

$$S_c(t) = x_1(t) A \cos(2\pi f_c t) + x_2(t) A \sin(2\pi f_c t). \quad \text{--- (1)}$$

$$S_r(t) = \text{---}$$

→ effect of phase and frequency errors in synchronous detection: -

ex frequency and phase error at locally generated carrier at receiver is $\Delta\omega$ and ϕ .

product of two signals at synchronous detector provides,

$$e_d(t) = x(t) \cos \omega_c t \cdot \cos[(\omega_c + \Delta\omega)t + \phi]$$

$$\text{or } e_d(t) = \frac{1}{2} x(t) \left\{ \cos[(\omega_c + \Delta\omega)t + \phi] + \cos[(2\omega_c + \Delta\omega)t + \phi] \right\}$$

pass this term through a LPF of cutoff $\frac{\omega_m}{2}$

$$e_o(t) = \frac{1}{2} x(t) \cos[\Delta\omega t + \phi]$$

Base band signal multiplied with slow-time varying function.

$\cos[\Delta\omega t + \phi]$ distort the message signal.

Case

(a) when $\Delta\omega = 0$, $\phi = 0$

$$e_o(t) = \frac{1}{2} x(t). \quad \text{No distortion}$$

(b) $\Delta\omega = 0$, $\phi \neq 0$ i.e

$$e_o(t) = \frac{1}{2} x(t) \cos \phi. \quad \text{when } (\phi \text{ is time varying, not})$$

No distortion only attenuation, as

ϕ vary b/w 0 to 90° then

$$e_o(t) = \text{max when } \phi = 0^\circ$$

$$e_o(t) = \text{min when } \phi = 90^\circ$$

and when $\phi \rightarrow$ time varying
 then it causes undesirable distortion at the
 detected output

(b) $\Delta\omega \neq 0, \phi = 0$ i.e.

$$e_{o}(t) = \frac{1}{2} x(t) \cos(\Delta\omega t)$$

$\cos(\Delta\omega t) \rightarrow$ time varying so distortion in output
 signal.

(c) $\Delta\omega \neq 0, \phi \neq 0$ i.e.

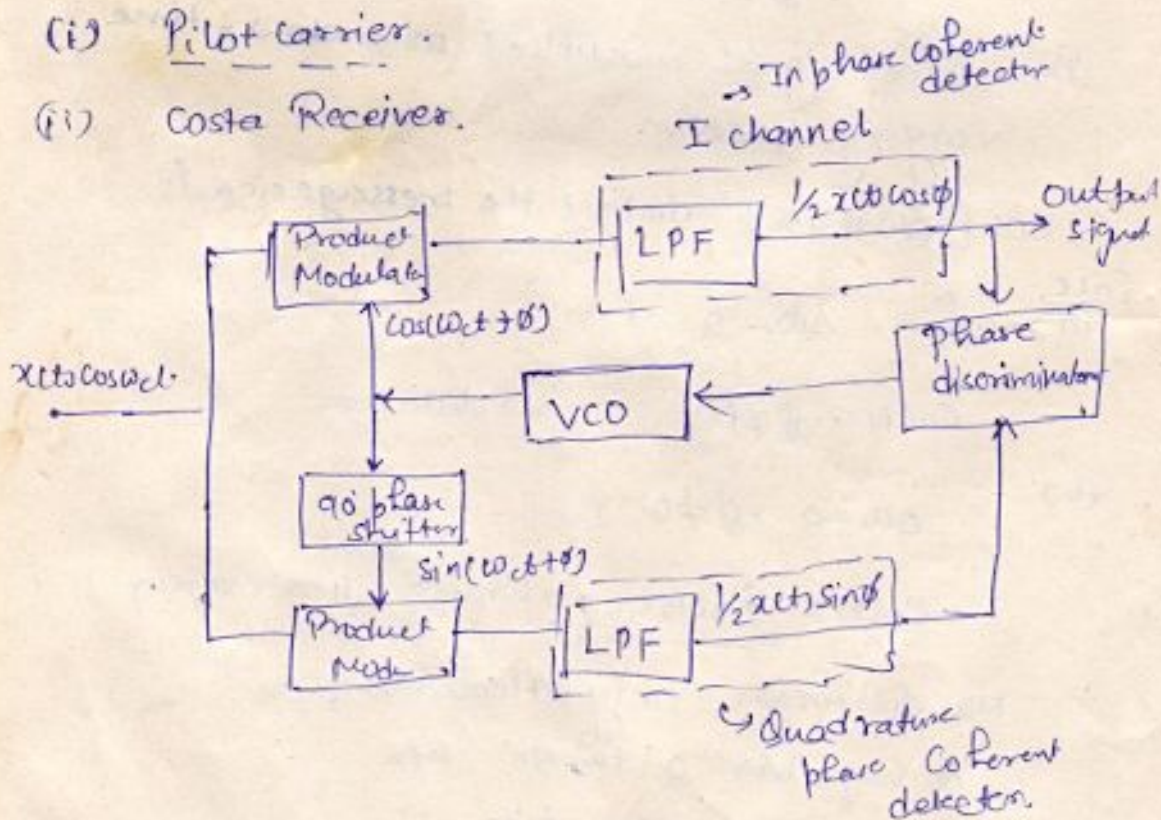
$$e_{o}(t) = \frac{1}{2} x(t) \cos[\Delta\omega t + \phi]$$

then we obtained attenuated and distorted
 output.

