Solution: Basic Electrical Engineering (KEE101)

Date of Exam: 3/10/19

Ans 1 a): Q-factor $= \tan\phi = \tan 45^\circ = 1$ Ans

Ans 1 b): $Z = 6 + j8$; $Y = \frac{6}{6^2+8^2} - j\frac{8}{6^2+8^2} = (0.06 - j\,0.08)\,\mho$

$\therefore$ conductance $= 0.06\,\mho$ and susceptance $= 0.08\,\mho$ Ans

Ans 1 c): given: $v = 100 \sin(314t + 55^\circ)$ volts ; $i = 15 \sin(314t + 325^\circ)$ Amp,

$\phi = 55 - 325 = -270^\circ$

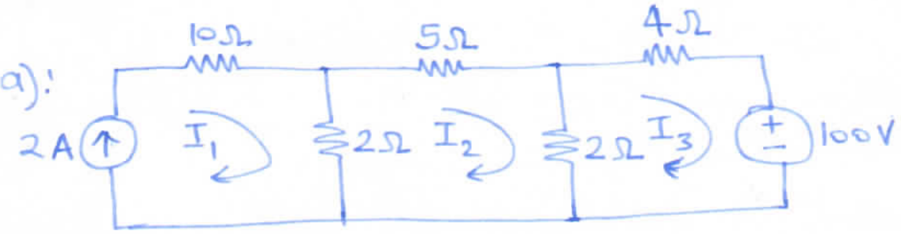
$\therefore$ Power $= \frac{V_{max}}{\sqrt{2}} \times \frac{I_{max}}{\sqrt{2}} \cos\phi = \frac{100}{\sqrt{2}} \times \frac{15}{\sqrt{2}} \cos(270) = 0$ Ans

Ans 1 d): power factor $= 1$ Ans

Ans 1 e): Internal resistance of ideal voltage source $= 0$ (zero)

Internal resistance of ideal current source $= \infty$ (infinite)

Ans 2 a):



$I_1 = 2$ A

KUL equation of mesh II : $-2(I_2 - 2) - 5I_2 - 2(I_2 - I_3) = 0$

$-9I_2 + 2I_3 = -4$ —— (I)

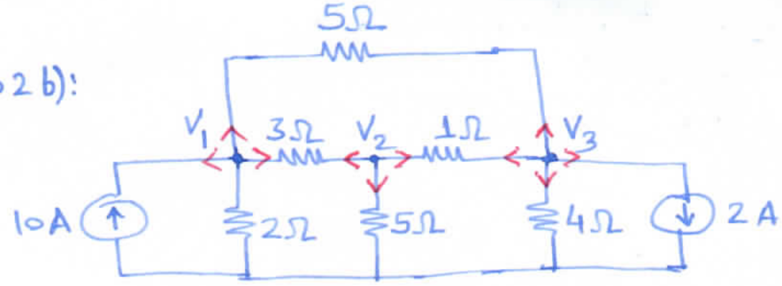
KUL equation of mesh III : $-2(I_3 - I_2) - 4I_3 - 100 = 0$

$2I_2 - 6I_3 = 100$ —— (II)

Solving eq. (I) & (II) we get, $I_2 = -3.52$ A $I_3 = -17.84$ A

$\therefore I = -I_2 = 3.52$ A Ans

Ans 2 b):



