Solution REE 501

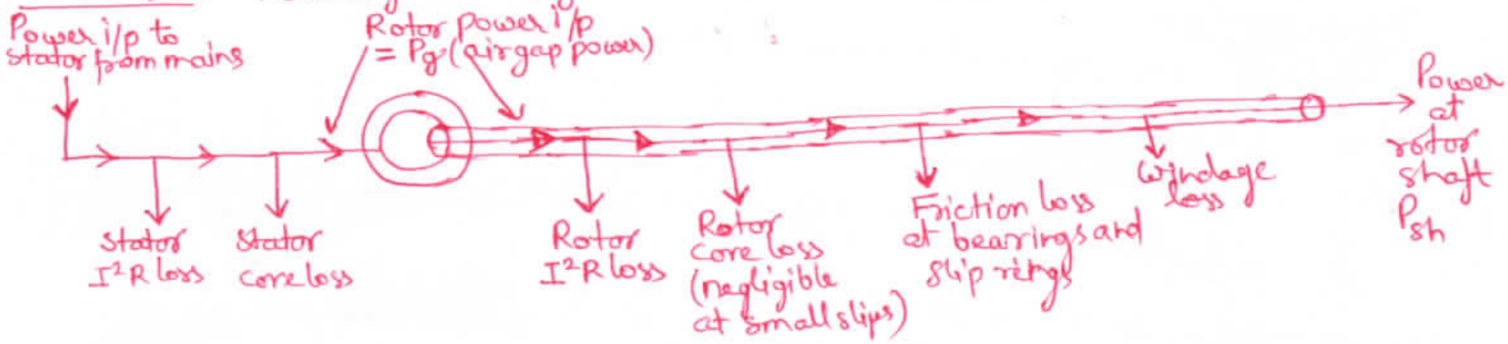
Date of Exam: 1/10/19

Ans 1(a): Condition for Max. torque $R_2 = sX_2$

Magnitude of max. torque $T_{max} = \frac{KE_2^2}{2X_2}$

where K = constant $= \frac{3}{\omega_s}$, R_2 = rotor resistance at standstill, X_2 = rotor reactance at standstill, E_2 = rotor induced emf at stand-still, s = full-load slip.

Ans 1(b): Power flow diagram:



Ans 1(c): zero

Ans 1(d): $\frac{T_{st}}{T_{Fl}} = \left(\frac{5I_{FL}}{I_{Fl}}\right)^2 \times 0.04 = 1$ i.e $T_{st} = T_{FL}$

Ans 1(e): i) The number of slots in rotor should not be equal to the number of slots in the stator.

ii) skewing of the rotor slots, that means the stack of the rotor is arranged in such a way that it angled with axis of rotation.

Ans 2(a): $T_{max} = \frac{KE_2^2}{2X_2}$ & $T_d = \frac{KsE_2^2 R_2}{R_2^2 + s^2X_2^2}$

$$\therefore \frac{T_{max}}{T_d} = \frac{\cancel{K}\cancel{E_2^2}}{2X_2} \times \frac{R_2^2 + s^2X_2^2}{\cancel{K}s\cancel{E_2^2}R_2} = \frac{R_2^2 + s^2X_2^2}{2sR_2X_2}$$

divide ~~and multiply~~ by X_2^2 in the numerator and denominator of the above equation in R.H.S only. $\left\{\because \frac{R_2}{X_2} = s_{max}\right\}$

$$\frac{T_{max}}{T_d} = \frac{\frac{R_2^2}{X_2^2} + \frac{s^2X_2^2}{X_2^2}}{\frac{2sR_2X_2}{X_2^2}} = \frac{s_{max}^2 + s^2}{2s\,s_{max}}$$

$$\Rightarrow \frac{T_{max}}{T_d} = \frac{1}{2}\left[\frac{s}{s_{max}} + \frac{s_{max}}{s}\right]$$ hence proved.

Ans 2(b): Necessity of starter:

i) To reduce the heavy starting current

ii) To provide overload and undervoltage protection.