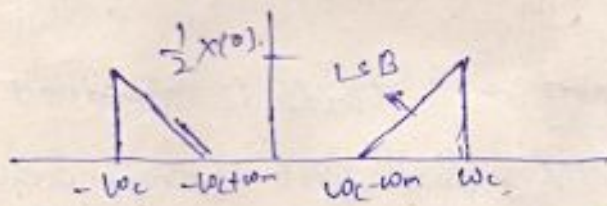
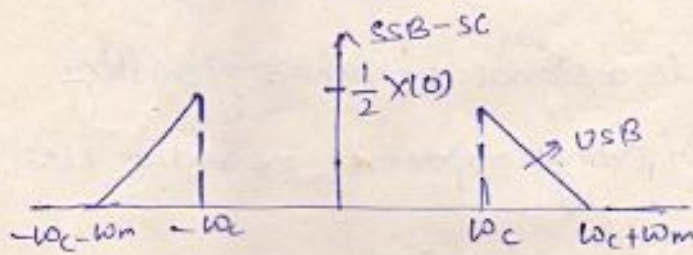
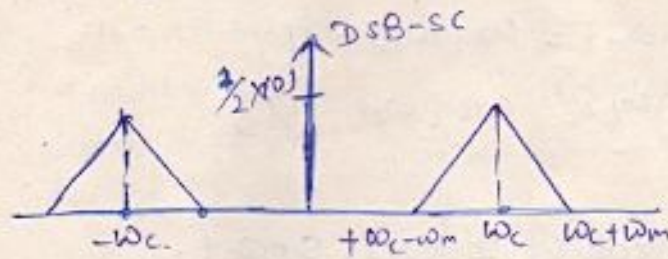
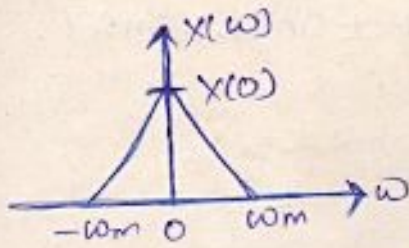
\rightarrow Carrier acquisition in DSB-SC system or Carrier
 synchronization technique in DSB-SC system:-

(i) Pilot carrier.

(ii) Costa Receiver.



SSB-SC \Rightarrow

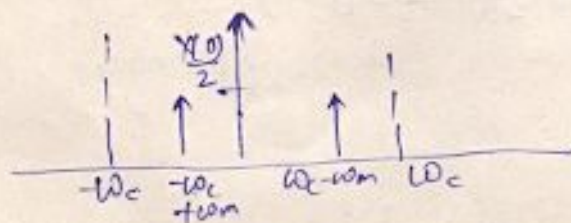
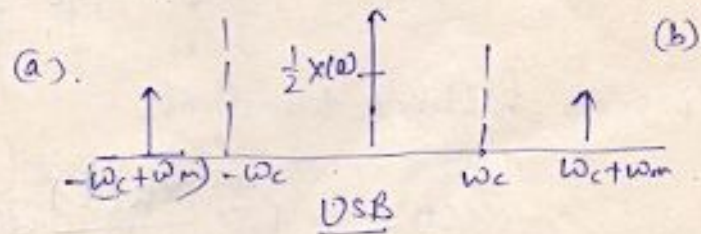
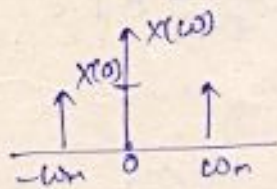


SSB-SC wave with single tone modulating signal \Rightarrow

$x(t) = \cos \omega_m t$ \rightarrow modulating signal
 $c(t) = \cos \omega_c t$ \rightarrow carrier signal.

$\cos(\omega_c + \omega_m)t = \cos \omega_c t \cdot \cos \omega_m t - \sin \omega_c t \cdot \sin \omega_m t$

$\cos(\omega_c - \omega_m)t = \cos \omega_c t \cdot \cos \omega_m t + \sin \omega_c t \cdot \sin \omega_m t$



Both these equation combine.

$$s(t)_{SSB} = \cos \omega_m t \cdot \cos \omega_c t \pm \sin \omega_m t \cdot \sin \omega_c t.$$

+ \rightarrow LSB

- \rightarrow USB.

We can write

$$\sin \omega_m t = \cos(\omega_m t - \frac{\pi}{2})$$

$$\sin \omega_c t = \cos(\omega_c t - \frac{\pi}{2}).$$

$$s(t)_{SSB} = x(t) \cos \omega_c t \pm x_{H}(t) \sin \omega_c t.$$

where $x_{H}(t)$ is a signal obtained by shifting the phase of every component present in $x(t)$ by $(-\pi/2)$.

Hilbert Transform \Rightarrow $x_{H}(t)$ is obtained

by providing $(-\pi/2)$ phase shift to every frequency component present in $x(t)$, actually represent the Hilbert transform of $x(t)$.

