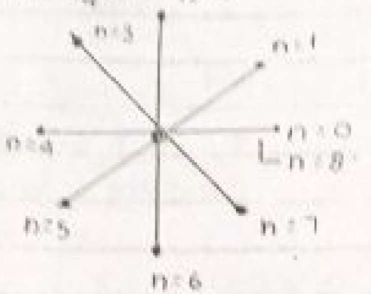
$e^{j(\pi)n}$

$e^{j(3\pi)n}$

$e^{j(5\pi)n}$

Plot of $e^{j\frac{\pi}{4}n}$ — Period (N)

$= 1\angle \frac{\pi}{4}n$



8

$\frac{2\pi}{\frac{\pi}{4}} = 8$

Plot of $e^{j\pi n}$

$= 1\angle \pi n$

2

$= \frac{2\pi}{\pi} = 2$

Plot of $e^{j3\pi n}$

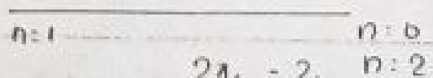
$= 1\angle 3\pi n$

2

$= \frac{2\pi}{3\pi} = 2/3$

$e^{j5\pi n}$

$= 1\angle 5\pi n.$

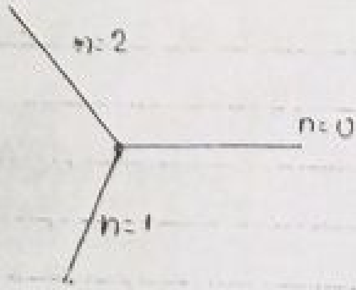


2.

$2\pi/5\pi = 2/5$

$e^{j\sqrt{2}\pi n}$

$= 1\angle\sqrt{2}\pi n$



→ Aperiodic Signal

A Discrete time may be periodic or non periodic.

- Addition of two saw tooth pulse with one with +ve slope and one with -ve slope then alway obtain a Rectangular pulse.

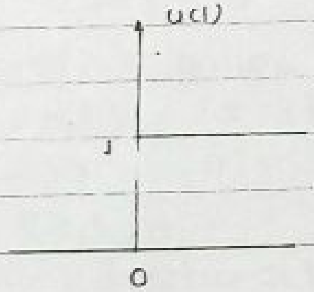
➡ Realize $f(-t+7)$ in above problem first by Reversing and then by shifting.

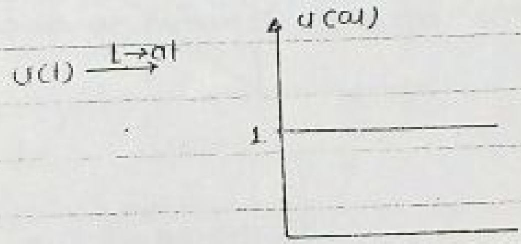
$t \cdot u(t) = r(t)$: Ramp signal

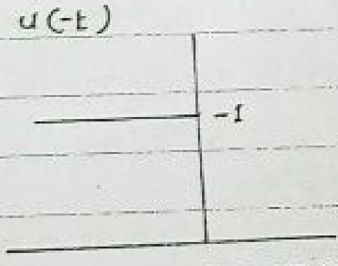
$(t-3)u(t-3) = r(t-3)$ Ramp signal

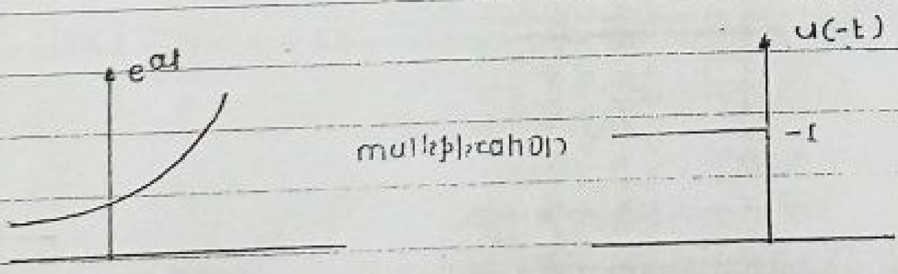
$(t-4)u(t-4) = r(t-4)$ Ramp signal.