Auto-transformer starting

→ Reduced voltage for starting can be obtained from three autotransformers connected in star as shown in figure below,

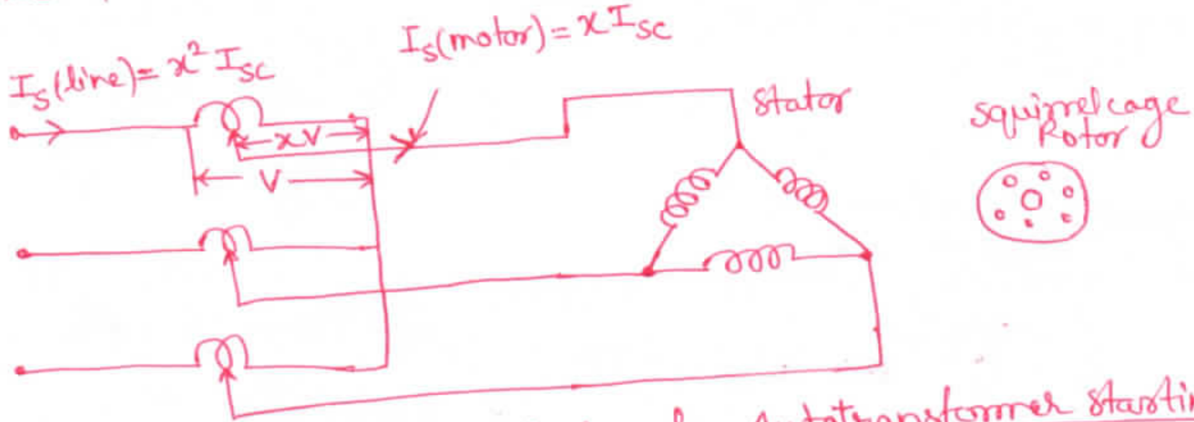


fig: Connection diagram for Autotransformer starting

→ If the voltage is reduced to a fraction "x" of the rated voltage V, the motor starting current is $I_s = \frac{xV}{Z_{sc}} = xI_{sc}$ —— (1)

→ If no-load current of autotransformer is neglected, then per phase output VA of autotransformer must be equal to its per phase input VA.

i.e $I_{st\,(line)} \times V = xV$ (per phase starting current in motor winding)

$I_{st(line)}\,V = xV(xI_{sc})$

∴ per phase starting current or line current from supply mains,

$I_{st} = x^2 I_{sc}$ —— (2)

$$\therefore \frac{T_{st}}{T_{FL}} = \left(\frac{I_{st}}{I_{FL}}\right)^2 s_{FL} = \left(\frac{xI_{sc}}{I_{FL}}\right)^2 = x^2\left(\frac{I_{sc}}{I_{FL}}\right)^2 s_{FL}$$

→ It is found that while starting torque is reduced to a fraction x^2 of that obtained in direct starting, the starting current is also reduced by the same fraction.

→ Smooth starting and high acceleration are possible by gradually raising the voltage to the full line value.

Ans 2(c) : Purpose of Double-cage rotor : It is used for obtaining high starting torque at a low value of starting current.

Double Cage rotors : (Working and Construction)

