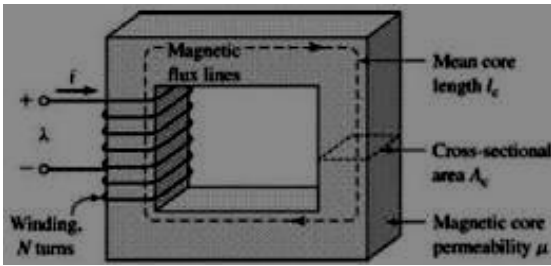


Magnetic Circuit Concepts:

- **Magnetic Circuit:** A closed path followed by magnetic flux is known as magnetic circuit (shown in diagram below), just like a closed path followed by current is known as electric circuit.



- **Magneto-Motive Force (MMF):** It is the force required for establishing magnetism or magnetic field in any conductor or coil or circuit. It is given by,

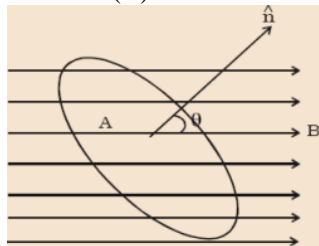
$$\text{MMF} = NI \text{ ampere-turn (AT)}$$

where, N = number of turn in the coil.

I = current in the coil.

AT= Ampere-turn, unit of MMF.

- **Magnetic Flux (ϕ):** The magnetic flux (ϕ) linked with a surface held in a magnetic field (B) is defined as the number of magnetic lines crossing a closed area (A). as shown below-



If θ is the angle between the direction of the field and normal to the area then $\phi = B \cdot A$

$$\phi = BA \cos \theta$$

$$\text{if } \theta = 0^\circ, \text{ then, } \phi = BA \text{ weber (Wb)}$$

- **Magnetic Flux Density:**

The flux per unit area, measured in a plane perpendicular to flux is defined as the flux density (B).

i.e

$$B = \phi / A$$

It has unit in weber per meter- square or Tesla

- **Magnetic Field Strength or Intensity or Magnetising force:**

MMF per unit length of a coil in magnetic field is defined as Magnetic Field Strength. It is denoted by ' H ' given by,

$$H = \frac{NI}{l} \text{ AT/m}$$

➤ **Permeability:**

It is the ability of a material to carry the flux lines.

- **High permeability material**- They allow the flux-lines to pass through them easily. e.g.-Iron, Steel etc.
- **Low Permeability material**- They don't allow the flux lines to pass through them easily. e.g. - Wood.

For any magnetic material, we can define-

- **Absolute Permeability (μ):** It is the ratio of magnetic flux density (B) in a particular medium to the magnetic field strength (H), which produces magnetic flux density (B).

$$\mu = \frac{B}{H} \quad \text{Henry per meter}$$

- **Permeability of the space/air/vacuum (μ_0):** If the medium in which the magnet is kept is air or vacuum, then the ratio of flux density (B) and magnetic field strength (H) is defined as the permeability of free space.

$$\mu_0 = \frac{B \text{ of air}}{H \text{ of air}} \quad \text{Henry per meter}$$

$$\text{value of } \mu_0 = 4\pi \times 10^{-7} \quad \text{Henry per meter}$$

- **Relative Permeability (μ_r):** The term relative permeability is defined as the ratio of the flux density in a particular medium produced by a magnet to the flux density in air or vacuum by the same magnet under the identical operating conditions.

$$\mu_r = \frac{B}{B_0}$$

$$B = \mu H \quad \text{for a given medium}$$

$$B_0 = \mu_0 H \quad \text{for air or freespace}$$

$$\mu_r = \frac{B}{B_0} = \frac{\mu H}{\mu_0 H} = \frac{\mu}{\mu_0}$$

$$\mu = \mu_r \mu_0$$

➤ **Reluctance:**

It is defined as the opposition to the flow of flux in material. It is denoted by '**RI**' and given by,

$$Rl = \frac{l}{\mu_0 \mu_r a}$$

where, l = length of the magnetic material in meters

$$\mu_0 = \text{permeability of free space or air} = 4\pi \times 10^{-7} \text{ H/M}$$