applying KCL at node-1

$$-10 + \frac{V_1}{2} + \frac{V_1 - V_2}{3} + \frac{V_1 - V_3}{5} = 0$$

$$\frac{-300 + 15V_1 + 10V_1 - 10V_2 + 6V_1 - 6V_3}{30} = 0$$

$$31V_1 - 10V_2 - 6V_3 = 300 \quad \text{--- (1)}$$

applying KCL at node-2

$$\frac{V_2 - V_1}{3} + \frac{V_2}{5} + \frac{V_2 - V_3}{1} = 0$$

$$\frac{5V_2 - 5V_1 + 3V_2 + 15V_2 - 15V_3 = 0}{15}$$

$$-5V_1 + 23V_2 - 15V_3 = 0 \quad \text{--- (2)}$$

applying KCL at node-3

$$\frac{V_3 - V_1}{5} + \frac{V_3 - V_2}{1} + \frac{V_3}{4} + 2 = 0$$

$$\frac{4V_3 + 4V_1 + 20V_3 - 20V_2 + 5V_3 + 40}{20} = 0$$

$$-4V_1 - 20V_2 + 29V_3 = -40 \quad \text{--- (3)}$$

solving eq. (1), (2) & (3) we get,

$V_1 = 12.06$ volts ; $V_2 = 5.1$ volts

$V_3 = 3.8$ volts

$$I_1 = \frac{V_1 - V_3}{5} = \frac{12.06 - 3.8}{5} = 1.652\text{ A}$$

$$I_2 = \frac{V_1 - V_2}{3} = \frac{12.06 - 5.1}{3} = 2.32\text{ A}$$

$$I_3 = \frac{V_1}{2} = \frac{12.06}{2} = 6.03\text{ A} \;;\; I_4 = \frac{V_2}{4} = \frac{5.1}{5} = 1.02\text{ A} \;;\; I_5 = \frac{V_3}{4} = \frac{3.8}{4} = 0.95\text{ A}$$

$$I_6 = \frac{V_2 - V_3}{1} = 5.1 - 3.8 = 1.3\text{ A}$$

Ans 2c): given $v = 100\sin(\omega t + 30^\circ)$ $\phi_1 = 30^\circ$

$i = 20\sin(\omega t + 60^\circ)$ $\phi_2 = 60^\circ$

$\therefore \phi = 30^\circ - 60^\circ = -30^\circ$ since ϕ is -ve current is leading

impedance $Z = \frac{V_{max}}{I_{max}} = \frac{100}{20} = 5\Omega$ Ans

Resistance $R = Z\cos\phi = 5\cos(30) = 4.33\Omega$ Ans

Reactance $X_C = Z\sin\phi = 5\sin(30) = 2.5\Omega$ (capacitive because current is leading)

power factor $= \cos\phi = \cos(30) = 0.866$ (leading) Ans

Power $= VI\cos\phi = \frac{100}{\sqrt{2}} \times \frac{20}{\sqrt{2}} \times \cos 30 = 866$ watt Ans

Ans 2 d): In a series resonant circuit, Quality factor is defined as, $Q\text{-factor} = \dfrac{\text{voltage across L or C at resonance}}{\text{voltage of the circuit at resonance.}}$

Consider case of inductor,

$$Q\text{-factor} = \frac{\text{voltage across 'L' at resonance}}{\text{voltage of the ckt at resonance}} = \frac{V_{L_0}}{V} = \frac{\cancel{I_{max}} X_{L_0}}{\cancel{I_{max}} R}$$

$$Q\text{-factor} = \frac{X_{L_0}}{R} = \frac{\omega_0 L}{R} = \frac{2\pi f_0 L}{R} \quad \text{using } f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

$$Q\text{-factor} = \frac{2\cancel{\pi} L}{R} \times \frac{1}{2\cancel{\pi}}\sqrt{\frac{1}{LC}} = \frac{1}{R}\sqrt{\frac{L^2}{LC}} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Now consider case of capacitor,

$$Q\text{-factor} = \frac{\text{voltage across 'C' at resonance}}{\text{voltage of the circuit at resonance}} = \frac{V_{C_0}}{V} = \frac{\cancel{I_{max}} X_{C_0}}{\cancel{I_{max}} R}$$

$$Q\text{-factor} = \frac{X_{C_0}}{R} = \frac{1}{\omega_0 CR} = \frac{1}{2\pi f_0 CR} = \frac{1}{2\cancel{\pi} CR \times \frac{1}{2\cancel{\pi}\sqrt{LC}}} = \frac{1}{CR}\sqrt{\frac{LC}{1}}$$

$$Q\text{-factor} = \frac{1}{R}\sqrt{\frac{LC}{C^2}} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Hence expression of quality factor in series resonant ckt $= \frac{1}{R}\sqrt{\frac{L}{C}}$

Ans 3a): $v_1 = 100 \sin 500t$; $v_2 = 200\sin\left(500t + \frac{\pi}{3}\right)$

$v_3 = -50\cos 500t = 50\sin\left(500t - \frac{\pi}{2}\right)$

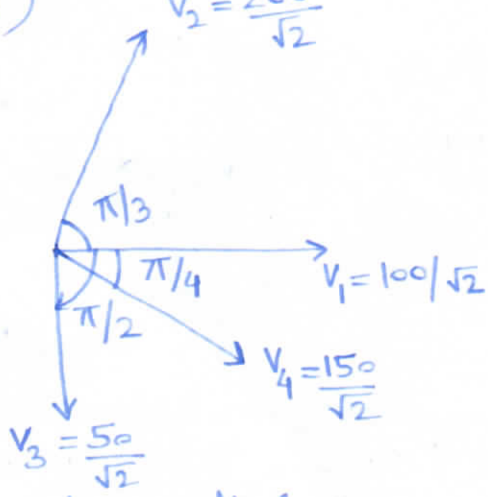
$v_4 = 150\sin\left(500t - \frac{\pi}{4}\right)$