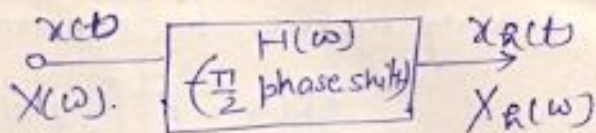
that means $x_{H}(t)$ is Hilbert transform of $x(t)$.

$$x_{H}(t) = \frac{1}{\pi} x(t) \otimes \frac{1}{\pm} = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau.$$

inverse Hilbert transform

$$x(t) = -\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x_{H}(\tau)}{t-\tau} d\tau.$$

* Characteristics of this system,

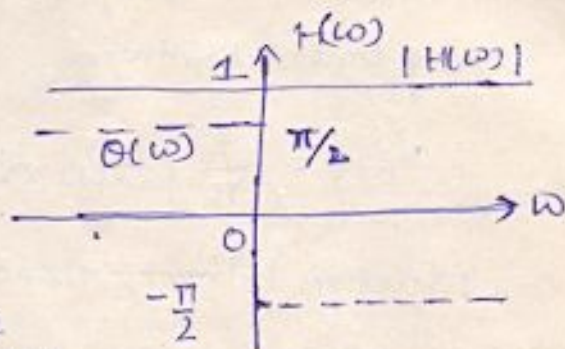


(a) $H(\omega) = 1$

magnitude unchanged when passed through system.

(b) the phase of positive freq component is shifted by $-\pi/2$.

the phase of negative freq component shifted by $\pi/2$
 $\theta(\omega) \rightarrow$ odd symmetry.



Transfer function of $-\pi/2$ phase shifter.

$$H(\omega) = |H(\omega)| e^{j\theta(\omega)}$$

$$H(\omega) = e^{j\theta(\omega)}$$

$$\theta(\omega) = \begin{cases} +\pi/2 & \text{for } \omega < 0 \\ -\pi/2 & \text{for } \omega > 0 \end{cases}$$

$$H(\omega) = \begin{cases} e^{j\pi/2}, & \omega < 0 \\ e^{-j\pi/2}, & \omega > 0 \end{cases}$$

$$H(\omega) = \begin{cases} j, & \omega < 0 \\ -j, & \omega > 0 \end{cases}$$

$$\frac{H(\omega)}{j} = \begin{cases} 1, & \omega < 0 \\ -1, & \omega > 0 \end{cases} = -\text{sgn}(\omega)$$

$$H(\omega) = j \text{sgn}(\omega)$$

$$\begin{aligned} X_R(\omega) &= H(\omega) X(\omega) \\ &= -j X(\omega) \text{sgn}(\omega) \end{aligned}$$

taking inverse fourier transform,

$$x_{R(t)} = \mathcal{F}^{-1}[-j x(\omega) \operatorname{sgn} \omega]$$

$$\frac{1}{\pi t} \leftrightarrow -j \operatorname{sgn}(\omega)$$

using convolution theorem.

$$x_{R(t)} = \frac{1}{\pi} \left[x(t) \otimes \frac{1}{t} \right] = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau$$

which is hilbert transform.

Pre envelope or analytic signal:-

pre envelope of real signal $x(t)$ is defined as

$$x_p(t) = x(t) + j x_R(t) \quad \text{--- (1)}$$

$$x_p^*(t) = x(t) - j x_R(t) \quad \text{--- (2)}$$

SSB-SC for general modulating signal:-

we know that pre envelope,

$$x_p(t) = x(t) + j x_R(t)$$

$$\mathcal{F}[x_p(t)] = \mathcal{F}[x(t)] + j \mathcal{F}[x_R(t)]$$

$$X_p(\omega) = X(\omega) + j [-j X(\omega) \operatorname{sgn}(\omega)]$$

$$X_p(\omega) = X(\omega) + X(\omega) \operatorname{sgn}(\omega)$$

We know that,

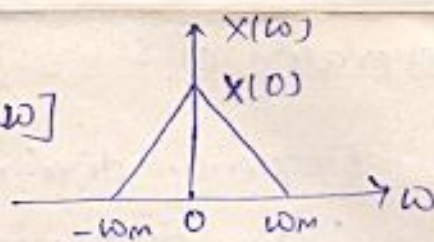
$$\operatorname{sgn}(\omega) = \begin{cases} 1 & \text{for } \omega > 0 \\ -1 & \text{for } \omega < 0 \end{cases}$$

therefore we have

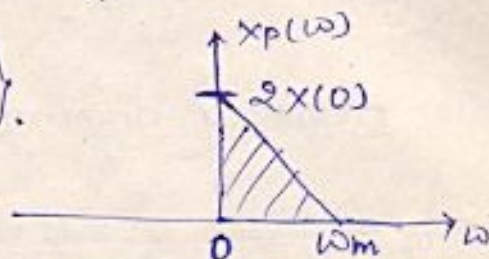
$$X_p(\omega) = \begin{cases} 2X(\omega), & \omega > 0 \\ 0, & \omega < 0 \end{cases}$$

