symmetrical portion below carrier frequency (ω_c) is known as the lower side band (LSB).

(b) We generally keep $\omega_c > \omega_m$ which ensures that the two sidebands do not overlap each other.

(c) From figure, it is obvious that for the positive side, the highest frequency component present in the spectrum of AM wave is "$\omega_c + \omega_m$" and lowest freq^n component "$\omega_c - \omega_m$".

So B.W of AM wave,

$$B = \omega_c + \omega_m - (\omega_c - \omega_m)$$

$$\boxed{B = 2\omega_m}.$$

Modulation Index:- "In AM system the modulation index is defined as the measure of extent of amplitude variation about an unmodulated maximum carrier." It is represented by "m_a". It also called depth of modulation, degree of modulation or modulation factor.

$$m_a = \frac{|x(t)|_{max}}{\text{Max. carrier amplitude}}.$$

$$m_a = \frac{|x(t)|_{max}}{A}.$$

$m_a \times 100$ = percentage modulation.

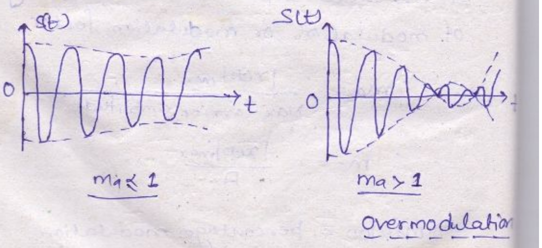
Overmodulation:- The baseband or modulating signal preserved in the envelope of AM signal only if we have

$$|x(t)|_{max} \leq A \quad \text{or} \quad m_a \leq 1$$

or percentage modulation $\leq 100\%$.

On the other hand if $m_a > 1$ or percentage modulation $> 100\%$, the baseband signal is not preserved in the envelope. It means that in this case, the baseband signal recovered from the envelope will be distorted.

This type of distortion is called envelope distortion and the AM signal with $m_a > 1$ or $m_a > 100\%$ is called overmodulated signal.



Overmodulation

Single Tone AM :- In this section, we shall discuss amplitude modulation in which the modulating or baseband signal consists of only one frequency.

This type of amplitude modulation is known as single tone amplitude Modulation.

Let $x(t) = V_m \cos\omega_m t$ —①

$c(t) = A\cos\omega_c t$ —②

AM signal $s(t) = [A + x(t)]\cos\omega_c t$

$s(t) = A\cos\omega_c t + x(t)\cos\omega_c t$ —③

$s(t) = A\cos\omega_c t + V_m\cos\omega_m t \cdot \cos\omega_c t$

or $s(t) = A\cos\omega_c t\left[1 + \frac{V_m}{A}\cos\omega_m t\right]$ —④

$= A\cos\omega_c t\left[1 + m_a\cos\omega_m t\right]$ —⑤

$\because m_a = \frac{V_m}{A}$.

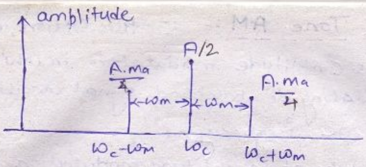
This is desired expression for single-tone AM signal.

$s(t) = A\cos\omega_c t + m_a A\cos\omega_c t \cdot \cos\omega_m t$.

$= A\cos\omega_c t + \frac{A \cdot m_a}{2}\left[2\cos\omega_c t \cdot \cos\omega_m t\right]$

$= A\cos\omega_c t + \frac{A \cdot m_a}{2}\left[\cos(\omega_c + \omega_m)t + \cos(\omega_c - \omega_m)t\right]$

or $s(t) = A\cos\omega_c t + \frac{A \cdot m_a}{2}\cos(\omega_c + \omega_m)t + \frac{A \cdot m_a}{2}\cos(\omega_c - \omega_m)t$.



Single sided frequency spectrum of Singtone

Power Content in AM:-

$$s(t) = A\cos\omega_c t + x(t)\cos\omega_c t$$

total power P of the AM wave is the sum of Carrier power P_c and Side band power P_s.