$\phi_1 = 0^\circ$, $\phi_2 = \pi/3$ $\phi_3 = -\pi/2$ $\phi_4 = -\pi/4$

Resolving in horizontal component,

$V_x = 100\cos 0^\circ + 200\cos(\pi/3) + 50\cos\left(-\frac{\pi}{2}\right) + 150\cos\left(-\frac{\pi}{4}\right)$

$V_x = 100\times 1 + 200\times\frac{1}{2} + 50\times 0 + 150\times\frac{1}{\sqrt{2}} = 306.066\,\text{volts}$



phasor diagram

Resolving verticle component;

$$V_y = 100\sin 0^\circ + 200\sin\left(\frac{\pi}{3}\right) + 50\sin\left(\frac{-\pi}{2}\right) + 150\sin\left(\frac{-\pi}{4}\right)$$

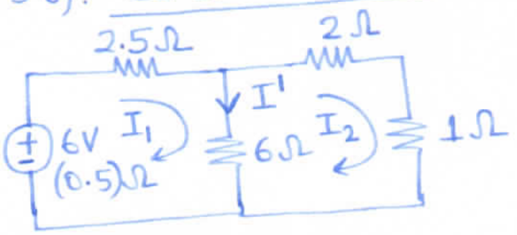
$$V_y = 100\times 0 + 200\times\frac{\sqrt{3}}{2} - 50\times 1 - 150\times\frac{1}{\sqrt{2}} = 17.139 \text{ volts}$$

Resultant magnitude; $V_{max_R} = \sqrt{V_x^2 + V_y^2} = \sqrt{(306.066)^2 + (17.139)^2}$

$V_{max_R} = 306.545$

$\therefore$ Resultant rms value, $V_{rms} = \frac{V_{max_R}}{\sqrt{2}} = \frac{306.545}{\sqrt{2}} = 216.76$ volts Ans

Ans 3 b): Consider 6V source:



KVL equation of mesh-I

$$-0.5I_1 + 6 - 2.5I_1 - 6(I_1 - I_2) = 0$$

$$-9I_1 + 6I_2 = -6 \quad \text{—— (1)}$$

KVL equation of mesh-II

$$-6(I_2 - I_1) - 2I_2 - 1I_2 = 0 \quad \text{—— (2)}$$

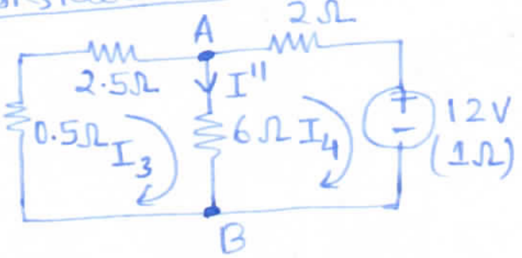
$$6I_1 - 9I_2 = 0$$

solving eq. (1) & (2) we get,

$I_1 = 1.2$ A , $I_2 = 0.8$ A

$\therefore I' = I_1 - I_2 = 1.2 - 0.8 = 0.4\text{A} (\downarrow)$

Consider 12 V source



KVL equation of mesh-I

$$-0.5I_3 - 2.5I_3 - 6(I_3 - I_4) = 0$$

$$-9I_3 + 6I_4 = 0 \quad \text{—— (3)}$$

KVL equation of mesh-II

$$-6(I_4 - I_3) - 2I_4 - 12 - 1I_4 = 0 \quad \text{—— (4)}$$

$$6I_3 - 9I_4 = 12$$

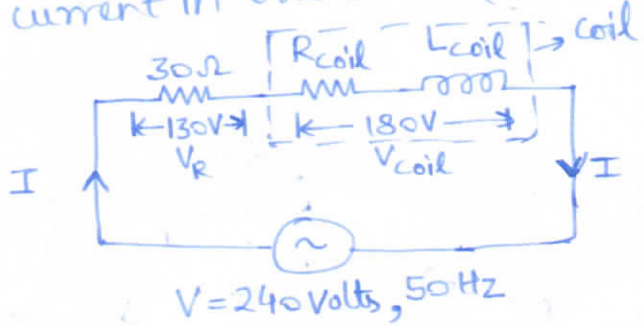
solving eq. (3) & (4) we get,

$I_3 = -1.6$ A , $I_4 = -2.4$ A

$\therefore I'' = I_3 - I_4 = -1.6 - (-2.4) = 0.8\text{A} (\downarrow)$

Hence current in branch AB (6Ω) $= I' + I'' = 0.4 + 0.8 = 1.2\text{A} (\downarrow)$ Ans