also,

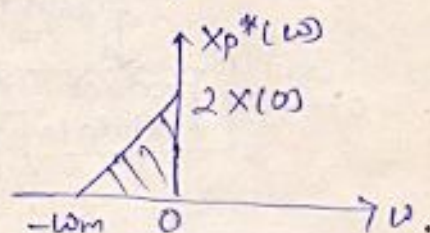
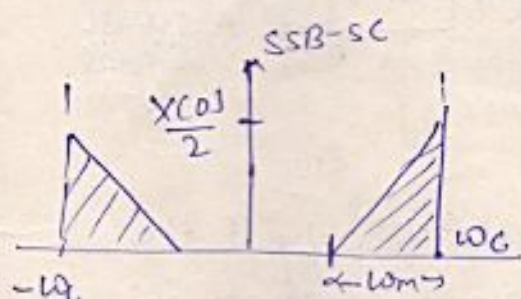
$$X_p^*(\omega) = X(\omega) - j[-jX(\omega)\text{sgn}\omega] \\ = X(\omega) - X(\omega)\text{sgn}\omega.$$



$$X_p^*(\omega) = \begin{cases} 0, & \omega > 0 \\ 2X(\omega), & \omega < 0 \end{cases}.$$



Let us consider SSB-SC
consist LSB.



- (i) Right hand portion is spectrum of $\frac{1}{4} X_p^*(\omega) e^{j\omega_c t}$.
 (ii) Left hand portion is spectrum of $\frac{1}{4} X_p(\omega) e^{-j\omega_c t}$.

So,

$$S(\omega)_{SSB} = \frac{1}{4} [X_p^*(\omega) e^{j\omega_c t} + X_p(\omega) e^{-j\omega_c t}] \\ = \frac{1}{4} [(X(\omega) - jX_R(\omega)) e^{j\omega_c t} + (X(\omega) + jX_R(\omega)) e^{-j\omega_c t}] \\ = \frac{1}{2} X(\omega) \left[\frac{e^{j\omega_c t} + e^{-j\omega_c t}}{2} \right] + \frac{j}{2} X_R(\omega) \left[\frac{e^{-j\omega_c t} - e^{j\omega_c t}}{2} \right]$$

$$S(\omega)_{SSB} = \frac{1}{2} [X(\omega) \cos \omega_c t + X_R(\omega) \sin \omega_c t] \rightarrow \text{for LSB.}$$

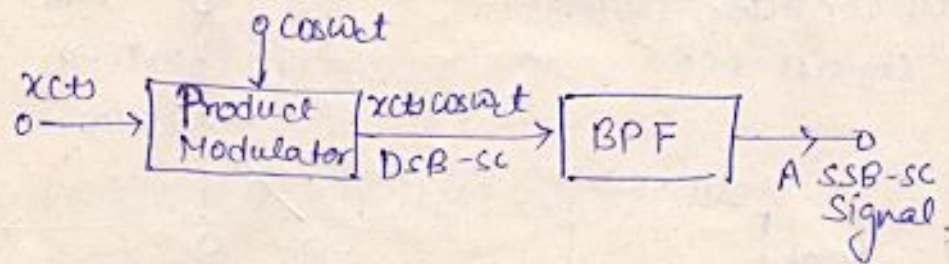
Similarly

$$S(\omega)_{SSB} = \frac{1}{2} [X(\omega) \cos \omega_c t - X_R(\omega) \sin \omega_c t] \rightarrow \text{for USB.}$$

Generation of SSB-Signal ⇒

- (i) Frequency discrimination method.
- (ii) Phase discrimination method.

(i) Frequency discrimination method:-



Limitation:-

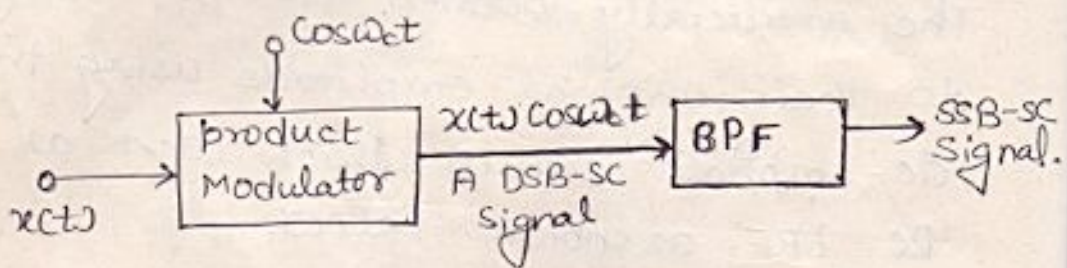
- (i) This method is useful only if the baseband signal is restricted at its lower edge due to which upper and lower sideband are non overlapping. speech communication.
- (ii) Another restriction of the frequency discrimination method is that the baseband signal must be appropriately related with the carrier frequency.

Generation of SSB-SC signal \Rightarrow

there are two methods used for generation of SSB-SC signal.

- (a) frequency discrimination or filter method
- (b) phase discrimination or phase shift method.

(a) Filter Method \Rightarrow



(b) Phase shift method for the SSB generation \Rightarrow

This system is used two balance modulators M_1 and M_2 and two phase shift networks which provide $-\frac{\pi}{2}$ phase shift.