Carrier power:-

P_c = mean square value of $A\cos\omega_c t$.

$$= [A\cos\omega_c t]^2$$

$$= \frac{1}{2\pi}\int_0^{2\pi} A^2\cos^2\omega_c t \cdot d(\omega t) = \frac{A^2}{2}.$$

Side band power:-

P_s = mean square value of $x(t)\cos\omega_c t$

$$P_s = [x(t)\cos\omega_c t]^2 = \frac{1}{2\pi}\int_0^{2\pi} x^2(t)\cos^2\omega_c t$$

$$= \frac{1}{2\pi}\int_0^{2\pi}\frac{1}{2}[2\cos^2\omega_c t]\,x^2(t)\,dt$$

$$= \frac{1}{2\pi}\int_0^{2\pi}\frac{1}{2}x^2(t)\,dt + \frac{1}{2\pi}\int_0^{2\pi}x^2(t)\cos 2\omega_c t\,d$$

In AM generation, a BPF tuned to carrier frequency ω_c is used to filter out the second integral form.

$$P_s = \frac{1}{2\pi}\int_0^{2\pi} \frac{\overline{x^2(t)}}{2}\,dt.$$

$$P_s = \frac{1}{2}\,\overline{x^2(t)}$$

$$P_s(USB) = P_s(LSB) = \frac{1}{4}\,\overline{x^2(t)}$$

therefore total power.

$$P_T = P_c + P_s = \frac{A^2}{2} + \frac{\overline{x^2(t)}}{2}$$

$$\boxed{P_T = \frac{1}{2}\left[A^2 + \overline{x^2(t)}\right]}$$

Transmission efficiency of AM signal:-

Out of this total power P_t, the useful message or baseband power is the power carried by the sideband P_s. The large carrier power P_c is a waste from the transmission point of view because it does not carry any information or message

In AM wave, the amout of useful message power P_s may be expressed by a term known as transmission efficiency η.

Mathematically, $\eta = \frac{P_s}{P_t} \times 100$

$$\eta = \frac{\frac{1}{2}\overline{x^2(t)}}{\frac{A^2}{2} + \frac{\overline{x^2(t)}}{2}} \times 100$$

$$\eta = \frac{100\ \overline{x^2(t)}}{A^2 + \overline{x^2(t)}}$$

The maximum transmission efficiency of AM is only 33.33%. This implies that only one third of the total power is carried by the sidebands and the rest two third power is wasted.

* for single tone AM wave.

total power, $P_t = P_c\left[1 + \frac{m_a^2}{2}\right]$.

* Current for single tone AM.

$$I_t = I_c\sqrt{1 + \frac{m_a^2}{2}}.$$

* Power content in multitone AM.

$$P_t = P_c\left[1 + \frac{m_1^2}{2} + \frac{m_2^2}{2} + \cdots + \frac{m_n^2}{2}\right].$$

* Total modulation index for multi tone modulation.

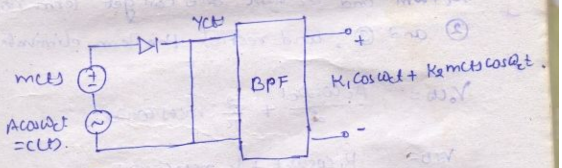
$$m_t = \sqrt{m_1^2 + m_2^2 + \cdots m_n^2}.$$

Double side Band with Carrier [DSB-C] Modulator

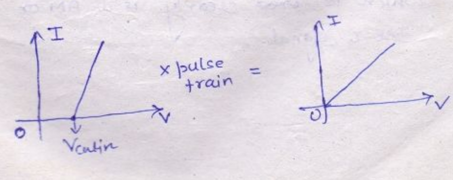
(a) **Switching Modulator**:- Consider the simple switching modulator circuits with only one diode as shown in fig(a). The BPF passes the frequency $\omega_c \pm \omega_m$, where ω_m is maximum frequency of message signal. Carrier is represented by $A\cos\omega_c t$.

The diode conducts when the combined signal (message plus carrier) is positive.
The switching action can be approximated by a pulse train as

$$s(t) = \frac{1}{2} + \frac{2}{\pi}\left(\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t - \dots\right) \quad -(1)$$



fig(a). An AM (DSB-C) Modulator.



