KAS-302

Roll No. | | | | | | | | | | |

SHAMBHUNATH INSTITUTE OF ENGINEERING & TECHNOLOGY

B. TECH. – Third Semester
First Sessional Examination, 2019-20
(Only for ME And EC)
MATHEMATICS IV

Time: 1 hour 30mins ***Max. Marks****: 30*

Section-A

1. Attempt **ALL** parts. Each questions carry equal marks. Write answers of each question in short: **(5x1=5)**

a) Write the normal equation for $y = a + \frac{b}{x}$.

Solution Normal equations are

$$\sum y = an + b\sum \frac{1}{x}, \sum \frac{y}{x} = a\sum \frac{1}{x} + b\sum \frac{1}{x^2}$$

b) A bag contains 8 red and 7 black balls. Find the probability of drawing a red ball.

Solution Find the probability of drawing a red ball$= \frac{c_1^8}{c_1^{15}} = \frac{8}{15}$

c) Explain mean, median and mode.

Solution

Mean: The mean, also called the arithmetic mean or average or average value, is the quantity obtained by summing two or more numbers or variables and then dividing by the number of numbers or variables.

Median: Measure of the central item when they are arranged in ascending or descending order of magnitude.

Mode: Mode is defined to be the size of the variable which occurs most frequently.

d) Write about Stratified Random Sampling.

Solution: Stratified Random Sampling involves categorizing the members of the population into mutually exclusive and collectively exhaustive groups.

e) If mean and variance for Binomial distribution is 9 and 3 respectively. Find the distribution.

Solution $m = np = 9, variance = npq = 3$

$$q = \frac{1}{3}, p = \frac{2}{3}$$

$$Binomial\ Distribution = \left(\frac{1}{3} + \frac{2}{3}\right)^n$$

Section-B

2. Attempt any **TWO** parts from this section **(2x5=10)**

a) Calculate the first four moments of the following distribution about the mean and hence find $\boldsymbol{\beta_1}$ and $\boldsymbol{\beta_2}$.

x	0	1	2	3	4	5	6	7
f	1	8	28	56	70	56	28	8

Section B

2 (a)

x	f	fx	$(x-A)$	$f(x-A)$	$f(x-A)^2$	$f(x-A)^3$	$f(x-A)^4$
0	1	0	−4	−4	16	−64	+256
1	8	8	−3	−24	72	−216	+648
2	28	56	−2	−56	112	−224	+448
3	56	168	−1	−56	56	−56	56
4	70	280	0	0	0	0	0
5	56	280	1	56	56	56	56
6	28	168	2	56	112	224	448
7	8	56	3	24	72	216	648
	256	1016		−4	496	−64	2560

$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{1016}{256} = 3.968 = 4$

$\mu_1' = \frac{\Sigma f(x-A)}{\Sigma f} = \frac{-4}{256} = -0.0156$

$\mu_2' = \frac{\Sigma f(x-A)^2}{\Sigma f} = \frac{496}{256} = 1.9375$

$\mu_3' = \frac{\Sigma f(x-A)^3}{\Sigma f} = \frac{-64}{256} = -0.25$

$\mu_4' = \frac{\Sigma f(x-A)^4}{\Sigma f} = \frac{2560}{256} = 10$

$\mu_2 = \mu_2' - \mu_1'^2 = 1.9375 - (-0.0156)^2$

$= 1.9375 - 0.000243.$

$= 1.9373 = 2$

$\mu_3 = 0, \quad \mu_4 = 11, \quad \beta_1 = 0, \quad \beta_2 = 2.75$

b) Find both the lines of regression of following data.

x	5.60	5.65	5.70	5.81	5.85
y	5.80	5.70	5.80	5.79	6.01

Solution:

Line y on x

$y = a + bx$

Normal equs

$\Sigma y = na + b\Sigma x$

$\Sigma xy = a\Sigma x + b\Sigma x^2$

x	y	xy	x^2	y^2
5.6	5.8	32.48	31.36	33.84
5.65	5.7	32.205	31.9225	33.49
5.7	5.8	33.06	32.49	33.64
5.81	5.79	33.6399	33.7561	32.5241
5.85	6.01	35.1585	34.2225	36.1201
28.61	29.1	166.5434	163.7511	168.4147

$a = 1.568$

$b = 0.743$

$y = 1.568 + 0.743x$

Line of x on y:

$x = a + by$

Normal equs

$\Sigma x = na + b\Sigma y$

$\Sigma xy = a\Sigma y + b\Sigma y^2$

$a = 5.93$

$b = -0.04$

$x = 5.93 - 0.04y$

c. A die is tossed twice. Getting a number greater than 4 is considered a success. Find the variance of the probability distribution of the number of successes.