The message signal $x(t)$ is applied directly to the product modulator M_1 and through a $-\frac{\pi}{2}$ phase shifter to the product modulator M_2 .

The carrier signal applied directly to the product modulator M_1 and a

$-\frac{\pi}{2}$ phase shifter to the product modulator M_2 .

Hence the o/p of product modulator M_1
 $= x(t) \cos(2\pi f_c t)$

and, the o/p of product modulator M_2
 $= \hat{x}(t) \sin(2\pi f_c t)$

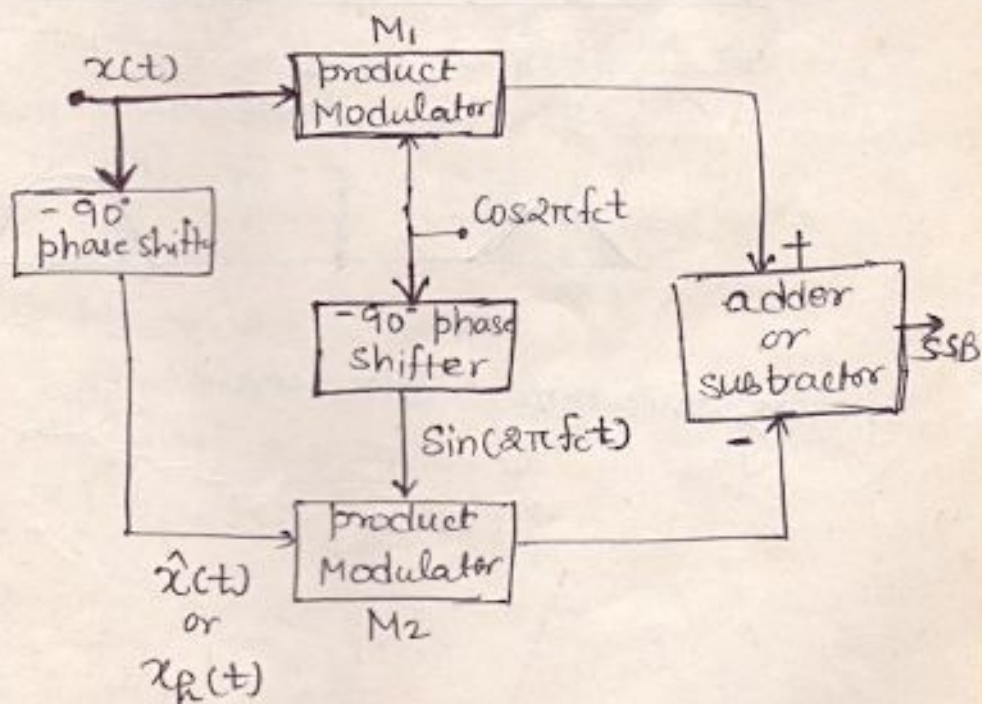
apply both o/p to the adder or subtractor circuit to obtain SSB-SC.

SSB-SC [LSB] when work as adder.

$$S(t)_{LSB} = x(t) \cos 2\pi f_c t + \hat{x}(t) \sin(2\pi f_c t)$$

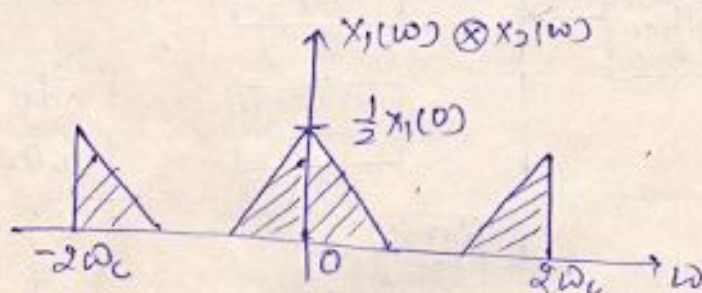
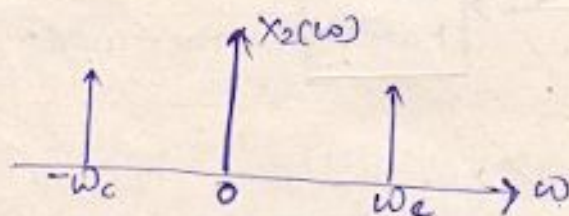
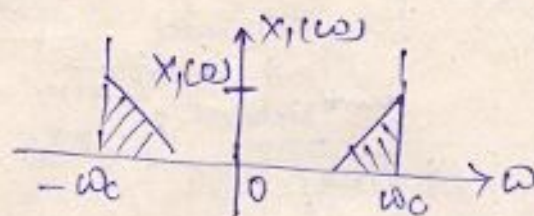
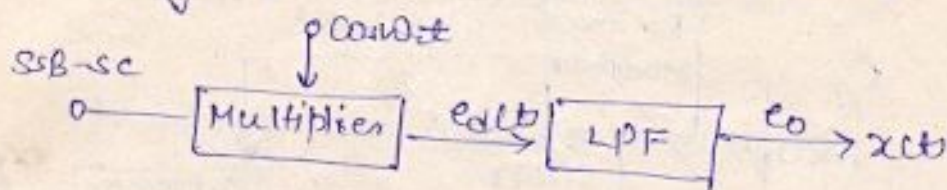
SSB-SC [USB], when work as subtractor,

$$S(t)_{USB} = x(t) \cos 2\pi f_c t - \hat{x}(t) \sin(2\pi f_c t)$$



phase shift method.