Now the combined signal message $m(t)+c(t)$ will appear at the output when diode is on,

Mathematically.

$$y(t) = [m(t)+c(t)]\, s(t).$$

$$y(t) = [m(t)+c(t)]\left[\frac{1}{2}+\frac{2}{\pi}\left(\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \cdots\right)\right]$$

$$y(t) = \overset{①}{\frac{m(t)}{2}} + \overset{②}{\frac{2}{\pi} m(t)\cos\omega_c t} + \overset{③}{\frac{c(t)}{2}} + \overset{④}{\frac{2}{\pi} c(t)\cos} \cdots$$

$$- \cdots$$

Thus $y(t)$ contain, the baseband signal (1^{st}), a dc term (4^{th}), carrier (3^{rd}), and high harmonics of carrier.

So by use of BPF of frequency b/w $\omega_c+\omega_m$ and $\omega_c-\omega_m$ one can get term no ② and ③, and rest of the term eliminate

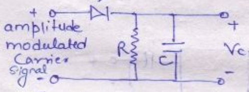
$$V_o(t) = \frac{A_c\cos\omega_c t}{2} + \frac{2}{\pi} m(t)\cos\omega_c t.$$

$$V_o(t) = K_1\cos\omega_c t + K_2\, m(t)\cos\omega_c t$$

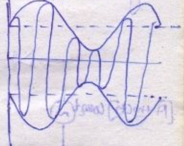
which is ~~area~~ clearly is an AM or DSB-C signal.

DSB-C Demodulator :-

Envelope detector or Diode Detector:- The merit of the amplitude modulated carrier is the ease with which the baseband signal is recovered. The recovery of the baseband signal, a process which is refered as demodulation or detection. is accomplished by the simple circuit of fig(a) which consist of a Diode D and Resistor-Capacitor RC combination.



fig(a)



In positive half of input diode will be ON and capacitor will be charge upto the peak value of the input voltage. In negative half, diode will be OFF and capacitor will discharge through resistance R.

Choice of RC value:- (i) If RC value is very high then discharging rate will be very slow, in this case small peaks of envelope can be miss out and this phenomea is known as diagonal clipping or peak clipping.

(ii) If RC value is kept low then voltage across capacitor will discharge at faster rate and in this case there will be high value of ripple or fluctuation.

* $RC < \frac{1}{\omega_m}$ or $\frac{1}{f_m}$ to avoid diagonal clipping.
* $RC > \frac{1}{\omega_c}$ or $\frac{1}{f_c}$ to avoid fluctuation.

So

$$\boxed{\frac{1}{f_c} < RC(\text{sec}) < \frac{1}{f_m}}.$$

<u>Rectifier Dectector</u>:- fig (b) describe the schem[e] we have a LPF with cutoff frequency ω_m follo[wed] by a capacitor which block dc component.

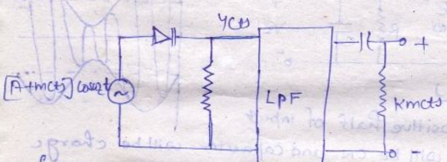


fig (b) <u>Rectifier Detector circuit for AM (DSB-C)</u>

The input to the LPF is the rectifier outpu[t] which is periodic pulse train multiplied by DSB-C modulated signal and can be written as

$$y(t) = [A + m(t)]\cos\omega_c t\left[\frac{1}{2} + \frac{2}{\pi}\left(\cos\omega_c t - \frac{1}{3}\cos\ldots\right.\right.$$
$$\left.+\ldots\right]$$

$$y(t) = \frac{A\cos\omega_c t}{2} + \frac{2A}{\pi}\cos^2\omega_c t + \frac{m(t)\cos\omega_c t}{2}$$
$$+ \frac{2m(t)}{\pi}\cos^2\omega_c t + \ldots$$

from portion ② and ④ we will obtained a D.C. term $\frac{A}{\pi}$ and m(t), which is passed by a DC blocker and LPF then we only get the demodulated message signal.