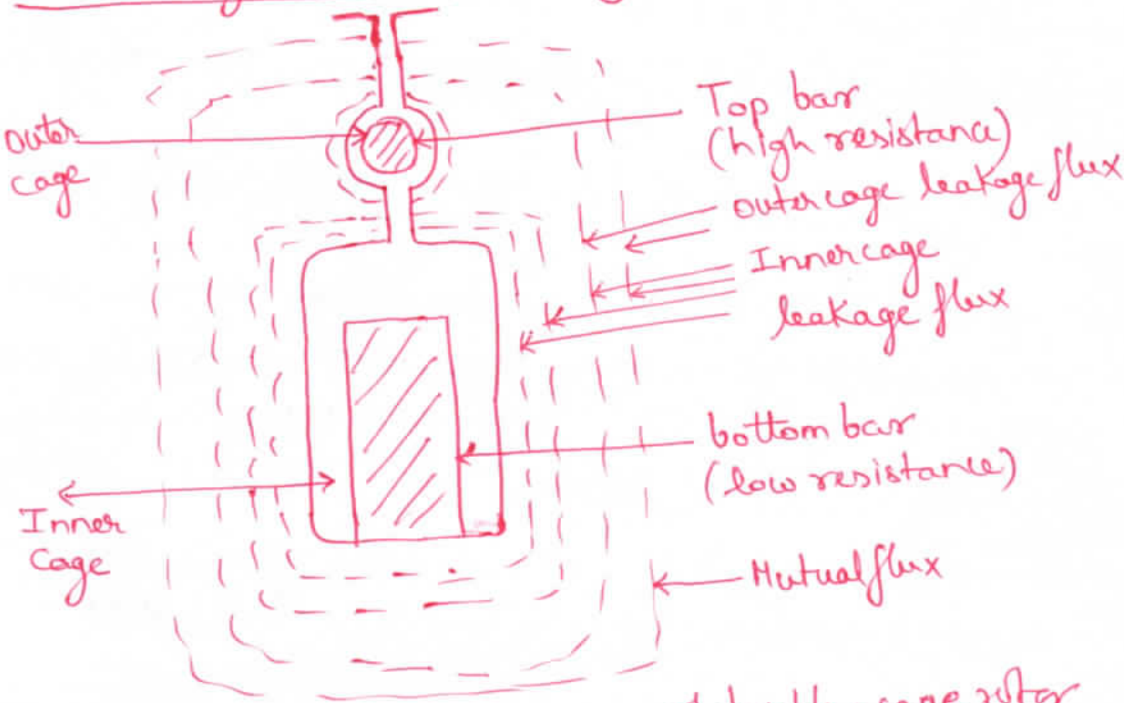


Fig : Constructional diagram of double-cage rotor

→ The stator of a double cage rotor of an induction motor is same as that of a normal induction motor. In double cage rotor of an induction motor, there are two layers of the bars. The figure of the double cage induction motor is shown above.

→ Each layer is short-circuited by the end rings. The outer cage bar have a smaller cross-sectional area than the inner bars and are made of high resistivity materials like brass, aluminium, broze etc. The bars of the inner cage are made of low resistance copper. Thus the resistance of the outer cage is greater than the resistance of the inner cage.

→ There is a slit between the top and the bottom slots. The slit increases the linking of leakage flux with the inner cage much larger than that of the outer cage. Thus inner cage has a greater self-inductance

→ At starting, the voltage induced in the rotor is same as the supply frequency that is ($f_2 = f_1$). Hence, the leakage reactance of the inner cage as compared to that of the outer cage winding is much larger. The outer cage winding carries most of the starting current which offers low impedance to the flow of current. The high resistance outer cage winding therefore, develops a high starting torque.

→ As the rotor speed increases, the frequency of rotor emf ($f_2 = sf_1$) decreases. At normal operating speed, the leakage reactance of both the windings becomes negligibly small. The current in the rotor divides between the two cages and is governed by their resistances. The resistance of the outer cage is about ·5 to 6 times that of the inner cage. Hence, the torque of the motor developed mainly by the low resistance inner cage and is developed under normal operating speed.

Ans 2(d): Slip-energy-Recovery: In this method, slip frequency, adjustable voltages are inserted in the rotor circuit for induction motor speed control

The slip power can either be returned to supply or added to main motor shaft output.

When slip power is returned to or taken from the supply mains, then the scheme is known as Constant-torque drive and when slip power is added or taken from the shaft power of the main motor then the scheme is said to be constant-power drive.

Constant-torque drive: For obtaining a constant-torque drive, the slip power sP_g is either returned to or taken from the supply mains.

Take a example; let motor input $P_i = 250\,KW = P_g$ and its synchronous speed $n_s = 250$ rad/sec

Then at slip $s = 0.2$ i.e below sub-synchronous speed, the power flow will be; (as shown in fig. below):