μ_r = relative permeability

a = area of cross section of magnetic material in meter²

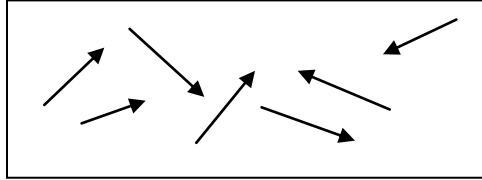
- Since reluctance of magnetic circuit is something similar to resistance of electric circuit, hence it may also be defined as,

$$Rl = \frac{MMF}{\text{flux}} = \frac{NI}{\phi} \quad \text{Ampere - turn/weber}$$

- **Permeance:** It is defined as reciprocal of reluctance. $\text{Permeance} = \frac{1}{\text{Reluctance}} = \frac{\text{weber}}{\text{Ampere-turn}}$

Magnetic Materials:**1. Paramagnetic Materials:**

- The materials, which are not strongly attracted by a magnet, such as aluminum, tin, platinum, magnesium etc., are known as paramagnetic materials.
- Their relative permeability is small but positive. Such materials are slightly magnetized when placed in a strong magnetic field and act in the direction of the magnetic field.
- In paramagnetic materials the individual atomic dipoles are oriented in a random fashion as shown in figure below.



The resultant magnetic field is, therefore, negligible.

- When an external magnetic field is applied, the permanent magnetic dipoles orient themselves parallel to the applied magnetic field and give rise to a positive magnetization.
- Since the orientation of the dipoles parallel to the applied magnetic field is not complete, the magnetization is small. These materials have little application in the field of electrical engineering.

2. Diamagnetic Materials:

- The materials which are repelled by a magnet such as zinc, mercury, lead, sulphur, copper, silver, bismuth etc., are known as diamagnetic materials.
- Their permeability is slightly less than unity. They are slightly magnetized when placed in a strong magnetic field and act in the direction opposite to that of applied magnetic field.
- Permanent magnetic dipoles are absent in diamagnetic materials. Hence these are unimportant from the point of view of application in the field of electrical engineering.

3. Ferromagnetic Materials:

- The materials which are strongly attracted by a magnet such as iron, steel, nickel, cobalt and some of their alloys, are known as ferromagnetic materials.
- The opposing magnetic effects of electron orbital motion and electron spin do not eliminate each other in an atom of such material. Their permeability is very high (varying from several hundreds to many thousands).
- There is a relatively large contribution from each atom which aids in the establishment of an internal magnetic field, so that when the material is placed in a magnetic field, its value is increased many times the value that was present in the free space before the material was placed there.
- Ferromagnetic materials are of two types: (a) those easily magnetized, called soft magnetic materials and (b) those retaining their magnetism with great tenacity, designated as hard magnetic materials.

Magnetisation Curve or B-H Curve:

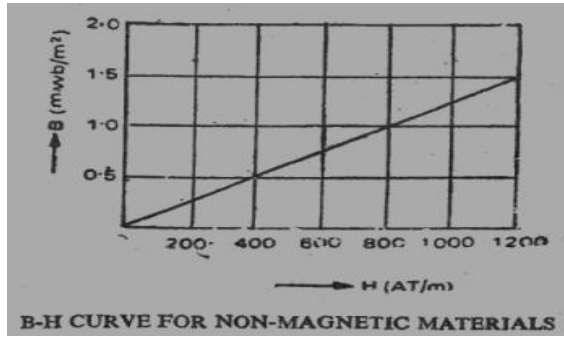
The B-H Curve (or magnetisation curve) indicates the manner in which the flux density (B) varies with the magnetising force or magnetic field intensity (H).

- **For non-magnetic materials:** For non magnetic materials (e.g. air, copper, rubber, wood etc.) the relation between B and H is given by,

$$B = \mu_0 H \quad , \text{ since } \mu_0 = 4\pi \times 10^{-7} \frac{H}{M} \text{ is constant}$$

therefore, $B \propto H$

Hence B-H curve of a non-magnetic material is a straight line passing through origin as shown in fig below.