Ans 4 a):

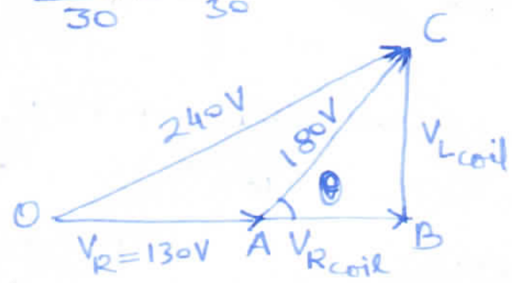


V=240 Volts, 50 Hz

current in the circuit,

$$I = \frac{V_R}{30} = \frac{130}{30} = 4.33 \text{ A}$$

Now draw phasor diagram of the circuit:



In ΔOBC: $OC^2 = OB^2 + BC^2$

$$OC^2 = (OA + AB)^2 + BC^2$$

$$(240)^2 = (130 + 180\cos\theta)^2 + (180\sin\theta)^2$$

$$(240)^2 = (130)^2 + (180)^2\cos^2\theta + 2\times130\times180\cos\theta + 180^2\sin^2\theta$$

$$(240)^2 = (130)^2 + (180)^2(\cos^2\theta+\sin^2\theta) + 2\times130\times180\cos\theta$$

$$(240)^2 = (130)^2 + (180)^2 + 2\times130\times180\cos\theta$$

$$\cos\theta = \frac{83}{468} = 0.17735$$

$$\theta = \cos^{-1}(0.177) = 79.78^\circ$$

$$\therefore V_{R_{coil}} = 180\cos\theta = 180\times0.177 = 31.92 \text{ volts}$$

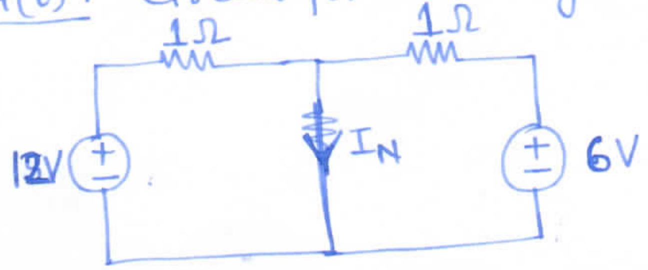
$$\therefore R_{coil} = \frac{V_{R\,coil}}{I} = \frac{31.92}{4.33} = 7.37\Omega \quad \underline{\text{Ans}}$$

$$V_{L_{coil}} = 180\sin(79.78) = 177.14 \text{ volts}$$

$$X_{L_{coil}} = \frac{V_{L\,coil}}{I} = \frac{177.14}{4.33} = 40.91\,\Omega$$

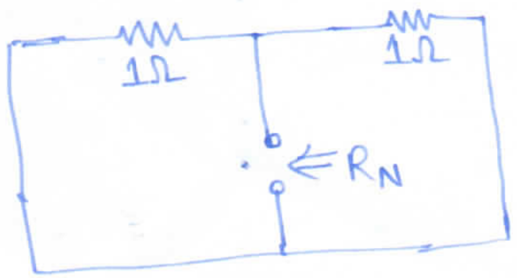
$$\therefore L_{coil} = \frac{X_{L\,coil}}{2\pi f} = \frac{40.91}{2\pi\times50} = 130.22 \text{ mH} = 0.1302\text{ H} \quad \underline{\text{Ans}}$$

Ans 4(b): Circuit for calculating I_N (norton's current)



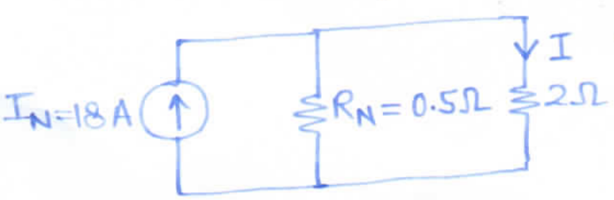
$$I_N = I_1 + I_2 = \frac{12}{1} + \frac{6}{1} = 18\text{ A}$$