Time Scaling & Time Reversal.

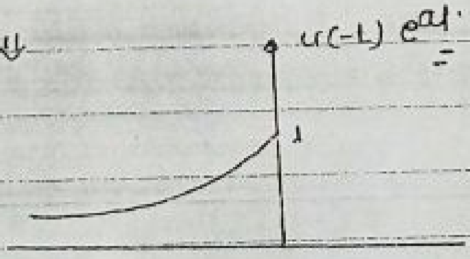








⇓



Sketch Graph of following signal

(i) $e^{at} u(t)$

(ii) $e^{-at} u(t)$

(iii) $e^{-at} u(-t)$

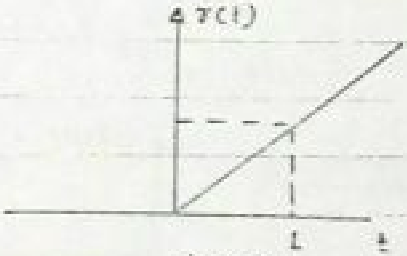
(iv) $\sin t \, u(t)$

(v) $\cos t \, u(-t)$

(vi) $u(t) + u(-t)$

(vii) $u(t) - u(-t)$.

Unit Ramp Signal

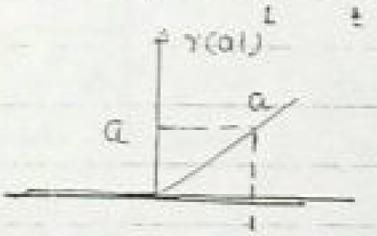


$$r(t) = t \quad t \geq 0$$
$$= 0 \quad t < 0.$$

Time scaling of Ramp

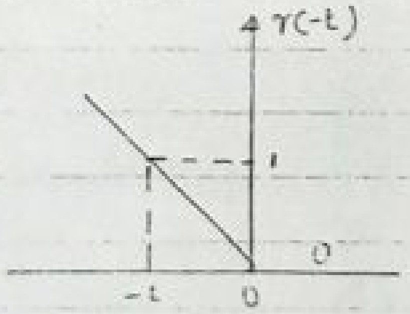
$$r(at) = at \quad at \geq 0$$
$$= 0 \quad at < 0$$
$$t < 0$$

$$\boxed{r(at) = a \cdot r(t)}$$



Time scaling is same as magnitude scaling only in Ramp signal.

$r(-t)$



mathematical way

$$r(-t) = 0. \quad t > 0$$
$$= -t \quad t \leq 0$$

Sketch the following graph.

(i) $r(2t+3)$

(ii) $r(-t+3)$

(iii) $r(-3t+2)$.

Conditionally periodic

$$\frac{2\pi}{\omega_0} : \text{Rational}$$

$$N : r \cdot \frac{2\pi}{\omega_0}$$

Discrete time phasor or Discrete sinusoidal signal are only conditionally periodic with a condition that $\frac{2\pi}{\omega_0}$ must be Rational if it is Rational there periodic with period

$$N : r \cdot \frac{2\pi}{\omega_0}$$

where

r = minimum possible integer such that the above product is an integer.

ω_0	$2\pi/\omega_0$
$\cos 2\pi n$	$\frac{2\pi}{2\pi} = 1$
$\cos 2n$	$\frac{2\pi}{2} = \pi$ = irrational
$\cos \sqrt{2} n$	$\frac{2\pi}{\sqrt{2}}$: irrational

$e^{j\omega_0 n}$

$\omega \to \omega_0 + 2\pi$

$$e^{j(\omega_0 + 2\pi)n}$$

$$= e^{j\omega_0 n} \cdot e^{j2\pi n}$$

$$= e^{j\omega_0 n}$$

$e^{j\omega_0 n} \xrightarrow{\omega \to \omega_0 + 2\pi K}$

$$e^{j(\omega_0 + 2\pi K)n}$$

$$= e^{j\omega_0 n} \cdot \underbrace{e^{j2\pi K n}}_{1}$$

$$= e^{j\omega_0 n} \quad 1.$$

$\omega + 2\pi K$

$\pi + 2\pi K = 3\pi \quad 5\pi \quad 7\pi \; \ldots$

For a Discrete phasor as well as the discrete Sinusoidal Signal there is no effect even if ω_0 is Replaced by $\omega_0 + 2\pi K$ where K is an integer that is:

A Discrete phasor $e^{j\frac{\pi}{4}n}$ is same as $e^{j\frac{\pi 17}{4}n}$, $e^{j\frac{25\pi}{4}}$ ... $e^{j\pi/4 n}$ - - - -

$e^{j\pi/4 n}$

$\omega_0 \cdot \pi/4$

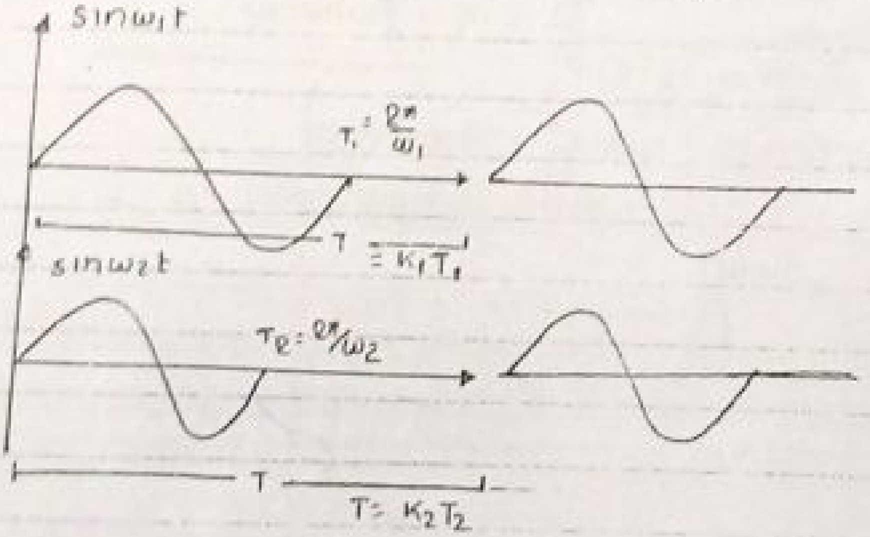
$\omega_0 + 2\pi K$

$$\frac{\pi}{4} + 2\pi K = \frac{9\pi}{4}, \frac{17\pi}{4}, \frac{25\pi}{4}, -\frac{7\pi}{4}$$

⇒ $\sin\omega_1 t$ + $\sin\omega_2 t$

$\sin\omega_1 t$ → Alway Periodic, $T_1 = \frac{2\pi}{\omega_1}$

$\sin\omega_2 t$ → Alway Periodic, $T_2 = \frac{2\pi}{\omega_2}$



Let after time T both have fresh cycle

So both have comple a integer multiple of cycle

For Periodic

$$K_1 T_1 = K_2 T_2$$

$$\boxed{\frac{T_1}{T_2} = \frac{K_2}{K_1}} \Rightarrow \text{Rational}$$

⇒ $\boxed{\frac{\omega_1}{\omega_2} : \text{Rational}}$

$\omega_1 \quad \omega_2 \quad \omega_3 \; - - - - \; \omega_K$

Fundamental Angular Frequency.

$$\omega_0 = \frac{HCF(\text{All numenator})}{LCM(\text{all Denominator})}$$

$$\begin{array}{ccc} \omega_1 & \omega_2 & \omega_3 \\ \frac{4}{5} & \frac{3}{2} & 2/3 \end{array}$$

$$\omega_0 = \frac{HCF(4,3,2)}{LCM(5,2,3)}$$

$$= \frac{1}{30}$$

Then time Period

$$\boxed{T = \frac{2\pi}{\omega_0}}$$

A signal $f(t)$ is Defined as $f(t) = 4 + 3\cos(\frac{\pi}{7}t + \pi/4) + 6\sin(5\pi/6 + \pi/3) + 3\cos(6\pi/5 + \pi/6)$

Is this signal periodic if it is find its period.

$$f(t) = 4 + 3\cos(\pi/7\,t + \pi/4) + 6\sin(5\pi t/6 + \pi/3) + 3\cos(6\pi t/5 + \pi/6)$$

$\sin\omega_0 t \longrightarrow T = 2\pi/\omega_0$

$\sin(\omega_0 t + \phi) \longrightarrow T = 2\pi/\omega_0$

$\cos\omega_0 t \qquad T = 2\pi/\omega_0$

$1 + \cos\omega_0 t \qquad T = 2\pi/\omega_0$

Addition of D.C signal not affect the period of signal the period is not change.