4

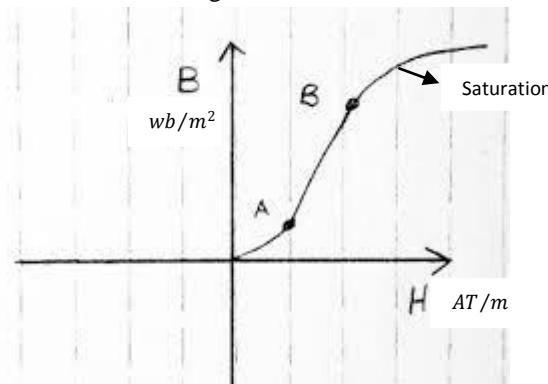
• For magnetic materials:

For magnetic materials (e.g. iron, steel etc.), the relation between B and H is given by,

$$B = \mu_0 \mu_r H$$

In above equation, relation between B and H is no more constant but, it varies with μ_r , as μ_r is not constant but varies with the flux density (B).

Consequently, the B-H curve of a magnetic material is not linear as shown below.



B-H curve of magnetic material

Magnetic Hysteresis:

When a ferromagnetic material is subjected to cycle of magnetisation (i.e it is magnetised first in one direction and then in the other direction), it is found that magnetic flux density B in the material lag behind the applied magnetising force or magnetic field intensity H. This phenomenon is known as **magnetic hysteresis**.

A typical B-H curve for magnetic material is shown below.

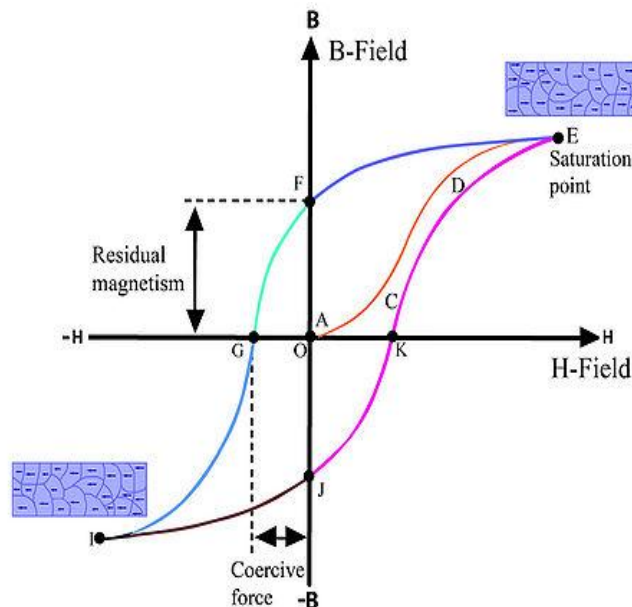


Fig Typical hysteresis loop of a ferromagnetic material

Note: A loop *O E F G I J K E* shown above is known as **Hysteresis Loop**

➤ **Important terms related to Hysteresis Loop:**

- **Residual Magnetism-** The value of B (magnetic flux density) that remains after magnetic field intensity (H) is removed called residual magnetism or residual flux density.
- **Retentivity:** The maximum possible value of the residual flux density is called retentivity.
- **Coercive Force:** The amount of magnetic field intensity (H) required to reduce the residual flux density to zero is called coercive force.

Hysteresis Loss:

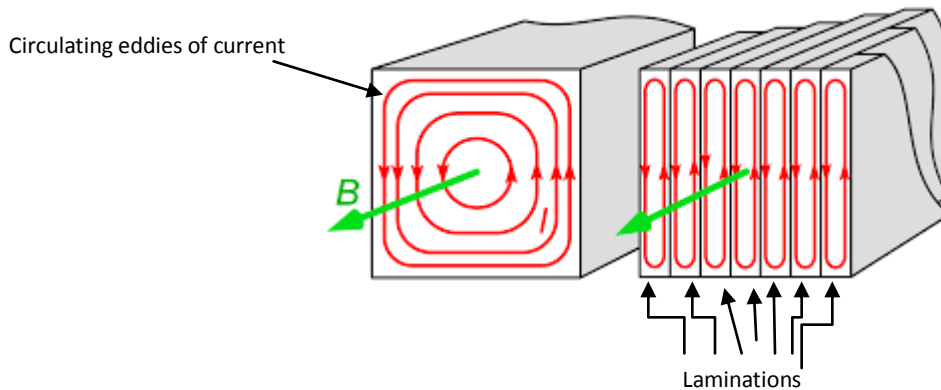
- When a magnetic material is subjected to a cycle of magnetisation (i.e it is magnetised first in one direction and then in the other direction), any energy loss takes place due to molecular friction in the material.
- That is, the domains* (or molecular magnets) of the material resist being turned first in direction and then in other.
- Energy is thus expended in the material in overcoming this opposition.
- This loss is in the form of heat and is called ***hysteresis loss***.
- Hysteresis loss is given by: $P_H = k B_{max}^{1.6} f V$ watts