Solution:

$$P(success) = \frac{2}{6} = \frac{1}{3}, P(failure) = 1 - \frac{1}{3} = \frac{2}{3}$$

Probability Distribution

X	0	1	2
P(X)	4/9	4/9	1/9

Variance= $\sum \boldsymbol{px^2} - \sum \boldsymbol{px} = \mathbf{2/9}$

d. Fit a Binomial distribution to the following frequency data:

x	0	1	3	4
f	28	62	10	4

Solution:

Mean= $\frac{\sum fx}{\sum f} = \frac{\mathbf{108}}{\mathbf{104}} = \mathbf{1.01}$

$\boldsymbol{np} = \mathbf{1.04}$

$\boldsymbol{p} = \mathbf{0.26}$ and $\boldsymbol{q} = \mathbf{1} - \boldsymbol{p} = \mathbf{1 - 0.26 = 0.74}$

Hence Binomial distribution= $\boldsymbol{N(q+p)^n} = \mathbf{104(0.74 + 0.26)^4}$

<u>Section-C</u>

Attempt **ALL** from this section.

3. Attempt any **ONE** part from the following**:** **(1x5=5)**

a. Find the moment generating function of the discrete Poisson distribution given by $P(x) = \frac{e^{-\lambda}.\lambda^x}{x!}$. Also find the first and second moments about the mean.

Solution. Here, we have

$$f(x) = \frac{e^{-m} m^x}{x!}$$

Moment generating function about the origin

$$M_0(t) = \sum \frac{e^{-m} m^x}{x!}$$

$$= e^{-m} \sum \frac{(me^t)^x}{x!}$$

$$= e^{-m}\left[1 + me^t + \frac{(me^t)^2}{2!} + \frac{(me^t)^3}{3!} + \ldots\right]$$

$$= e^{-m} . e^{me^t} = e^{m(e^t - 1)}$$

$$\mu'_1 = \nu_1 = \left[\frac{d}{dt} M_0(t)\right]_{t=0} = \left[\frac{d}{dt} e^{m(e^t-1)}\right]_{t=0}$$

$$= \left[e^{m(e^t-1)} me^t\right]_{t=0} = e^{m(1-1)} . m = m$$

$$\mu'_2 = \nu_2 = \left[\frac{d^2}{dt^2} M_0(t)\right]_{t=0} = \frac{d}{dt}\left[e^{m(e^t-1)} me^t\right]_{t=0}$$

$$= \left[e^{m(e^t-1)} (me^t)^2 + e^{m(e^t-1)} . me^t\right]_{t=0}$$

$$= \left[e^{m(1-1)} m^2 + e^{m(1-1)} . m\right] = m^2 + m$$

$$\mu_2 = \nu_2 - \bar{x}^2 = \nu_2 - \nu_1^2 = (m^2 + m) - m^2 = m$$

b. The pressure of the gas corresponding to various volume V is measured, given by the following data:

V (cm^3)	50	60	70	90	100
P (kg cm^{-3})	64.7	51.3	40.5	25.9	78

Fit the data to the equation $PV^r = c$

Solution:

Fit the data to the equ...

Sol.

$$PV^{\gamma} = C$$

$\Rightarrow$

$$P = CV^{-\gamma}$$

Taking log on both sides, we get

$$\log P = \log C - \gamma \log V$$

$\Rightarrow$

$$Y = A + BX$$

where, $Y = \log P$, $A = \log C$, $B = -\gamma$, $X = \log V$

Normal equations are

$$\Sigma Y = mA + B\Sigma X$$

and

$$\Sigma XY = A\Sigma X + B\Sigma X^2$$

Here $m = 5$

The table is as below:

V	P	$X = \log V$	$Y = \log P$	XY	X^2
50	64.7	1.69897	1.81090	3.07666	2.88650
60	51.3	1.77815	1.71012	3.04085	3.16182
70	40.5	1.84510	1.60746	2.96592	3.40439
90	25.9	1.95424	1.41330	2.76193	3.81905
100	78.0	2	1.89209	3.78418	4
		$\Sigma X = 9.27646$	$\Sigma Y = 8.43387$	$\Sigma XY = 15.62954$	$\Sigma X^2 = 17.27176$

