**Unit 2 (Differential Calculus-II)**

# Important Questions

1. Expand $tan^{-1}\frac{y}{x}$ in the neighbourhood of $(1,1)$ upto and inclusive of second degree terms. Hence compute $f(1.1, 0.9)$.
   **Ans: 0.6857**
2. Expand $(x^2y + \sin y + e^x)$ in powers of $(x-1)$ and $(y-\pi)$ taylor's theorem.
   **Ans:** $\boldsymbol{\pi + e + (x-1)(2\pi + e) + \frac{1}{2}(x-1)^2(2\pi + e) + 2(x-1)(y-\pi) + \cdots}$
3. If $y_{1=}\frac{x_2x_3}{x_1}$ , $y_{2=}\frac{x_1x_3}{x_2}$ , $y_{3=}\frac{x_1x_2}{x_3}$ then show that $\frac{\partial(y_1, y_2, y_3)}{\partial(x_1, x_2, x_3)} = 4$.
4. If $u, v, w$ are the roots of the equation $(x-a)^3 + (x-b)^3 + (x-c)^3 = 0$, then find $\frac{\partial(u, v, w)}{\partial(a, b, c)}$.
   **Ans:** $\boldsymbol{-\left[\frac{2(a-b)(b-c)(c-a)}{(u-v)(v-w)(w-u)}\right]}$
5. If $u = x + 2y + z$, $v = x - 2y + 3z$ and $w = 2xy - xz + 4yz - 2z^2$, show that they are not independent. Find the relation between $u, v \; and \; w$.
   **Ans:** $\boldsymbol{u^2 - v^2 = 4w}$
6. **(i)** Find the possible percentage error in computing the parallel resistance $r$ of three resistances $r_1, r_2, r_3$ from the formula $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$ if $r_1, r_2, r_3$ are each in error by +1.2%.
   **Ans: 1.2%**
   **(ii)** If the base radius and height of a cone are measured as 4 and 8 inches with a possible error of 0.04 and 0.08 inches respectively, calculate the percentage error in calculating volume of the cone.
   **Ans: 3%**
7. In estimating the cost of a pile of bricks measured as 6m×50m×4m, the tape is stretched 1% beyond the standard length. If the count is 12 bricks in 1 $m^3$ and bricks cost Rs. 100 per 1,000. Find the approximate error in the cost.
   **Ans: 43.20 Rs.**
8. Examine for extreme values of $x^3 + y^3 - 63(x + y) + 12xy$.
   **Ans: maximum value 784, minimum value -216.**
9. In a plane triangle ABC, find the maximum value of $\cos A \cos B \cos C$.
   **Ans:** $\boldsymbol{\frac{1}{8}}$
10. Find the dimensions of a rectangular box of maximum capacity whose surface area is given when
    (i) box is open at the top
    (ii) box is closed.

    **Ans: (i)** $\boldsymbol{x = y = \sqrt{\frac{S}{3}}, z = \frac{1}{2}\sqrt{\frac{S}{3}}}$ **(ii)** $\boldsymbol{x = y = z = \sqrt{\frac{S}{6}}}$