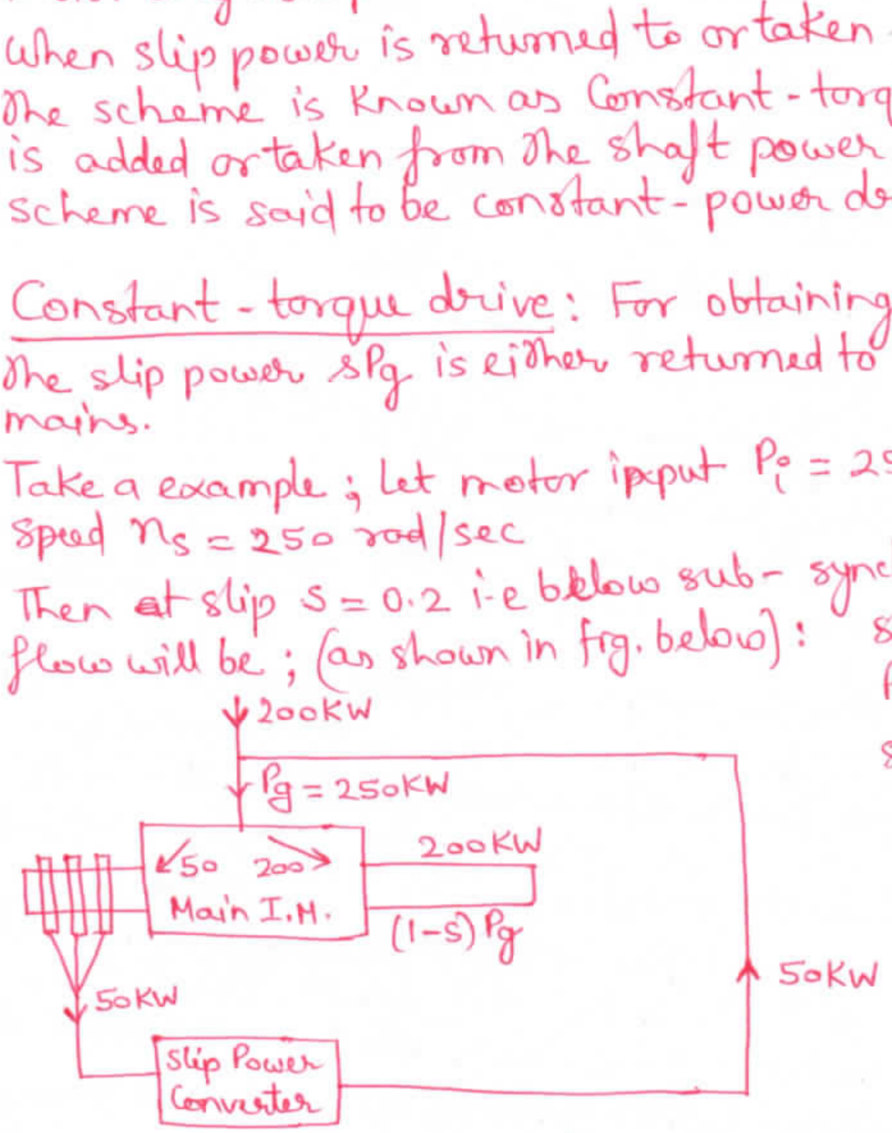


fig: Constant-torque drive: Slip power returned to supply.

shaft power o/p $P_m = (1-s)P_g$

$P_m = (1-0.2)250 = 200\,KW$

shaft speed $\omega_r = (1-s)\,\omega_s$

$\omega_r = (1-0.2)250$

$\omega_r = 200$ rad/sec

$\therefore$ Torque $= \dfrac{P_g}{\omega_s} = \dfrac{250 \times 1000}{250}$

Torque $= 1000$ N·m

Now, At slip $s = -0.2$ i.e above synchronous speed, the power is shown in fig below: In this figure –

Shaft power output, $P_m = (1-s)P_g = (1+0.2)250 = 300\,KW$

shaft speed $\omega_r = (1-s)\,\omega_s = (1+0.2)250 = 300$ rad/sec

$\therefore$ Torque $= \dfrac{P_g}{\omega_s} = \dfrac{(1-s)P_g}{(1-s)\,\omega_r} = \dfrac{300 \times 1000}{300} = 1000$ rad/sec

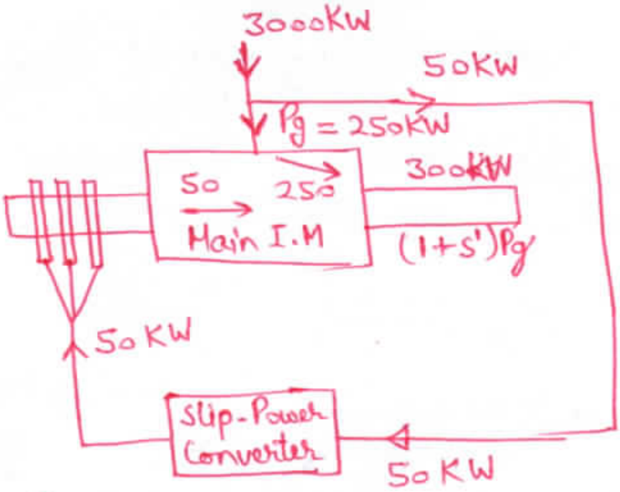


fig: Constant-torque drive: slip power taken from the supply mains.