Circuit for calculating R_N (Norton's resistance)



$$R_N = 1\,||\,1 = \frac{1}{2} = 0.5\Omega$$

Draw Norton's Equivalent circuit



$$\therefore I = \frac{18\times0.5}{0.5+2} = 3.6\text{ A} \quad \underline{\text{Ans}}$$

Ans 5a): given $R = 2\Omega$, $L = 2mH = 2\times10^{-3}H$, $C = 10\mu F = 10\times10^{-6}F$

resonant frequency $f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = \frac{1}{2\pi}\sqrt{\frac{1}{2\times10^{-3}\times10\times10^{-6}}} = 1125.39\ Hz$ Ans

$Q\text{-factor} = \frac{2\pi f_0 L}{R} = \frac{2\pi\times1125.39\times2\times10^{-3}}{2} = 7.071$ Ans

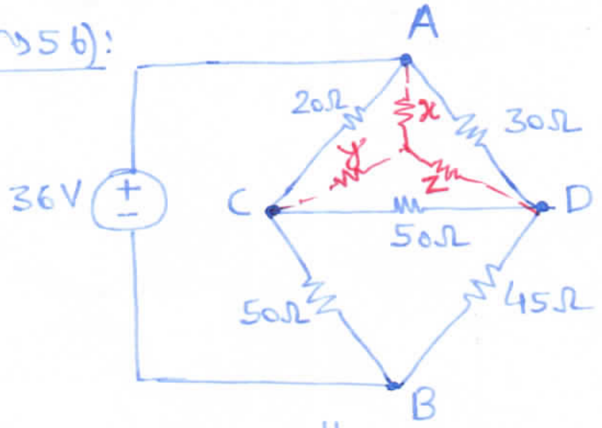
$\text{Bandwidth}(\Delta f) = \frac{R}{2\pi L} = \frac{2}{2\pi\times2\times10^{-3}} = 159.15\ Hz$ Ans

Half power frequencies,

$f_1 = f_0 - \frac{R}{4\pi L} = 1125.39 - \frac{159.15}{2} = 1045.81\ Hz$ Ans

$f_2 = f_0 + \frac{R}{4\pi L} = 1125.39 + \frac{159.15}{2} = 1204.96\ Hz$ Ans

Ans 5b):



using delta to star conversion in network ADC,

$x = \frac{20\times30}{20+30+50} = 6\Omega$; $y = \frac{20\times50}{20+30+50} = 10\Omega$

$z = \frac{30\times50}{20+30+50} = 15\Omega$

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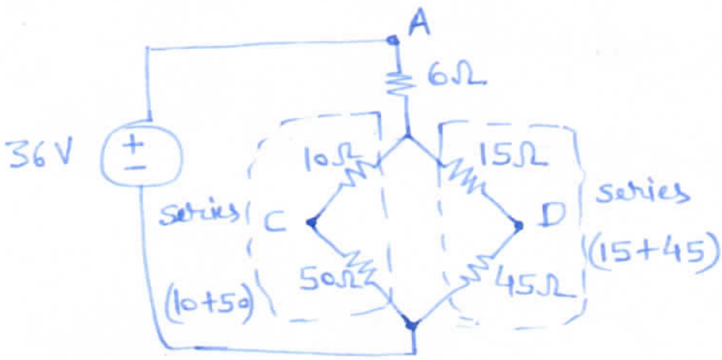

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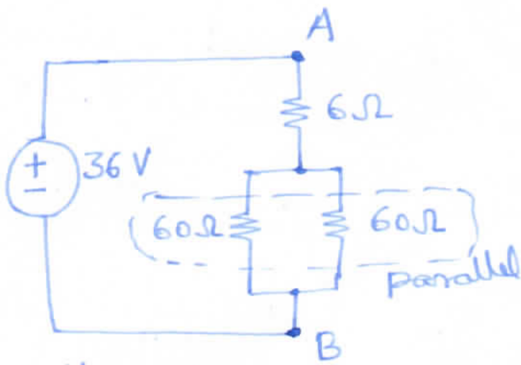
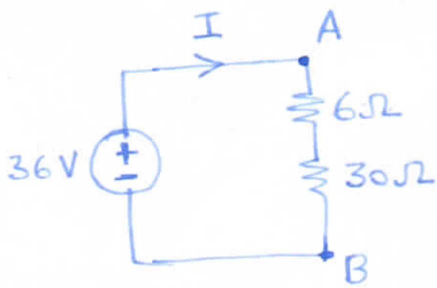


$60\|60 = \frac{60\times60}{60+60} = 30\Omega$



$I = \frac{36}{6+30} = 1\ A$ Ans