Square Law Detector :- The square law detector circuit is used for detecting modulating signal of small amplitude (i.e. 1v) so that the operating region may be restricted to the non-linear portion of the V-I characteristics of the ~~device~~ device. fig (c) shows the circuit of square law detector.

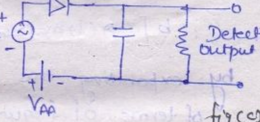
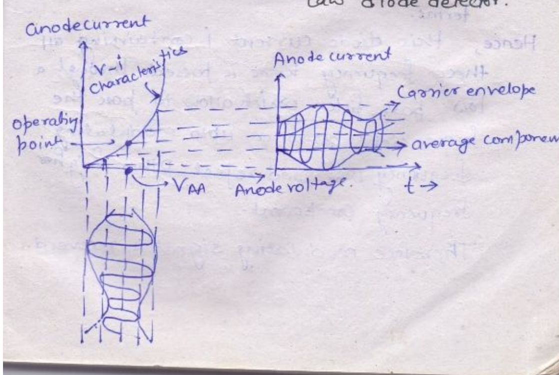


fig (c).
Basis circuit for square law diode detector.



In this circuit the dc supply voltage V_{AA} is used to get the fixed operating point to the non linear portion of the diode V-I characteristics. Since the operation is limited to non-linear portion of diode the lower half portion of the modulated waveform is compressed.

$$i = av + bv^2 \quad \text{—(1)}$$

where v is the input modulated voltage

$$V = A[1 + m_a \cos\omega_m t]\cos\omega_c t \quad \text{—(2)}$$

then

$$i = a[A(1 + m_a \cos\omega_m t)\cos\omega_c t] + b[A(1 + m_a \cos\omega_m t)\cos\omega_c t]^2$$

by expanding this we ~~obtaine~~ observe that presence of terms of frequencies like $2\omega_c$, $2(\omega_c \pm \omega_m)$, ω_m and $2\omega_m$ besides the input frequency terms.

Hence, this diode current i containing all these frequency terms is passed through a low pass filter which allow to pass the frequencies below or upto modulating frequency ω_m and reject other higher frequency component.

Therefore modulating signal is recoverd.

Double sideband Suppressed Carrier [DSB-SC] system :-

Single tone amplitude modulation is expressed as

$$s(t) = A\cos\omega_c t + \frac{A \cdot m_a}{2}\left[\cos(\omega_c+\omega_m)t + \cos(\omega_c-\omega_m)t\right] \quad (1)$$

From this expression it is clear that carrier component in AM wave remains constant in amplitude and frequency.

For 100% modulation about 67% of the total power is required for transmitting the carrier which does not contain any information. Hence if the carrier is suppressed, only the sideband remains and in this way a saving of 2/3rd may be achieved at 100% modulation.

The resulting signal obtained after suppressing the carrier from AM wave called Double side band suppressed carrier [DSB-SC] system.

