(51) If $x = e^v \sec u$, $y = e^v \tan u$, then evaluate $\frac{\partial(x,y)}{\partial(u,v)}$

(52) If $x = r\cos\theta$, $y = r\sin\theta$, find $\frac{\partial(r,\theta)}{\partial(x,y)}$

(53) If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$, find $\frac{\partial(r,\theta,\phi)}{\partial(x,y,z)}$

(54) Verify $JJ' = 1$ if $x = u$, $y = u\tan v$, $z = w$

(55) If $u = xyz$, $v = x^2+y^2+z^2$, $w = x+y+z$, find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$

(56) If $x = \sqrt{vw}$, $y = \sqrt{wu}$, $z = \sqrt{uv}$ and $u = r\sin\theta\cos\phi$, $v = r\sin\theta\sin\phi$, $w = r\cos\theta$, then calculate the Jacobian $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$

(57) If $x+y+z = u$, $y+z = uv$, $z = uvw$, show that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2v$

(58) If $u_1 = x_1+x_2+x_3$, $u_1^2u_2 = x_2+x_3$, $u_1^3u_3 = x_3$, find $\frac{\partial(u_1,u_2,u_3)}{\partial(x_1,x_2,x_3)}$

(59) If u, v, w are the roots of the cubic $(\lambda-x)^3+(\lambda-y)^3+(\lambda-z)^3 = 0$ in λ, then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.

(60) If u, v, w are the roots of the equation $(x-a)^3+(y-b)^3+(z-c)^3 = 0$ then find $\frac{\partial(u,v,w)}{\partial(a,b,c)}$.

(61) If $u = x+2y+z$, $v = x-2y+3z$, $w = 2xy-xz+4yz-2z^2$, show that they are not independent. Find the relation between them.

(62) If $u = \frac{x+y}{z}$, $v = \frac{y+z}{x}$, $w = \frac{y(x+y+z)}{xz}$, then show that u, v, w are not independent and find the relation between them.

(63) Use the Jacobian to prove that the function $u = xy+yz+zx$, $v = x^2+y^2+z^2$, $w = x+y+z$ are not independent of one another. Find the relation between them.

(64) Are the function $u = \frac{x-y}{x+z}$, $v = \frac{x+z}{y+z}$ functionally dependent? If so, find the relation between them.

(65) Show that $u = y+z$, $v = x+2z^2$, $w = x-4yz-2y^2$ are not independent. Find the relation between them.

(66) If $u = x+y+z$, $v = x^2+y^2+z^2$, $w = x^3+y^3+z^3-3xyz$, prove that u, v, w are not independent, hence find the relation between them.

(67) Find extreme values of function x^3+y^3-3axy.

(68) Examine for extreme values of $x^3+y^3-63(x+y)+12xy$.

(69) In a plane triangle ABC, find the maximum value of $\cos A \cos B \cos C$

(70) A rectangular box, open at the top, is to have a given capacity. Find the dimensions of the box requiring least material for its construction.

(71) Divide 24 into three parts such that the continued product of the first, square of the second and the cube of the third may be maximum.

(72) Divide a number 120 into three parts so that the sum of their product taken two at a time shall be maximum.

(73) Locate the stationary point of $x^4+y^4-2x^2+4xy-2y^2$ and determine their nature.

(74) Examine for extreme values of the following:

(a) $3x^2-y^2+x^3$ (b) $\sin x \sin y \sin(x+y)$ (c) $2(x-y)^2-x^4-y^4$

---

(75) Use Lagrange's method to determine the minimum distance from the origin to the plane $3x+2y+z=12$.

(76) Find, by using Lagrange's method, the minimum distance from the point $(1,2,0)$ to the cone $z^2=x^2+y^2$.

(77) Find the maximum and minimum distances of the point $(3,4,12)$ from the sphere $x^2+y^2+z^2=1$.

(78) Find the maximum and minimum distances from the origin to the curve $x^2+4xy+6y^2=140$

(79) Find the shortest and longest distances from the point $(1,2,-1)$ to the sphere $x^2+y^2+z^2=24$

(80) The temperature T at any point (x,y,z) in space is $T(x,y,z)=Kxyz^2$ where K is constant. Find the highest temperature on the surface of the sphere $x^2+y^2+z^2=a^2$

(81) Find the extreme value of $x^2+y^2+z^2$ subject to the condition $xy+yz+zx=p$

(82) Find the minimum value of $x^2+y^2+z^2$, given that $ax+by+cz=p$

(83) Find, using Lagrange's method, the volume of the largest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

(84) Using Lagrange's method, find the dimensions of a rectangular box of maximum capacity whose surface area is given when
(a) box is open at the top (b) box is closed.