From Given Signal.

$\omega_1 = \frac{\pi}{7}$ $\omega_2 = \frac{5\pi}{6}$ $\omega_2 = \frac{6\pi}{5}$

For Periodic

$\frac{\omega_1}{\omega_2}$ $\frac{\omega_2}{\omega_3}$ $\frac{\omega_3}{\omega_1}$ = will be Rational

All are rational so, it is periodic.

Now calculation of period

Fundamental angular Frequency

$$\omega_0 = \frac{HCF(\pi\ 5\pi,\ 6\pi)}{LCM(7\ 6\ 5)}$$

$$\omega_0 = \frac{\pi}{210}$$

So, Time Period

$$= \frac{2\pi}{\omega_0}$$

$$\boxed{T = 420}$$

Discreate Time signal

⇒ $\sin\omega_1 n + \sin\omega_2 n$.

$\frac{2\pi}{\omega_1}$ = Rational

Then Period is

$N_1 = r_1 \frac{2\pi}{\omega_1}$

Integer.

$2\pi/\omega_2$ = Rational

Then Period of Periodic

$N_2 = r_2 \cdot 2\pi/\omega_2$

Integer

Then

$\frac{N_1}{N_2}$ = always Rational.

The Period

$N = LCM(N_1\ N_2)$

Sum of two continuous time periodic signal may or may not be periodic but sum two Discrete time Periodic Signal is always periodic.

when two continuous time signal which are Sinusoidal or complex exponential are added then they are always individually periodic. The issue is only with periodic nature of Combination where as two DT Sinusoidal or complex exponential signal are added the issue is with the individual periodic nature.

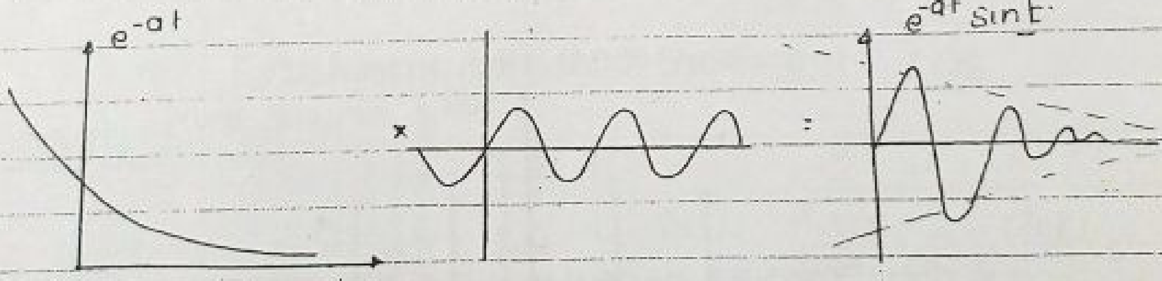
10. $x_1(t) = e^{j20t}$ Always periodic $T_1 = 2\pi/20$

$x_2(t) = e^{(-2+j)t}$

$= e^{-2t} \cdot e^{jt}$

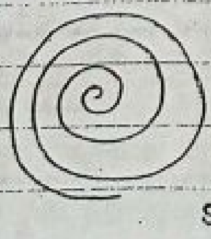
↳ it is periodic

↳ exponential decaying signal.



when periodic signal is multiply with exponential decaying singnal then Resultant signal is not a periodic signal.

Gaph of $e^{-2t} e^{jt}$



so not periodic

Graph of $e^{-2t} e^{jt}$

↳ it Graph is spiral Graph.

11. $\omega_1 = 10\pi$

$\frac{2\pi}{10\pi} = \frac{1}{5}$