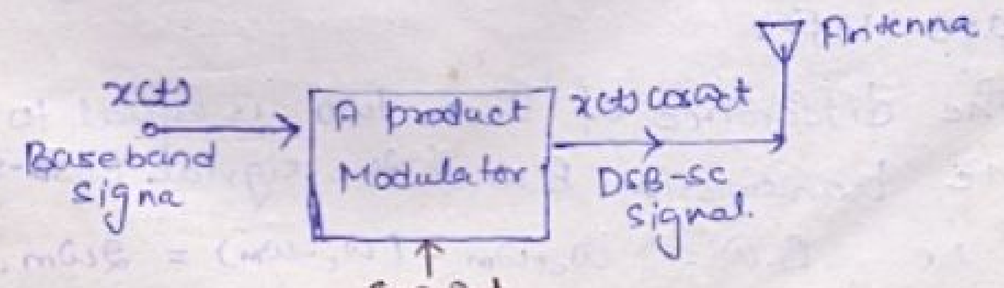
we know that AM wave equation,

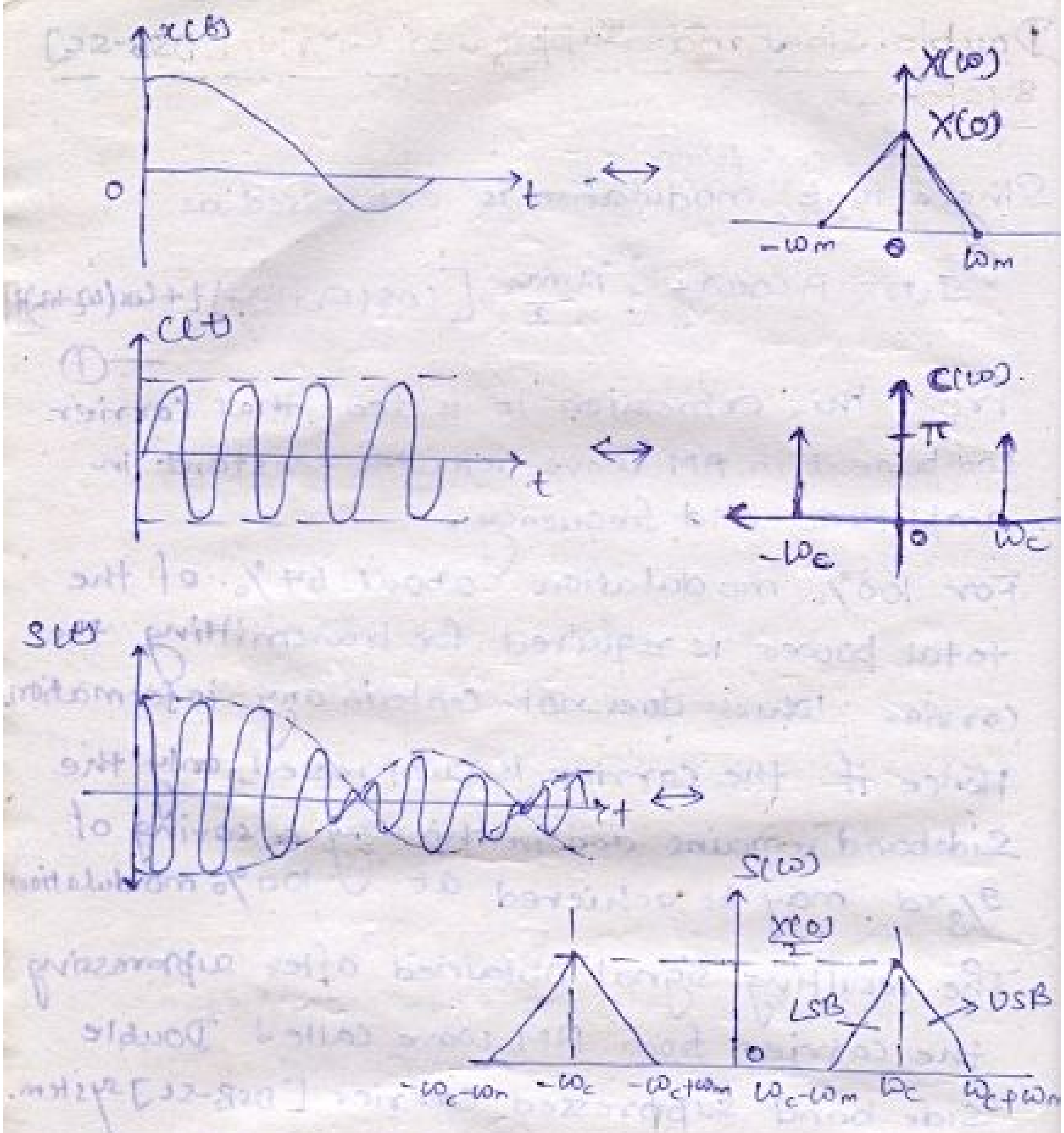
$$s(t) = [A + x(t)]\cos\omega_c t$$

$$= A\cos\omega_c t + x(t)\cos\omega_c t$$

if carrier is suppressed then "$A\cos\omega_c t$" term vanish,

so, $s(t) = x(t)\cos\omega_c t$ equation for DSB-SC.