Demodulation of SSB-SC :- The baseband or modulating signal recovered from SSB by using synchronous detection method.



$$e_{dtb} = x_{c(t)} \cdot \cos \omega_c t$$

$$= [x_{R(t)} \cos \omega_c t \pm x_{L(t)} \sin \omega_c t] \cos \omega_c t$$

$$= x_{R(t)} \cos^2 \omega_c t \pm x_{L(t)} \sin \omega_c t \cdot \cos \omega_c t$$

$$= \frac{1}{2} [2x_{R(t)} \cos^2 \omega_c t] \pm \frac{1}{2} x_{L(t)} \sin 2\omega_c t$$

$$= \frac{1}{2} x_{c(t)} + \frac{1}{2} [x_{R(t)} \cos 2\omega_c t \pm x_{L(t)} \sin 2\omega_c t]$$

$$\xrightarrow{\text{L.P.F.}} e_o = \frac{1}{2} x_{c(t)}$$

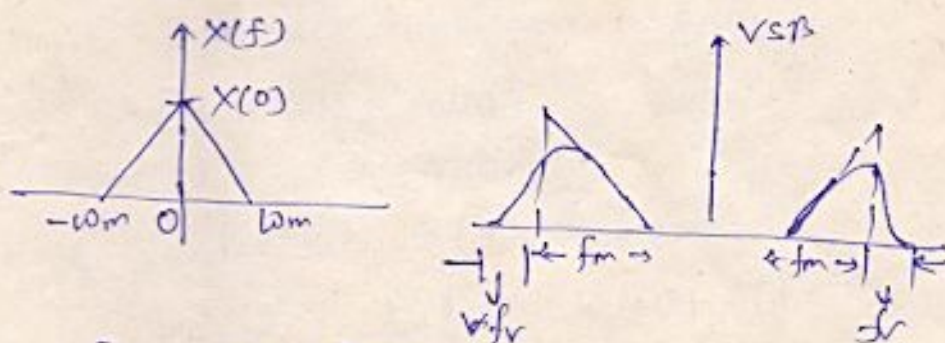
VSB [Vestigial sideband Modulation System] :-

A VSB system is compromise b/w DSB-SC and SSB modulation system, because it can take advantage of both.

- (a) The generation of VSB is easier than other modulated signals.
- (b) Its B.W is slightly (25%) higher than SSB but considerable less than DSB-SC.
- (c) SSB when signals contains very low frequency component the USB and LSB are of the translated signal tend to meet at the carrier frequency. So it become very difficult to isolate one side band to other side band.
SSB become unscitable for that.

This difficulties overcome by VSB in which gradual cutoff of one side band is allowed.

This gradual cutoff is compensated by a vestige or portion of the other sideband.

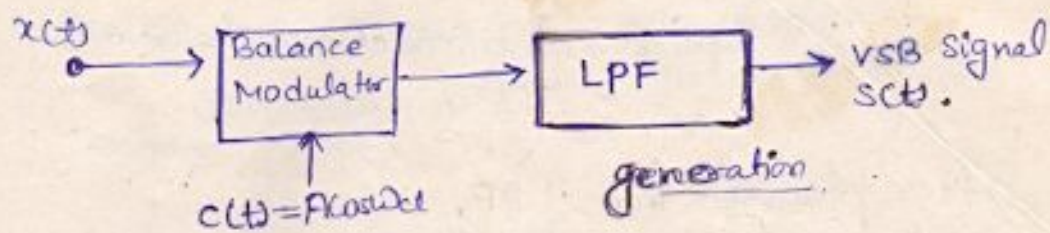


$f_v = \text{width of VSB}$

$$\text{B.W} = f_c + f_v - f_c + f_m$$

$$\underline{\text{B.W}} = f_m + f_v$$

Generation of VSB Signal → Basically VSB can be generated by passing a DSB-SC signal through an appropriate filter having transfer function $H(f)$.

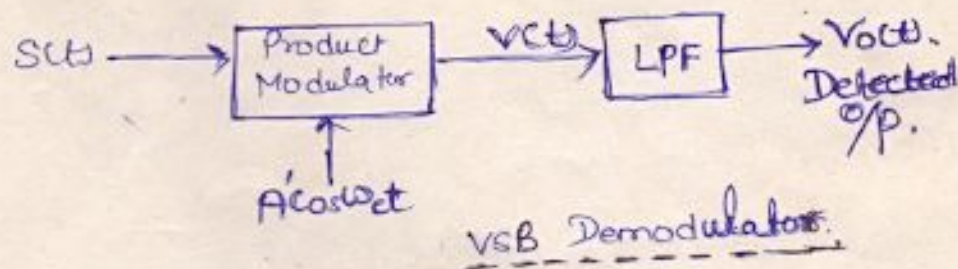


the frequency spectrum of VSB signal.

$$S(f) = F[s(t)]$$

$$= \frac{A}{2} [X(f-f_c) + X(f+f_c)] H(f). \quad \text{--- (1)}$$

where $X(f) = F[x(t)]$.