→ Hence constant-torque drive can be used to obtain sub-synchronous as well as super-synchronous speeds.

Ans 3a): Double-Revolving Field Theory:

→ It states that a stationary pulsating magnetic field can be resolved into two rotating magnetic fields, each of equal magnitude but rotating in opposite directions at synchronous speed.

→ The induction motor responds to each magnetic field separately and the net torque in the motor is equal to the sum of the torques due to each of the two magnetic fields.

→ The equation for an alternating magnetic field whose axis is fixed in space is give by: $b(\alpha) = B_{max} \sin \omega t \cos \alpha$ —— ①

where B_{max} is the maximum value of the sinusoidally distributed air gap flux density produced by a properly distributed stator winding carrying an alternating current of frequency ω. And α is space-displacement angle measured from the axis of the stator winding.

→ Eq. ① can be written as: $b(\alpha) = \frac{B_{max}}{2} 2 \sin \omega t \cos \alpha$

$$b(\alpha) = \frac{B_{max}}{2}\left[\sin(\omega t - \alpha) + \sin(\omega t + \alpha)\right]$$

$$b(\alpha) = \underbrace{\frac{1}{2} B_{max} \sin(\omega t - \alpha)}_{\text{1st term}} + \underbrace{\frac{1}{2} B_{max} \sin(\omega t + \alpha)}_{\text{2nd term}} \quad \text{—— ②}$$

where, $\frac{1}{2} B_{max} \sin(\omega t - \alpha)$ = forward field revolving in the +ve "α" direction at synchronous speed.

$\frac{1}{2} B_{max} \sin(\omega t + \alpha)$ = backward field revolving in the −ve "α" direction at synchronous speed.

→ Both terms (1st and 2nd) has same frequency as the stationary magnetic field alternates.

→ When the rotor is stationary (i.e at standstill), the induced voltages are equal and opposite. Consequently, the two torques are also equal and opposite. Hence at standstill the net torque is zero. Hence 1-ϕ I.M. with single-stator winding inherently has no starting torque.

Ans 3(b): $N_S = \frac{120 \times 50}{4} = 1500$ rpm. $I_{2st} = \frac{E_2}{\sqrt{R_2^2+X_2^2}} = \frac{120}{\sqrt{(0.2)^2+(1)^2}} = 117.67A$ Ans

$$\cos\phi_{2st} = \frac{R_2}{\sqrt{R_2^2+X_2^2}} = \frac{0.2}{\sqrt{(0.2)^2+(1)^2}} = 0.196 \text{ Ans}$$

$$\omega_s = \frac{2\pi N_s}{60} = \frac{2\pi(1500)}{60} = 50\pi \text{ rad/sec.}$$

$$T_{st} = \frac{3}{\omega_s}(I_{2st})^2 \frac{r_2}{1} = \frac{3}{50\pi}(117.67)^2 \times (0.2) = 52.9 \text{ N-m Ans}$$

$$s_{F.L} = \frac{1500-1440}{1500} = 0.04 \quad (N_r = 1440 \text{ given})$$

$$I_{2F.L} = \frac{sE_2}{\sqrt{R_2^2+(sX_2)^2}} = \frac{(0.04)(120)}{\sqrt{(0.2)^2+(0.04\times 1)^2}} = \frac{4.8}{0.204} = 23.53A \text{ Ans}$$

$$\cos\phi_{2F.L} = \frac{R_2}{\sqrt{R_2^2+(sX_2)^2}} = \frac{0.2}{\sqrt{(0.2)^2+(0.04\times 1)^2}} = 0.98 \text{ Ans}$$