In fig, ~~considering only positive sideband suppressed carrier~~, it is clear that the impulse at $\pm\omega_c$ are missing which means that the carrier term is suppressed and only two side band terms USB (upper side band) and lower side band (LSB) are left.

The difference of these two is equal to the transmission B.W of the signal (DSB-SC)

i.e. $B.W = \omega_c + \omega_m - (\omega_c - \omega_m) = 2\omega_m$.

Generation of DSB-SC Signal:-

$s(t) = x(t) \cos\omega_c t$.

$x(t)$ = Base band signal.

$\cos\omega_c t$ = carrier signal.

A circuit used to achieve the generation of DSB-SC signal is called a product modulator.

Balanced Modulator:- We know that a non-linear resistance or a non-linear device may be used to produce amplitude modulation i.e. one carrier and two sidebands. However, a DSB-SC signal contains only two sidebands. Thus if two non-linear devices such as diodes, transistors etc. are connected in balanced mode so as to suppress the carriers of each other the only sidebands are left i.e. a DSB-SC signal is generated.

So "Balance modulator may be defined as a circuit in which two non linear devices are connected in a balance mode to produce a DSB-SC signal."

A non liear V-i relationship is given as.

$$i = av + bv^2 \quad — (1)$$

$$V_1 = \cos\omega_c t + x(t) \quad — (2)$$

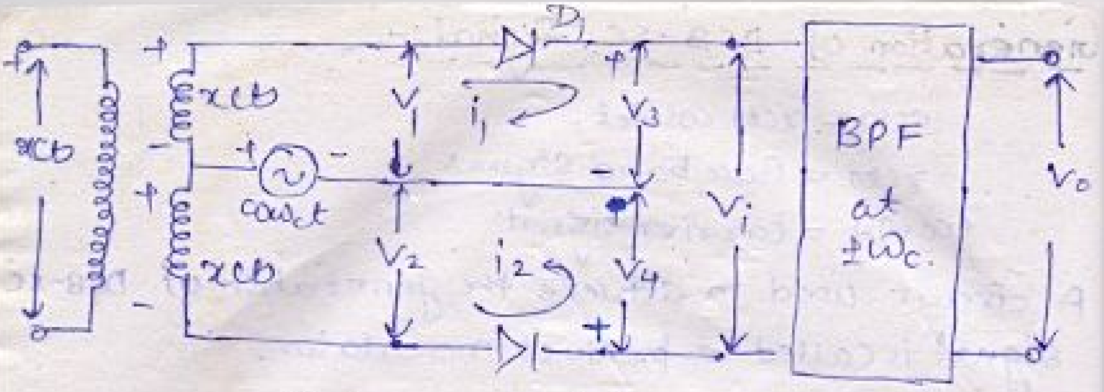
$$V_2 = \cos\omega_c t - x(t) \quad — (3)$$

for Diode D_1, $i_1 = av_1 + bv_1^2$

$$i_1 = a[\cos\omega_c t + x(t)] + b[\cos\omega_c t + x(t)]^2$$

$$i_1 = a\cos\omega_c t + ax(t) + b[\cos^2\omega_c t + x^2(t) + 2x(t)\cos\omega_c t]$$

$$i_1 = a\cos\omega_c t + ax(t) + b\cos^2\omega_c t + bx^2(t) + 2b\,x(t)\cos\omega_c t \quad — (4)$$



Balance modulator using diode.

Simillarly for Diode D_2,

$$i_2 = a v_2 + b v_2^2$$

$$i_2 = a[\cos\omega_c t - x(t)] + b[\cos\omega_c t - x(t)]^2$$

$$i_2 = a\cos\omega_c t - a x(t) + b\cos^2\omega_c t + b x^2(t) - 2b\,x(t)\cos\omega_c t \quad \text{— (5).}$$

Due to current i_1 and i_2 the net voltage V_i at the input of BPF is expressed as.