Where B_{max} = Maximum flux density in the material
 f = Frequency of magnetic reversals
 V = volume of material in m^3
 k = Steinmetz hysteresis coefficient

*Note: *Domains is group of atoms (nearly 10^{11}) which have come together and aligned there magnetic dipole moment in one direction.*

Eddy-Current Loss:

- Eddy current is formed due to change in magnetic flux linked with a metal.
- Consider a solid iron core (shown below in figure), with which a time varying flux is linking.
- The core may be considered to be made up of concentric shells. Since iron is conducting material, each shell may be treated as a closed coil.
- Each shell links with some portion of the flux.



- Hence emf induced in each shell, as per faraday's law of electromagnetic induction.
- Each induced emf causes current to flow in its closed shell.
- Hence circulating eddies of current is formed over the surface of iron core.
- As per Lenz's law this induced current will oppose change in magnetic field and also heat is generated due to the flow of eddy currents through the core.
- The power loss due to eddy currents is called ***eddy-current loss***.
- The eddy-current loss per unit volume of a magnetic core subjected to a time varying flux is given by:

$$P_e = \frac{1.645}{\rho} t^2 f^2 B_{max}^2 \quad \text{watt/meter}^3$$

Where B_{max} = Maximum flux density in the material
 f = Frequency of magnetic reversals
 t = thickness of core
 ρ = resistivity of core

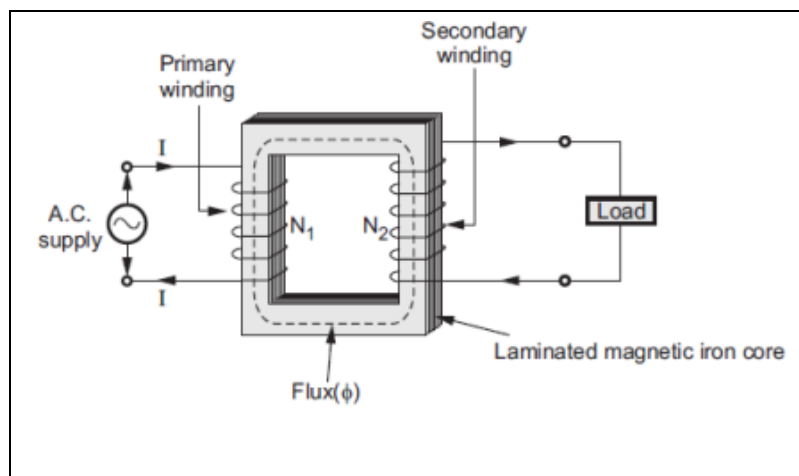
➤ **Methods of reducing eddy-currents:**

- By making holes or slots on the metallic plate, thus reducing the area available to flow of eddy currents.
- Eddy currents are minimized by using laminations of metal to make a metal core. The lamination must be separated by an insulating material like lacquer.

➤ **Application of Eddy-currents:**

- Eddy current heating is used for heating metals, for example melting, hardening and other heat-treatment processes.
- Eddy current damping is used in permanent-magnet moving-coil instrument for damping.
- Eddy current braking is used in induction energy meters.

Transformer



Basic Diagram of Single-Phase Transformer

Important points:

- Transformer is a static device that transfers electrical energy from one electrical circuit to another electrical circuit through the medium of magnetic field and without a change in the frequency.
- The electrical circuit which receives energy from the supply mains is called primary circuit, and winding connected is known as primary winding. And other which delivers electrical energy to the load is called secondary circuit, winding connected is known as secondary winding.
- The transformer is an electromagnetic energy conversion device, since the energy received by the primary is first converted to magnetic energy and it is then reconverted to useful electrical energy in the secondary circuit.
- Thus, primary and secondary winding of a transformer are not connected electrically, but are coupled magnetically.
- Since transformer has no moving or rotating parts, it has efficiency up to 98-99%.