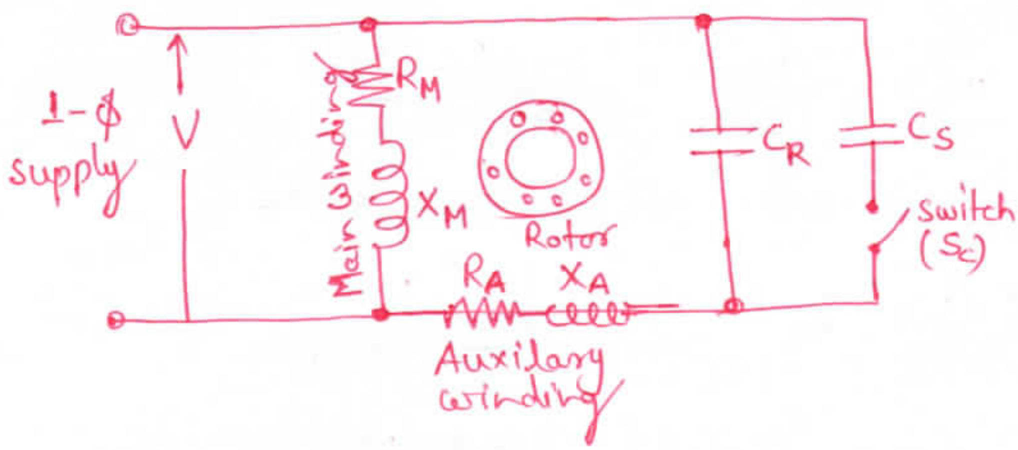
$$T_{F.L} = \frac{3}{\omega_s}(I_{2F.L})^2 \frac{R_2}{s} = \frac{1}{50\pi} \times 3\,(23.53)^2 \times \frac{0.2}{0.04} = 52.87 \text{ Nm Ans}$$

$$\frac{I_{2st}}{I_{2F.L}} = \frac{117.67}{23.53} \simeq 5.00 \text{ Ans} \qquad \frac{T_{st}}{T_{F.L}} = \frac{52.90}{52.87} \cong 1.00 \text{ Ans}$$

Ans 4(a): Construction and Working of Capacitor start and Capacitor run motor

→ It has a cage rotor and its stator has two windings namely the main winding and auxilary winding which are displaced 90° in space.

→ The motor uses two capacitors C_S (starting capacitor) and C_R (running capacitor). The two capacitors are connected in parallel at starting.

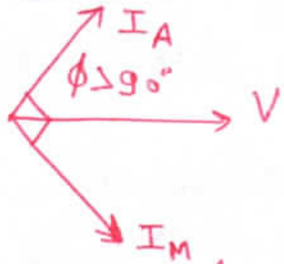


→ In order to obtain a high starting torque, a large current is required, for this purpose, the capacitive reactance in the starting winding should be low. For this pupose the value of C_s must be large. $\left(X \propto \frac{1}{C_S}\right)$. The C_s is short-time rated and it almost always elect--rolytic.

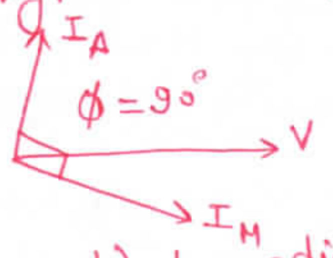
→ During normal operation, the rated line current is smaller than the starting current. Hence capacitive reactance should be large, so for this purpose value of C_R should be small.

→ As the motor approaches synchronous speed, C_s is disconnected by a centrifugal switch "S_c".

→ C_R is permanently connected in the circuit. It is long time rated for continuous running. It is usually of oil-filled paper construction.



a) phasor diagram when C_s & C_R both connected in the circuit



b) phasor diagram when C_s is disconnected from the circuit.

Ans 4(b): $Z_{1m} = (2.2 + j3.1)\Omega$

$$Z_f = \left(\frac{R_2'}{2s} + j\frac{X_2'}{2}\right) \| \left(\frac{jX_M}{2}\right) = \left(\frac{4.5}{2\times 0.03} + j\frac{2.6}{3}\right) \| \left(\frac{j80}{2}\right)$$

$$= (75 + j1.3) \| j40 = 16.369 + j30.985 = R_f + jX_f$$

$$Z_b = \left\{\frac{R_2'}{2(2-s)} + j\frac{X_2'}{2}\right\} \| \left(\frac{jX_M}{2}\right) = \left(\frac{4.5}{2(2-0.03)} + j\frac{2.6}{2}\right) \| \frac{j80}{2}$$

$$= (1.142 + j1.3) \| j40 = (1.070 + j1.288)\Omega = R_b + jX_b$$

Input current $I_m = \dfrac{V_m}{Z_{1m} + Z_f + Z_b} = \dfrac{230}{2.2 + j3.1 + 16.369 + j30.985 + 1.070 + j1.288}$

$$I_m = \frac{230}{19.639 + j35.373} = \frac{230\angle 0^\circ}{40.459\angle 60.961} = 5.684\angle -60.961\,A \text{ Ans}$$

developed Power, $P_m = (1-s)(R_f - R_b) I_m^2$

$$= (1 - 0.03)(16.369 - 1.070)(5.684)^2$$

$$P_m = 479.449 \text{ watts Ans}$$

Ans 5(a): Phenomenon of Crawling in 3-ϕ I.M.