$$V_i = V_3 - V_4 \quad \text{— (6)}$$

$$V_3 = i_1 R \quad \text{— (7)}$$

$$V_4 = i_2 R \quad \text{— (8)}$$

$$V_i = V_3 - V_4 = R(i_1 - i_2)$$

$$V_i = R[2a\,x(t) + 4b\,x(t)\cos\omega_c t] \quad \text{— (9)}$$

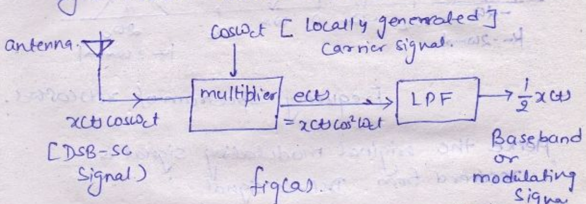
A BPF is centred around $\pm\omega_c$ it will pass a narraoband of frequencies centered at $\pm\omega_c$ with a small bandwidth of $2\omega_m$ to preserve the bandwidth.

So $V_0 = 4b\,x(t)\cos\omega_c t = K\,x(t)\cos\omega_c t$

which is DSB-SC Signal.

Demodulation of DSB-SC Signal:-

Synchronous detection method:- A method of DSB-SC detection is known as synchronous detection. fig(a) shows the block diagram of synchronous detection method.



fig(a)

Synchronous detection method.

Mathematically, $e(t) = x(t)\cos\omega_c t \cdot \cos\omega_c t$

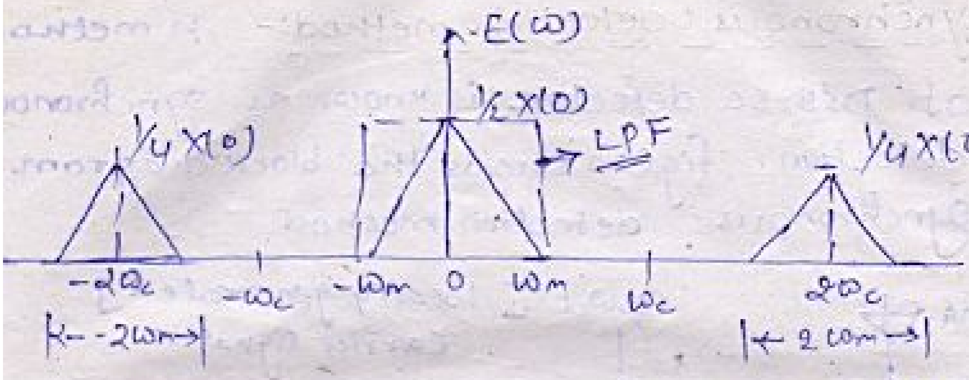
$$e(t) = x(t)\cos^2\omega_c t = \frac{1}{2}x(t)[2\cos^2\omega_c t]$$

$$= \frac{1}{2}x(t)[1+\cos 2\omega_c t]$$

$$e(t) = \frac{1}{2}x(t) + \frac{1}{2}x(t)\cos 2\omega_c t.$$

Now, it may be observed that when $e(t)$ passed through a LPF, then the term $\frac{1}{2}x(t)\cos 2\omega_c t$, centered at $\pm 2\omega_c$ is suppressed by LPF and thus at the output of the LPF, the original modulating signal $\frac{1}{2}x(t)$ is obtained.

$x(t)\cos^2\omega_c t \longleftrightarrow \frac{1}{2}X(\omega) + \frac{1}{4}[X(\omega+2\omega_c) + X(\omega-$

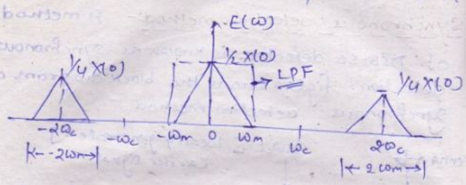


Frequency spectrum of $x(t)\cos^2\omega$

Hence the original modulating signal is recoverd from DSB-SC signal.