Working Principle of single-phase Transformer: Transformer works on the principle of mutual induction between two inductively coupled coil or circuits.

Working of Single-phase Transformer:

- An alternating voltage is applied to the primary winding of a transformer and an alternating current flows through the primary winding.
- Flow of alternating current in the primary winding sets up the alternating (variable) flux (direction of which is given by right-hand grip rule) in the core of the transformer.
- This variable flux links the primary winding and hence an emf is induced in the primary winding of a transformer according to faraday law of electromagnetic induction. magnitude of emf is given by: $e_1 = -N_1 \frac{d\phi}{dt}$
- This variable flux also links with the secondary winding and mutually induced emf appears across secondary winding of a transformer, magnitude of which is also given by: $e_2 = -N_2 \frac{d\phi}{dt}$

Types of Single-Phase Transformer:

➤ On the basis of working transformers are of two types-

(i) **Step-up Transformer:** If the secondary winding has more turns than the primary winding ($N_2 > N_1$), then the secondary voltage is higher than the primary voltage and transformer is called a step-up transformer.

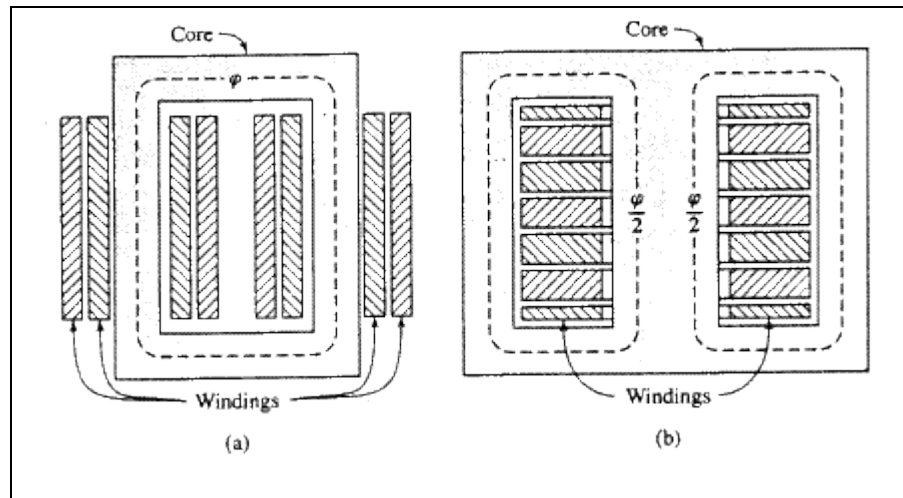
(ii) **Step-down Transformer:** In case secondary winding has less turns ($N_2 < N_1$), then the secondary voltage will be lower than the primary voltage and transformer is called a step-down transformer.

➤ On the basis of construction transformer is also of two types-

(i) **Core-Type Transformer:** It is used for high-voltage level and high-power level.

(ii) **Shell-Type Transformer:** It is used for low-voltage level and low-power level.

Construction diagram for both is shown below,



Core-type

Shell-type

EMF Equation of a Single-Phase Transformer:

Since flux set up in the core of a transformer is from alternating current, hence instantaneous value of flux can be written as,

$$\phi = \phi_{\max} \sin \omega t \quad \text{----- (1)}$$

Magnitude of e.m.f induced in primary winding of transformer, according to faraday's law of electromagnetic induction, is given by,

$$e_1 = -N_1 \frac{d\phi}{dt} \quad \text{----- (2)}$$

put the value of ϕ from equation (1) into equation (2), we have,

$$\begin{aligned} e_1 &= -N_1 \frac{d\phi_{\max} \sin \omega t}{dt} \\ e_1 &= -N_1 \phi_{\max} \omega \cos \omega t \\ e_1 &= N_1 \phi_{\max} \omega \sin\left(\omega t - \frac{\pi}{2}\right) \quad \text{----- (3)} \end{aligned}$$

Since, $e_1 = e_{1_{\max}}$, when $\sin\left(\omega t - \frac{\pi}{2}\right) = 1$

Hence equation (3) can be written as,

$$e_1 = e_{1_{max}} \sin\left(\omega t - \frac{\pi}{2}\right)$$

Where, $e_{1_{max}} = N_1 \phi_{max} \omega$ ----- (4)