(85) Using Lagrange's method, find the dimensions of a rectangular box which is open at the top, has a capacity of 32 c.c. such that the least material is required for the construction of the box.

(86) A rectangular box, which is open at the top, has a capacity of 256 cubic feet. Determine the dimensions of the box such that the least material is required for the construction of the box. Use Lagrange's method to obtain solution.

(87) The period T of a simple pendulum is $T = 2\pi\sqrt{\frac{l}{g}}$. Find the maximum percentage error in T due to possible errors of 2% in l and 5% in g.

(88) The time T of a complete oscillation of a simple pendulum of length L is governed by the equation $T = 2\pi\sqrt{\frac{L}{g}}$, g is constant. Find the approximate error in calculated value of T corresponding to an error of 2% in the value of L.

(89) The power P required to propel a steamer of length l at a speed u is given by $P = \lambda u^3 l^3$ where λ is constant. If u is increased by 3% and l is decreased by 1%, find the corresponding increase in P.

(90) The diameter and height of a right circular cylinder are found by measurement to be 8 cm. and 12.5 cm. respectively with possible errors of 0.05 in each measurement. Find the maximum possible approximate error in the volume.

(91) Find the possible percentage error in computing the parallel resistance r of two resistances r_1 and r_2 from the formula $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2}$ where the error in both r_1 and r_2 is 2% each.

(92) Find the percentage error in the area of an ellipse when an error of +1% is made in measuring the major & minor axes.

(93) If the base radius and height of a cone are measured as 4 and 8 inches with a possible error of 0.04 and 0.08 innches respectively. Calculate the percentage error in calculating volume of the cone.

(94) A balloon is in the form of right circular cylinder of radius 1.5 m and length 4 m and is surmounted by hemispherical ends. If the radius is increased by 0.01 m and length by 0.05 m find the percentage change in the volume of balloon.

(95) The period of a simple pendulum is $T = 2\pi\sqrt{\frac{l}{g}}$. Find the max. percentage error in T due to the possible errors upto 1% in l and 2.5% in g.

(96) The angles of a triangle are calculated from the sides a, b, c. If small changes $\delta a, \delta b, \delta c$ are made in the sides, find $\delta A, \delta B, \delta C$ approximately, where Δ is the area of the triangle and A, B, C are angles opposite to sides a, b, c respectively. Also show that $\delta A + \delta B + \delta C = 0$

(97) In estimating the cost of a pile of bricks measured as 6 m × 50 m × 4 m, the tape is stretched 1% beyond the standard length. If the count is 12 bricks in 1 m³ and bricks cost ₹ 100/1000, find the approximate error in the cost.

(98) If Δ is the area of a triangle, prove that the error in Δ resulting from a small error in c is given by

$$\delta\Delta = \frac{\Delta}{4}\left[\frac{1}{s} + \frac{1}{s-a} + \frac{1}{s-b} + \frac{1}{s-c}\right]\delta c$$

(99) The height h and the semi-vertical angle α of a cone are measured and from them, A, the total surface area of the cone, including the base, is calculated. If h and α are in error by small quantities δh and $\delta\alpha$ respectively, find the corresponding error in the area. Show further that if $\alpha = \pi/6$, an error of +1% in h will be approximately compensated by an error of −0.33% in α.

(100) The rate of flow Q of water per second over the sharp-edged notch of length l, the height of the general level of the water above the bottom of the notch being h, is given by the formula $Q = c\left(l - \frac{h}{5}\right)h^{3/2}$, where c is constant. Show that for small error δh in the measurement of h, the error δQ in Q is $\frac{1}{2}c(3l - h)h^{1/2} \cdot \delta h$

(101) The two sides of a triangle are measured as 50 cm and 70 cm and the angle between them is 30°. If there are possible errors of 0.5% in the measurement of the sides and 0.5 degree in that of the angle, find the maximum approximate percentage error in measuring the area of the triangle.

(102) The resistance R of a circuit was found by formula $I = \frac{E}{R}$. If there is an error of 0.1 amp. in reading I, 0.5 volts in E, find the corresponding possible percentage ~~change~~ error in R when readings are I = 15 amp. and E = 100 volts

(103) What error in common logarithm of a number will be produced by an error of 1% in the number?

(104) Compute an approximate value of $(1.04)^{3.01}$

(105) compute an approximate value of $[(3.82)^2 + 2(2.1)^3]^{1/5}$

(106) If $f(x,y) = x^2 y^{1/10}$, compute the value of f when $x = 1.99, y = 3.01$

(107) Evaluate $\log[(1.01)^{1/3} + (0.99)^{1/4} - 1]$ approximately.

(108) Evaluate $\log[(1.03)^{1/3} + (0.98)^{1/4} - 1]$ approximately.