In this detection process a local oscillator is required at the receiver end. The frequency and phase of the locally generated carrier signal and carrier signal at the trans ~~carrier~~ must be identical. This means th the local oscillator signal must be exactly coherent or synchronized with the carrier signal at the transmitter both in frequ and phase, otherwise the detected signal would get distorted. Therefore this method of recovery called synchronous detection or coherent detection.

$$x(t)\cos^2\omega_c t \longleftrightarrow \frac{1}{2}X(\omega)+\frac{1}{4}\left[X(\omega+2\omega_c)+X(\omega\ldots\right.$$



Frequency spectrum of $x(t)\cos^2\omega_c t$

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QAM ⇒

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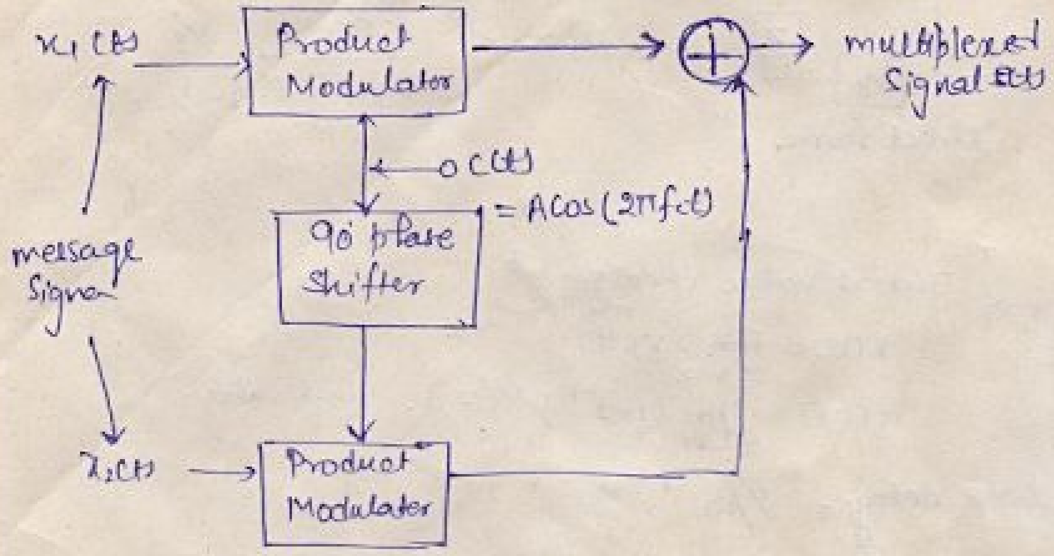
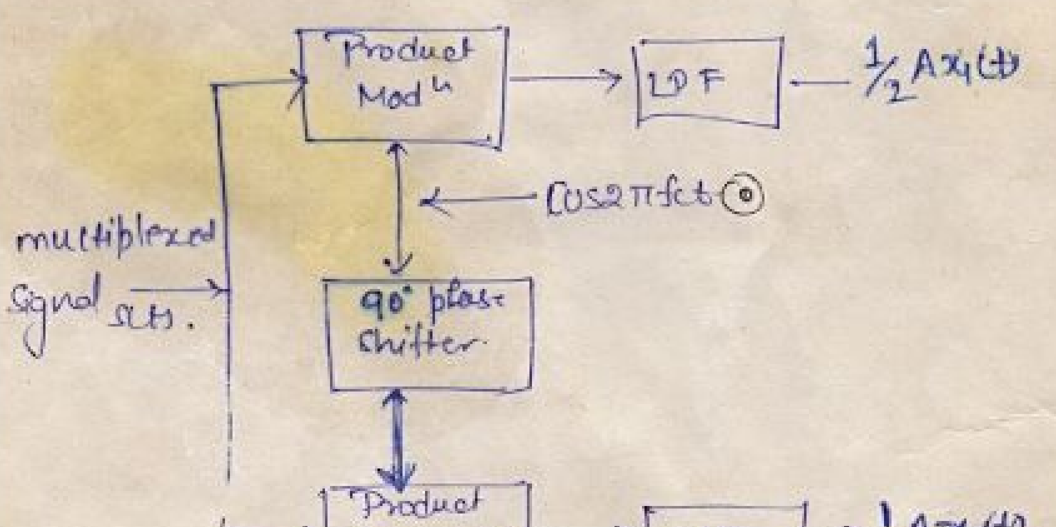


QAM Transmitter.



Receiver