Hence the r.m.s value of the e.m.f induced in the primary winding is,

$$E_{1_{rms}} = \frac{e_{1_{max}}}{\sqrt{2}}$$

$$\text{Or } E_1 = \frac{N_1 \phi_{max} \omega}{\sqrt{2}}$$

$$E_1 = \frac{N_1 \phi_{max} 2\pi f}{\sqrt{2}}$$

$$E_1 = 4.44 N_1 \phi_{max} f \quad \text{----- (4)}$$

Similarly, r.m.s value of e.m.f induced in secondary winding is given by,

$$E_2 = 4.44 N_2 \phi_{max} f \quad \text{----- (5)}$$

Equation (4) and (5) are the e.m.f equation of the transformer.

Dividing equation (5) from equation (4) we have,

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = k \quad \text{----- (6)}$$

Where 'k' is known as transformation ratio.

If transformer is assumed to be ideal,

$$\text{Then } V_1 I_1 = V_2 I_2 \quad \text{or } \frac{V_2}{V_1} = \frac{I_1}{I_2} \quad \text{----- (7)}$$

Since $V_1 = E_1$ and $V_2 = E_2$ in case of ideal transformer ----- (8)

Using equation (6),(7) and (8) we can write,.....

$$\frac{V_2}{V_1} = \frac{E_2}{E_1} = \frac{I_1}{I_2} = \frac{N_2}{N_1} = k$$

Where $k = \text{transformation ratio}$

Ideal Transformer:

Following assumptions are made for ideal transformer-

- The primary and secondary winding has zero resistance. It means that there is no ohmic power loss and no resistive voltage drop in an ideal transformer.
- There is no leakage flux and all the flux set up is confined to the core and links both the windings.
- Hysteresis and eddy current losses in transformer core are zero.
- The core has infinite permeability and zero reluctance so that zero magnetizing current is required for establishing the requisite amount of flux in the core.

Note: From above mentioned points an ideal transformer is supposed to consist of two pure inductive coils wound on a loss-free core.

Losses in Transformer:

There are basically two types of losses occurs in transformer-

(i) Core or Iron Losses: This loss includes Hysteresis loss* and eddy current loss*. Both losses occur in the core of a transformer. These losses are generally constant losses.

(ii) Copper Losses or Ohmic Losses (I^2R losses): This loss occurs in the winding of a transformer. This loss is a variable loss and depends on the load current and varies with square of the load current.

Testing of Transformer:

For evaluating core losses and copper losses in the transformer, two important tests is performed on transformer. These two tests are:

➤ Open-Circuit Test-

- This test is performed for calculating core losses in the core of a transformer.
- While performing this test, secondary winding of transformer is kept open that is transformer is on no-load.

➤ Short-Circuit Test-

- This test is performed for calculating copper losses in the windings of a transformer.

- While performing this test, secondary winding of transformer is kept short circuited.

Efficiency of a Transformer:

In a simple way, efficiency of a transformer, $\eta = \frac{\text{output in kW}}{\text{input in kW}}$ per-unit

or in percentage, $\% \eta = \frac{\text{output in kW}}{\text{input in kW}} \times 100$

Now, we will analyze efficiency in per-unit rather in percentage for simplicity.

Since, input = output + losses

Hence efficiency of transformer can also written as,

$$\eta = \frac{\text{output in kW}}{\text{output in kW} + \text{Losses}}$$

Since, output of transformer in kW = $V_2 I_2 \cos \phi_2$

Losses = Core Loss (Iron Loss) + Full load Copper Loss

Losses = $P_i + P_{cu_{FL}}$

Where, P_i = Iron or core loss

And, $P_{cu_{FL}} = I_2^2 R_{02}$ = Full load copper loss, I_2 = full load current in secondary winding,

R_{02} = equivalent resistance of transformer winding refer to secondary.

$$\text{Now, } \eta = \frac{V_2 I_2 \cos \phi_2}{V_2 I_2 \cos \phi_2 + P_i + I_2^2 R_{02}} \quad \text{----- (1)}$$

Condition for Maximum Efficiency:

Since efficiency of transformer is given by,

$$\begin{aligned} \eta &= \frac{V_2 I_2 \cos \phi_2}{V_2 I_2 \cos \phi_2 + P_i + I_2^2 R_{02}} \\ &= \frac{V_2 \cos \phi_2}{V_2 \cos \phi_2 + P_i / I_2 + I_2 R_{02}} \quad \text{----- (1)} \end{aligned}$$

For a normal transformer, V_2 is approximately constant. Hence for a load of given power factor, efficiency depends upon load current I_2 , it is clear from equation (1) that numerator is constant and for the efficiency to be maximum, the denominator should be minimum i.e.

$$\frac{d}{dI_2} (\text{denominator}) = 0$$

$$\text{or } \frac{d}{dI_2} (V_2 \cos \phi_2 + P_i / I_2 + I_2 R_{02}) = 0$$

$$\text{or } 0 - \frac{P_i}{I_2^2} + R_{02} = 0$$

$$\text{or } P_i = I_2^2 R_{02} \quad \text{----- (2)}$$

i.e. **Iron losses = Copper losses**

Hence efficiency of a transformer will be maximum, when copper losses are equal to constant or iron losses.

From equation (2), the load current I_2 corresponding to maximum efficiency is given by;

$$I_{2max} = \sqrt{\frac{P_i}{R_{02}}} \quad \text{----- (3)}$$

Note: In a transformer, iron losses are constant whereas copper losses are variable. In order to obtain maximum efficiency, the load current should be such that copper losses become equal to iron losses.

Multiply by V_2 in both sides of equation (3), we have-

$$V_2 I_{2max} = V_2 \sqrt{\frac{P_i}{R_{02}}}$$

Multiply and divide by I_2 in right hand side of the above equation we have,

$$V_2 I_{2max} = V_2 \frac{I_2}{I_2} \sqrt{\frac{P_i}{R_{02}}}$$

or

$$V_2 I_{2max} = V_2 I_2 \sqrt{\frac{P_i}{I_2^2 R_{02}}}$$

or

$$kVA_{max} = kVA_{rated} \sqrt{\frac{P_i}{I_2^2 R_{02}}}$$

| | | |
|----|---|-----------|
| or | $kVA_{max} = kVA_{rated} \sqrt{\frac{P_i}{P_{cuFL}}}$ | ----- (4) |
|----|---|-----------|

Where $kVA_{max} = V_2 I_{2max}$ and $kVA_{rated} = V_2 I_2$

Formula for calculating transformer Efficiency:

Since efficiency of transformer is given by, $\eta = \frac{V_2 I_2 \cos \phi_2}{V_2 I_2 \cos \phi_2 + P_i + I_2^2 R_{02}}$

Above formula can be modify as,

$V_2 I_2 = kVA$, $\cos \phi_2 = p.f$ (load power factor), $P_i =$ Iron losses in kW, $I_2^2 R_{02} = P_{cuFL}$, Full load copper losses in kW

Now formula for efficiency can be written as,

$$\eta = \frac{(kVA \times p.f)}{(kVA \times p.f) + P_i + P_{cuFL}} \quad \text{----- (1)}$$

If transformer is operating at x times the full load, **wherex = fraction of full load at which transformer is operating,**

Then generally transformer efficiency is expressed as,

$$\eta = \frac{x(kVA \times p.f)}{x(kVA \times p.f) + P_i + x^2 P_{cuFL}}$$

| |
|--|
| or $\% \eta = \frac{x(kVA \times p.f)}{x(kVA \times p.f) + P_i + x^2 P_{cuFL}} \times 100$ |
|--|