→ It has been observed that squirrel cage type I.M. has a tendency to run at very low speed compared to its synchronous speed, this phenomenon is known as crawling.

→ The resultant speed is nearly 1/7th of its synchronous speed.

→ This is because of the fact that harmonics fluxes produces in the gap of the stator winding of odd harmonics like 3rd, 5th, 7th etc. These harmonics create additional torque fields in addition to the synchronous torque.

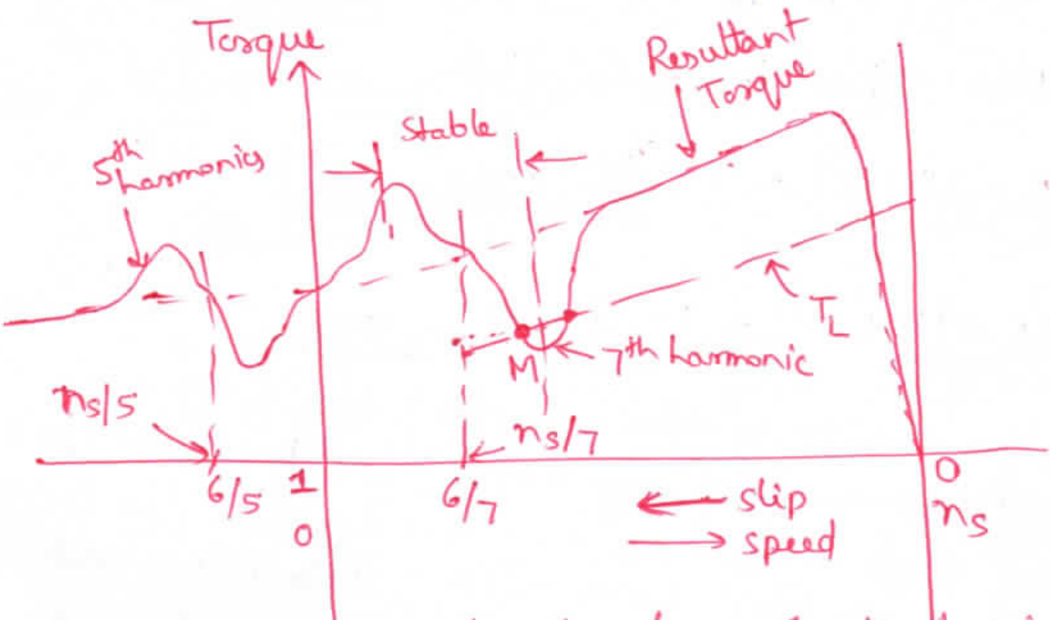
→ The torque produced by these harmonics rotates in forward or backward directions at $\frac{N_s}{3}, \frac{N_s}{5}, \frac{N_s}{7}$ speed respectively.

→ Since supply is assumed 3-ϕ balanced thats why 3rd harmonics may be neglected. Only 5th and 7th harmonics are considered.

→ The torque produced by the 5th harmonics rotates in the backward direction. Torque by 5th harmonics works as a braking torque and is small in magnitude, so it can be neglected.

→ where as seventh harmonics produces a forward rotating torque at synchronous speed $N_s/7$.

→ Hence the net forward torque is equal to the sum of the torque produced by 7^{th} harmonics and fundamental torque. As shown below.



Torque - slip characteristics of a 3-ϕ I.M. showing the effect of harmonics asynchronous (induction) torques.

→ The torque produced by 7^{th} harmonic reaches its maximum +ve value just below $\frac{1}{7}$ of N_s and at this point slip is high. At this stage motor does not reach upto its normal speed and continue to rotate at a speed which is much lower than its normal speed.

→ This causes crawling of the motor at just below $1/7\,N_s$ and create noise.

→ Crawling can be reduce by using short pitch winding.