Output of product modulator is given by,

$$v(t) = s(t) A' \cos \omega t$$

$$v(t) = A' s(t) \cos \omega t$$

taking Fourier transform, we have

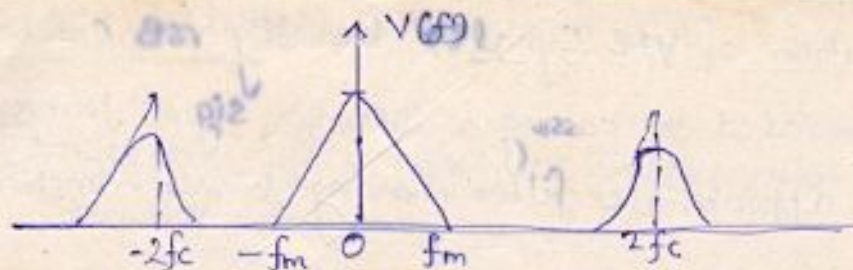
$$V(f) = \frac{A'}{2} [s(f-f_c) + s(f+f_c)]$$

$$V(f) = \frac{AA'}{4} [X(f-2f_c) + X(f)] H(f-f_c)$$

$$+ \frac{AA'}{4} [X(f) + X(f+2f_c)] H(f+f_c)$$

$$\text{or } V(f) = \frac{AA'}{4} [H(f-f_c) + H(f+f_c)] X(f)$$

$$+ \frac{AA'}{4} [X(f-2f_c) H(f-f_c) + X(f+2f_c) H(f+f_c)]$$



Spectrum of product modulator output $V(f)$.

after passing from LPF,

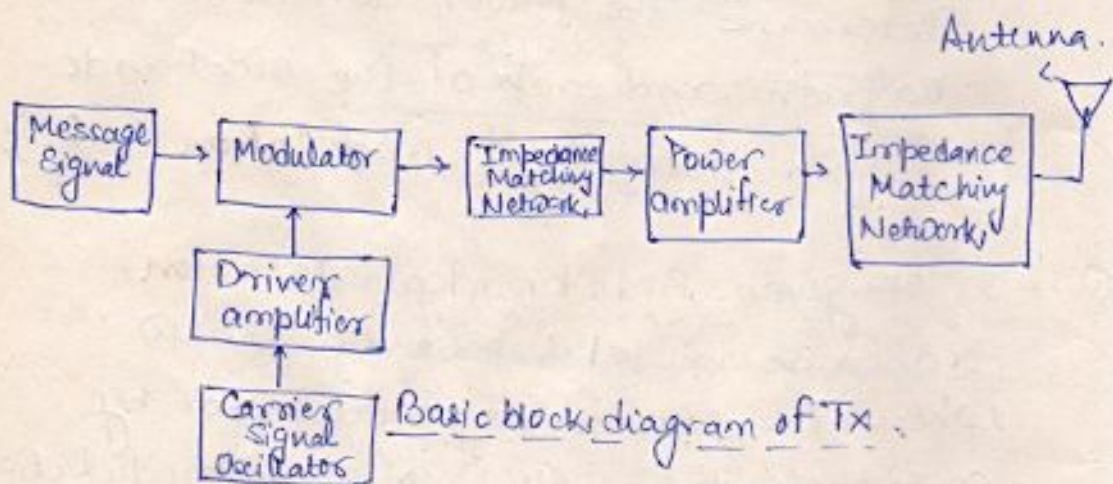
$$V_o(f) = \frac{AA'}{4} [H(f+f_c) + H(f-f_c)] X(f) H'(f)$$

where,

$$H'(f) = \frac{k}{H(f+f_c) + H(f-f_c)}$$

Basic functions of transmitter

- (a) The transmitter must generate a signal of correct frequency at a desired point in the spectrum.
- (b) Secondly, it must provide some form of modulation to modulate the carrier.
- (c) Third, it must provide sufficient power amplification in order to carry the modulated signal to a long distance.



Three important blocks of Tx section are,

- (i) Power amplifier.
- (ii) Impedance Matching network
- (iii) Antenna.

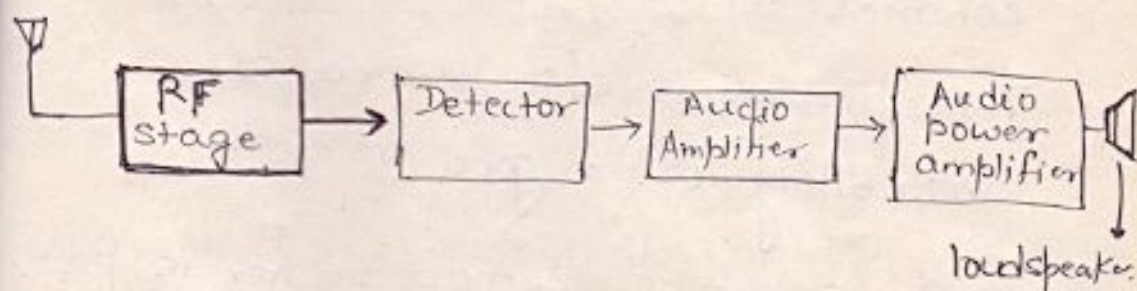
Functions of a Receiver :->

- ① Intercept the incoming modulated signal by the receiving antenna.
- ② Select the desired signal and reject the unwanted signal.
- ③ Amplify the selected RF signal.
- ④ Detect the modulated signal to get back the original modulating or baseband signal.
- ⑤ Amplify the modulating frequency signal.