For example, $x = 1$ for full load

and $x = 1/2$ for half – full load

- **kVA corresponding to maximum efficiency is given by:**

| |
|---|
| $kVA_{max} = kVA_{rated} \sqrt{\frac{P_i}{P_{cuFL}}}$ |
|---|

- **Load corresponding to maximum efficiency is given by:**

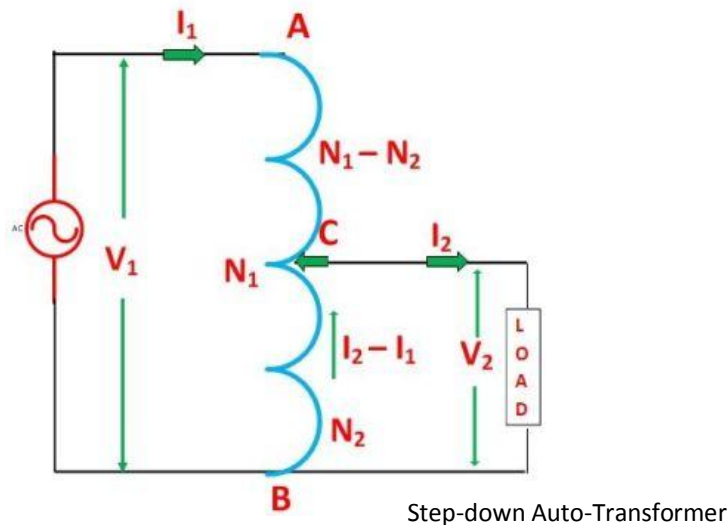
| |
|--|
| $Load_{max} = kVA_{rated} \times p.f. \sqrt{\frac{P_i}{P_{cuFL}}}$ |
|--|

- **Maximum efficiency can be calculated from expression given below:**

| |
|---|
| $\% \eta_{max} = \frac{kVA_{max} \times p.f}{kVA_{max} \times p.f + 2P_i} \times 100$ |
|---|

SINGLE-PHASE AUTO-TRANSFORMER:

- Auto-transformer is a transformer in which a part of winding is common to both primary and secondary circuits.(shown in figure below)
- Its principle of working is same as that of two-winding transformer i.e mutual induction.
- The total power transfer in auto-transformer is consisting of inductive transfer and conductive transfer.
- In a two-winding transformer, there is only inductive transfer of power which is transfer due to transformer action.
- In auto-transformer there is additional power transfer on account of physical connection between the source and the load through the series winding.
- This conductive transfer is the main factor responsible for massive copper saving and increased kVA output in auto transformer, as compared to a two winding transformer of same capacity and voltage rating.



In above figure, Primary winding = AB of N_1 number of turns

Secondary winding = BC of N_2 number of turns

Transformation ratio, for an auto-transformer is given by:

$$\frac{V_2}{V_1} = \frac{E_{BC}}{E_{AB}} = \frac{I_1}{I_2} = \frac{N_2}{N_1} = k \quad \text{----- (1)}$$

$$\text{Power delivered to load} = V_2 I_2 \quad \text{----- (2)}$$

Power transformed = Power in winding BC

$$\begin{aligned} &= V_2 (I_2 - I_1) \\ &= V_2 I_2 \left(1 - \frac{I_1}{I_2}\right) \end{aligned}$$

Since $\frac{I_1}{I_2} = k$ put this value in above equation we have

$$\text{Power transformed} = V_2 I_2 (1 - k) \quad \text{----- (3)}$$

Hence power conducted directly = power delivered to load - power transformed

$$\begin{aligned} &= V_2 I_2 - V_2 I_2 (1 - k) \quad \text{(from equation (2) and (3))} \\ &= k V_2 I_2 = k (\text{Power output or kVA}) \end{aligned}$$

Advantage of Auto-transformers:

- Continuously varying voltage can be obtained.
- An auto-transformer requires less copper and is more efficient than a two-winding transformer of a same rating.

Applications of Auto-transformers:

- Auto-transformers with a number of tappings are used for starting induction motors and synchronous motors. When auto-transformers are used for this purpose, these are known as auto-starters.
- A continuously variable auto-transformer finds useful application in electrical testing laboratory.
- Auto-transformers are used as regulating transformers.
- Auto-transformers are also used as boosters to raise the voltage in ac feeders.

- As furnace transformer for getting convenient supply to suit the furnace winding from 240V.

Disadvantage of Auto-Transformer:

- Since in distribution transformers, the transformation ratio is low and auto-transformer loses much of their advantage when the ratio of transformation is low, it is not advantageous to use auto transformer as distribution transformer.
- In case there is break in conductor at common portion then more voltage say equal to supply voltage will appear at output, which will damage the equipment connected through transformer.

Note: Few new topics in this module is introduced in current academic session which is not included in this notes. For all those topics kindly see your class notes. Those topics are:

- ***Practical Transformer, Equivalent circuit of transformer, voltage regulation of transformer and Three-phase transformer connections.***