Ans 5(b): PAM Technique: This technique is used where speed ratio other than 2:1 are required. Since mmf distribution in the air gap due to 3-ϕ balanced supply is given by:

$$F_A = F_{mA} \sin p\theta \quad \text{——} (1)$$

$$F_B = F_{mB} \sin(p\theta - 2\pi/3) \quad \text{——} (2)$$

$$F_C = F_{mC} \sin(p\theta - 4\pi/3) \quad \text{——} (3)$$

where p is the number of pairs of poles and θ is the mechanical angle in radians. Let 3-ϕ carrier wave which is balance and is given by:

$$F_{mA} = F \sin K\theta \quad \text{——} (4)$$

$$F_{mB} = F \sin(K\theta - \alpha) \quad \text{——} (5)$$

$$F_{mC} = F \sin(K\theta - 2\alpha) \quad \text{——} (6)$$

where F is a constant, K is the number of modulating cycles in one complete perimeter of the motor, $\alpha = \pm 2\pi/3$

put eq. 4, 5, 6 in eq. 1, 2 & 3 we get,

$F_A = F \sin p\theta \sin K\theta$ —— 7

$F_B = F \sin(p\theta - 2\pi/3) \sin(K\theta - \alpha)$ —— 8

$F_C = F \sin(p\theta - 4\pi/3) \sin(K\theta - 2\alpha)$ —— 9

Using trignometry using $\left[\sin A \cos B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]\right]$

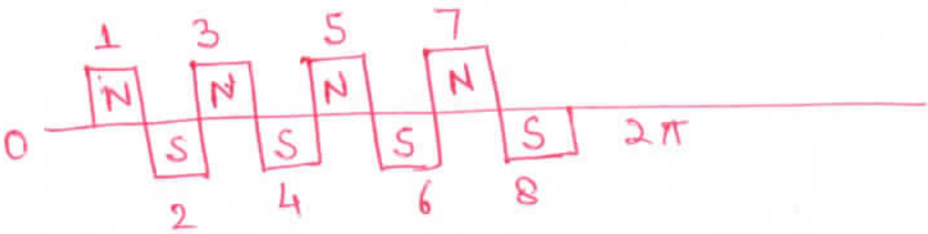
$F_A = \frac{F}{2}\left[\cos(p-K)\theta - \cos(p+K)\theta\right]$ —— 10

$F_B = \frac{F}{2}\left\{\cos\left[(p-K)\theta - \frac{2\pi}{3} + \alpha\right] - \cos\left[(p+K)\theta - \frac{2\pi}{3} - \alpha\right]\right\}$ —— 11

$F_C = \frac{F}{2}\left\{\cos\left[(p-K)\theta - \frac{4\pi}{3} + 2\alpha\right] - \cos\left[(p+K)\theta - \frac{4\pi}{3} - 2\alpha\right]\right\}$ —— 12

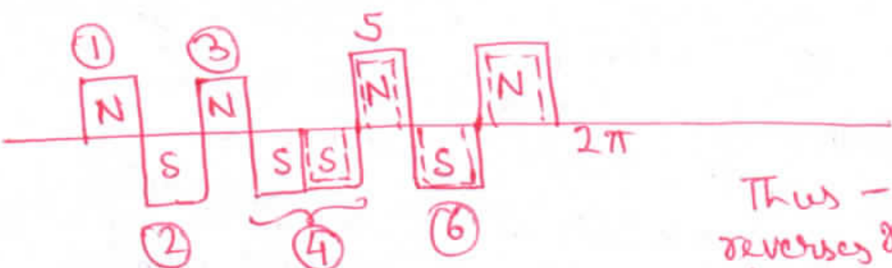
Thus, by modulating the amplitudes of the mmf in a 3-ϕ I.M. having "p" pair of poles, produces two sets of 3-ϕ mmfs with (p−K) and (p+K) poles. These two sets of poles will produce torques in opposite directions. To obtain steady torques in one direction only, one of these pole pairs must be suppressed and the other pair should be retain.

Coil Inversion method: In this method, a rectangular space mmf wave of unit amplitude and of period equal to the length of the stator periphery is used for modulation. Windings of each phase are divided into two parts. The current through the second half of winding in each phase is reversed.



mmf wave of a stator wound for 8-poles

2- pole modulating wave



Resultant modulated wave, it has 6 pole.

Thus −ve cycle of ~~resultant~~ modulating wave reverses the polarities of the main poles 5, 6, 7, & 8