(1)

Assignment

If $y = \cos(m \sin^{-1} x)$, prove that

$$(1-x^2)\, y_{n+2} - (2n+1)\, x\, y_{n+1} + (m^2 - n^2)\, y_n = 0$$

(2) If $x = \sin\left(\frac{\log y}{a}\right)$ then evaluate the value

$$(1-x^2)\, y_{n+2} - (2n+1)\, x\, y_{n+1} - (n^2+a^2)\, y_n = 0$$

(3) If $y = \log(x + \sqrt{1+x^2})$, then prove that

$$(1+x^2)\, y_{n+2} + (2n+1)\, x\, y_{n+2} + n^2 y_n = 0$$

(4) If $y = e^{\tan^{-1} x}$, prove that

$$(1+x^2)\, y_{n+2} + [2(n+1)x - 1]\, y_{n+1} + n(n+1)\, y_n = 0$$

(5) If $y^{1/m} + y^{-1/m} = 2x$, prove that

$$(x^2-1)\, y_{n+2} + (2n+1)\, x\, y_{n+1} + (n^2 - m^2)\, y_n = 0$$

(6) If $y = (\sin^{-1} x)^2$, prove that $(y_n)_0 = 0$, for n odd

$$= 2 \cdot 2^2 \cdot 4^2 \cdot 6^2 \cdots (n-2)^2, \; n \neq 2 \text{ for } n \text{ even}$$

(7) Find the value of the nth derivative of

$$y = e^{m \sin^{-1} x}, \text{ for } x = 0$$

(8) If $z = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$, prove that $\frac{\partial^2 z}{\partial y \partial x} = \frac{x^2-y^2}{x^2+y^2}$

(9) If $x^x y^y z^z = c$, show that at $x = y = z$

$$\frac{\partial^2 z}{\partial x \partial y} = -(x \log ex)^{-1}$$

(10) If $u = \log(x^3+y^3+z^3 - 3xyz)$, show that

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$$

(11) If $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}\cot u = 0$

(12) If $u = \log\left(\frac{x^2+y^2}{x+y}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$

13. If $u(x,y,z) = \log(\tan x + \tan y + \tan z)$, prove that

$$\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} + \sin 2z \frac{\partial u}{\partial z} = 2$$

14. If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, prove that

i, $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$

ii, $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 2\cos 3u \sin u$

15. If $z = f(x,y)$, $x = e^{u} + e^{-v}$, $y = e^{-u} - e^{v}$, prove that

$$\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$$

16. If $u = u(y-z, z-x, x-y)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$

17. If $V = f(2x-3y, 3y-4z, 4z-2x)$, compute the value

$$6V_x + 4V_y + 3V_z$$

18. If $u = u\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$, show that $x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z} = 0$

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19. Expand $e^{x}\cos y$ in power of x and $\left(y - \frac{\pi}{2}\right)$ upto terms of degree 3.

20. Obtain Taylor's expansion of $\tan^{-1}\frac{y}{x}$ about $(1,1)$ upto and including the second degree terms. Hence compute $f(1.1, 0.9)$

21. Expand x^y in powers of $(x-1)$ and $(y-1)$ upto the third degree terms.

22. Find the Jacobian $J\left(\frac{u,v}{x,y}\right)$ for $u = e^{x}\sin y$ and $v = x\log\sin y$

23. If $y_1 = \frac{x_2 x_3}{x_1}$, $y_2 = \frac{x_3 x_1}{x_2}$, $y_3 = \frac{x_1 x_2}{x_3}$

then show that Jacobian $\frac{\partial(y_1, y_2, y_3)}{\partial(x_1, x_2, x_3)} = 4$

24. Calculate the Jacobian $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ of the following

$u = x+2y+z$, $v = x+2y+3z$, $w = 2x+3y+5z$

(25) If $x+y+z=u$, $y+z=uv$, $z=uvw$, then show that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2 v$.

(26) If u, v, w are the roots of the cubic $(\lambda-x)^3+(\lambda-y)^3+(\lambda-z)^3=0$ in λ then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$

(27) If u, v, w are the roots of the eqn $(x-a)^3+(x-b)^3+(x-c)^3=0$, then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$

(28) Determine the functional dependence and find reln b/w

(i) $u=\frac{x+y}{x-y}$, $v=\frac{xy}{(x-y)^2}$ (ii) $u=\frac{x-y}{x+y}$, $v=\frac{x+y}{x}$

(29) If $u=\frac{x+y}{z}$, $v=\frac{y+z}{x}$, $w=\frac{y(x+y+z)}{xz}$, then show that u, v, w are not independent and find the relation b/w them.

(30) If $u=\sin^{-1}x+\sin^{-1}y$, $v=x\sqrt{1-y^2}+y\sqrt{1-x^2}$, find $\frac{\partial(u,v)}{\partial(x,y)}$. Is u, v functionally related? If so, find the relationship.

(31) If the base radius and height of a cone are measured as 4 and 8 inches with a possible error of 0.04 and 0.08 inches respectively, calculate the percentage (%) error in calculating volume of the cone. Ans:- 3%

(32) A balloon is in the form of right circular cylinder of radius 1.5 m and length 4 m and is surrounded by hemispherical ends. If the radius is increased by 0.01 m and length by 0.05 m find the percentage change in the volume of balloon.
Ans:- 2.389%

(33) compute an approximate value of $(1.04)^{3.01}$, Ans:- 1.12

(34) In estimating the cost of a pile of bricks measured as $6m \times 50m \times 4m$, the tape is stretched 1% beyond the standard length. If the count is 12 bricks in 1 m^3 and bricks cost 100 Rs per 1000, find the approximate error in the cost. Ans :- 43.20 Rs.

(35) The height h and the semi-vertical angle α of a cone are measured and from them, A, the total surface area of cone, including the base, is calculated. If h and α are in error by small quantities δh and $\delta\alpha$ respectively, find the corresponding error in the area. Show further that if $\alpha = \frac{\pi}{6}$, an error of +1% in h will be approximately compensated by an error of -0.33° in α. Ans :- $\delta A = 2\pi h(\tan^2\alpha + \sec\alpha\tan\alpha)\delta h$

$$+\pi h^2(2\tan\alpha\sec^2\alpha + \sec^3\alpha + \sec\alpha\tan^2\alpha)\delta\alpha$$

$$\& \ \delta\alpha = -0.33^\circ \quad (\because 1^c = 57.3^\circ \text{ nearly})$$

(36) Examine for extreme values:

(i) $x^2+y^2+6x+12$ (ii) $x^3+y^3-63(x+y)+12xy$

(37) Examine for minimum and maximum values:

$\sin x + \sin y + \sin(x+y)$.

(38). A rectangular box, open at the top, is to have a given capacity. Find the dimensions of the box requiring least material for its construction.

(39) The sum of three positive numbers is const. Prove that their product is maximum when they are equal

(40) Find the volum of the largest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

(41) Find the maximum and minimum distances of the pt (3,4,12) from the sphere $x^2+y^2+z^2=1$

(42) Find the minimum value of $x^2+y^2+z^2$, given that $ax+by+cz=p$