Depending upon the fundamental aspects, the radio receivers may also be classified as :-

- (a) Tuned Radio Frequency [TRF] Receiver
- (b) Superheterodyne Receiver.

Tuned Radio Frequency Receivers :->



Drawback of TRF receivers:-

(i) The TRF receiver suffers from a tendency to oscillate at higher frequency from the multistage RF amplifiers ~~and~~ with high gain and operating at same frequency. If such an amplifiers has a gain of 20,000 then if a small portion of the output leaked back to the input of the RF stage, the positive feedback and Oscillation will result.

This type of leakage occur due to power supply coupling, stray capacitance coupling, radiation ~~and~~ coupling or coupling through any other element common to the input and output stages.

This problem is also termed as instability of a good receivers.

(ii) The selectivity of TRF receiver is poor. In fact, it is difficult to achieve sufficient selectivity at high frequencies due to the enforced use of single tuned circuits.

(iii) Another problem associated with TRF receiver is bandwidth variation over tuning range. For example in AM broadcast system, let us consider that a tuned circuit is required to have bandwidth of 10 kHz at a frequency of 540 kHz.

Quality factor (Q) of this tuned circuit, $Q = \frac{f_r}{B.W} = \frac{540}{10} = 54.$

At the other end of this AM broadcast band (1640 kHz),

$$Q = \frac{1640}{10} = 164.$$

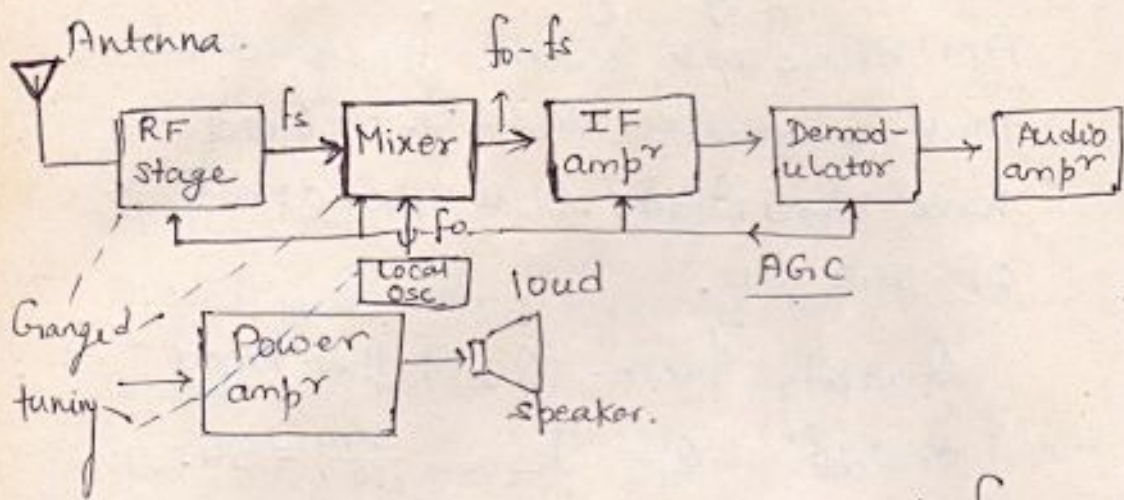
However due to several losses dependent upon frequency would prevent such a large increase. Thus practically the Quality factor Q of this tuned circuit is unlikely to exceed 120 and thus.

~~$$B.W = Q \times f_r = 120 \times 1640$$~~

$$B.W = \frac{f_r}{Q} = \frac{1640}{120} = 13.8 \text{ kHz.}$$

due to this increase bandwidth 13.8 kHz receiver would pick up or select adjacent frequency.

Superhetrodyne Receivers :-



$$f_{IF} = f_o - f_s = \text{Intermediate frequency.}$$

advantages of superhetrodyne receivers

- (i) No variation in bandwidth. The B.W. remains constant over the entire operating range.
- (ii) High sensitivity and selectivity.
- (iii) High adjacent channel rejection.

Receiver characteristics:-

- (a) Selectivity
- (b) sensitivity.
- (c) Fidelity
- (d) Double spotting.
- (e) Tracking.

Image frequency and Its Rejection \rightarrow

$$f_o = f_{IF} + f_s$$

if a frequency f_i manages to reach the mixer, such that

$$f_i = f_o + f_{IF}$$

$$f_o = f_s + f_{IF}$$

then $f_i = f_s + 2f_{IF}$

f_i = Image frequency.

The rejection of an image frequency signal by a single tuned circuit may be defined as the ratio of the gain at the signal frequency to the gain at the image frequency.

$$d = \sqrt{1 + Q^2 \rho^2}$$

$$\rho = \frac{f_{si} - f_s}{f_i}$$

mt