From Normal equations, we have

$$8.43387 = 5A + 9.27646\,B$$

and

$$15.62954 = 9.27646\,A + 17.27176\,B$$

Solving these, we get

$$A = 2.22476,\quad B = -0.28997$$

$\therefore$

$$\gamma = -B = 0.28997$$

$$C = \text{antilog}(A) = \text{antilog}(2.22476) = 167.78765$$

Hence, the required equation of curve is

$$PV^{0.28997} = 167.78765.$$

4. Attempt any **ONE** part from the following: **(1x5=5)**

a. The number of accidents in a year involving taxi drivers in a city follows a Poisson distribution with mean equal to 3. Out of 1000 taxi drivers, find approximately the number of drivers with

I. No accident in a year.

II. More than 3 accidents in a year.

Solution:

Here mean $m = 3$

I. $P(0) = \frac{e^{-3}3^0}{0!} = 0.0498$

No of Accidents= $0.0498 * 1000 = 49.8 \approx 50$

II. $P(More\ than\ 3) = P(4) + P(5) + \cdots$

$$= 1 - P(0) - P(1) - P(2) - P(3)$$

$$= 1 - \frac{e^{-3}3^0}{0!} - \frac{e^{-3}3^1}{1!} - \frac{e^{-3}3^2}{2!} - \frac{e^{-3}3^3}{3!} = 0.3528$$

No of accidents=$1000 * 0.3528 = 352.8 \approx 353$

b. Urn A contains 2 white, 1 black and 3 red balls, urn B contains 3 white, 2 black and 4 red balls and urn C contains 4 white, 3 black and 2 red balls. One urn is chosen at random and 2 balls are drawn from the urn. If the chosen balls happen to be red and black, what is the probability that both balls come from urn B?

Solution:

Let E_1, E_2, E_3 and A denote the events

E_1 =urn A is chosen, E_2 =urn B is chosen, E_3 =ur
n C is chosen

A = two balls drawn at random are red and black

$$P(E_1) = \frac{1}{3} = P(E_2) = P(E_3)$$

Probability of drawing a red and a black ball= $\frac{C_1^3 \times C_1^1}{C_2^6}$

So $P(A/E_1) = \frac{1}{5}, P(A/E_2) = \frac{2}{9}, P(A/E_3) = \frac{1}{6}$

Now, $P(E_2/A) = \frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)} = \frac{20}{53}$

5. Attempt any **ONE** part from the following. **(1x5=5)**

a. Sample of two types of electric bulbs were tested for lengths of life and the following data were obtained:

	Type 1	Type 2
Number in the sample	**8**	**7**
Mean of the sample (in hours)	**1134**	**1024**
Standard deviation of the sample (in hours)	**35**	**40**

Is there a significant difference in the two means? Given that $t_{(0.05,14)} = 2.15$

Solution:

H_0 = there is no significant in the average life

$s_1 = 35, s_2 = 40$

$$\sum(x_1 - \overline{x_1})^2 = 9800$$

$$\sum(x_2 - \overline{x_2})^2 = 11200$$

$$s^2 = \frac{1}{n_1 + n_2 - 2}[\sum(x_1 - \overline{x_1})^2 + \sum(x_2 - \overline{x_2})^2] = 1615.38$$

$$s = 40.19$$

$$t = \frac{\overline{x_1} - \overline{x_2}}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = 5.288$$

As the calculated value of $|t| = 5.288 > t_{(0.05,13)}$

H_0 is rejected and hence, there is significant difference in the average.

b. Ten individuals are chosen at random from a normal population of students and their marks are found to be 63, 63, 66, 67, 68, 69, 70, 70, 71, 71. In the light of these data, discuss the suggestion that mean marks of the population of students is 66. Given that $t_{(0.01,9)} = 3.36$.

Solution:

Null Hypothesis H_0 there is no significant difference in the sample mean and population mean i.e. $\mu = 66$

Alternate Hypothesis H_1 : $\mu \neq 66$

$$\bar{x} = \frac{\sum x}{n} = 67.8$$

x	63	63	66	67	68	69	70	70	71	71
$(x - \bar{x})^2$	23.04	23.04	3.24	0.64	0.04	1.44	4.84	4.84	10.24	10.24

$$s = \sqrt{\frac{\sum(x - \bar{x})^2}{n - 1}} = 3.01$$

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = 1.89$$

Since calculated value of $|t| = 1.89 < t_{(0.01,9)}$

So, H_0 is accepted and hence there is no difference in the mean.