FLUID MECHANICS AND FLUID MACHINES (KME-302)

B.Tech. / III- SEMESTER

FIRST SESSIONAL EXAMINATION, ODD SEMESTER, (2019-20)

Branch: Mechanical Engineering

Time – 90 Minutes Maximum Marks – 30

Section-A

Q-1) Attempt all the parts- (5*1 = 05)

a) Differentiate between the model and prototype.

Answer: The model is the small scale replica of the actual structure or machine. The actual structure or machine is called Prototype. It is not necessary that the models should he smaller than the prototypes (though in most of cases it is), they may be larger than the prototype. The study of models of actual machines is called Model analysis. Model analysis is actually an experimental method of finding solutions of complex flow problems. Exact analytical solutions are possible only for a limited number

b) What is the difference between a notch and a weir?

Answer: A notch is a device used for measuring the rate of flow of a liquid through a small channel or a tank. It may be defined as an opening in the side of a tank or a small channel in such a way that the liquid surface in the tank or channel is below the top edge of the opening.

A weir is a concrete or masonary structure, placed in an open channel over which the flow occurs. It is generally in the form of vertical wall, with a sharp edge at the top, running all the way across the open channel. The notch is of small size while the weir is of a bigger size. The notch is generally made of metallic plate while weir is made of concrete or masonary structure

c) Explain the Rotational and Irrotational flow.

Answer: Rotational flow is that type of flow in which the fluid particles while flowing along stream-lines, also rotate about their own axis. And if the fluid particles while flowing along stream-lines, do not rotate about their own axis then that type of flow is called irrotational flow.

d) What is dynamic similarity?

Answer: Dynamic similarity means the similarity of forces between the model and prototype. Thus dynamic similarity is said to exist between the model and the prototype if the ratios of the corresponding forces acting at the corresponding points are equal. Also the directions of the corresponding forces at the corresponding points should be same.

e) What do you mean by isotropic turbulence?

Answer: When the intensity components in all the directions are equal *i.e.*,

$$\overline{\overline{u'^2}} = \overline{v'^2} = \overline{w'^2}$$

the turbulence is said to be. isotropic..The homogeneity of turbulence implies that intensity components are not a function of position in space.

Section-B

Q-2) Attempt any two parts from this section- (2*5 = 10)

a) Define and distinguish between the following sets:-

(i) Dynamic Viscosity and Kinematic Viscosity,

Answer:

Dynamic viscosity (also known as absolute viscosity) is the measurement of the fluid's internal resistance to flow while kinematic viscosity refers to the ratio of dynamic viscosity to density. Based on the expression above, two fluids with the same dynamic viscosities can have very different kinematic viscosities depending on density and vice versa. As a result, grasping the physical meaning of these two material properties may not always be so easy. The internal resistance of a fluid to flow (dynamic viscosity) implies that there is a force involved in displacing a fluid. That force (*F*) is proportional to:

1) Shear Rate (SR)
2) Surface Area (*A*)
3) Dynamic Viscosity (η)

Another difference between these two properties is the uniqueness of their units. Dynamic viscosity units are well established mPa-s in SI units or the equivalent cP (centipoises) in CGS. On the other hand, most common kinematic viscosity units are cm2/s in SI units and cSt (centistokes) in CGS

ii) Newtonian Fluids and Non-Newtonian Fluids

Answer:

Newtonian Fluid:A real fluid, in which the shear stress_ is directly proportional to the rate- of shear strain (or velocity gradient), is known as a Newtonian fluid.

Non-Newtonian Fluid: A real fluid, in which the shear stress is not proportional to the rate of shear strain (or velocity gradient), known as a Non-Newtonian fluid.

b) What are the distorted and undistorted models? Write different scale ratios for distorted models.

Answer:

Undistorted Models: Undistorted models are those models which are geometrically similar to their prototypes or in other words if the scale ratio for the linear dimensions of the model and its prototype is same, the model is called undistorted model. The behaviour of the prototype can be easily predicted from the results of undistorted model.

Distorted Models: A model is said to be distorted if it is not geometrically similar to its prototype. For a distorted model different scale ratios for the linear dimensions are adopted. For example, in case of rivers, harbours, reservoirs etc., two different scale ratios, one for horizontal dimensions and other for vertical dimensions are taken. Thus the models of rivers, harbours and reservoirs will become as distorted models. If for the river, the horizontal and vertical scale ratios are taken to be same so that the model is undistorted, then the depth of water in the model of the river will be very-very small which may not be measured accurately. The following are the advantage of distorted models :

1. The vertical dimensions of the model can be measured accurately.
2. The cost of the model can be reduced.
3. Turbulent flow in the model can be maintained.

Though there are some advantages of the distorted model, yet the results of the distorted model cannot be directly transferred to its prototype. But sometimes from the distorted models very useful information can be obtained.

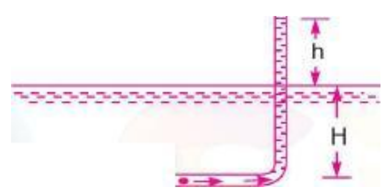
c) Write short notes on:

(i) Pitot Static Tube

Answer:

It is a device used for measuring the velocity of flow at any point in a pipe or a channel. It is based on the principle that if the velocity of flow at a point becomes zero, the pressure there is increased due to the conversion of the kinetic energy into pressure energy. In its simplest form, the pitot-tube consists of a glass tube, bent at right angles as shown in Fig.

The lower end, which is bent through 90° is directed in the upstream direction as shown in Fig. 6.13. The liquid rises up in the tube due to the conversion of kinetic energy into pressure energy. The velocity is determined by measuring the rise of liquid in the tube.

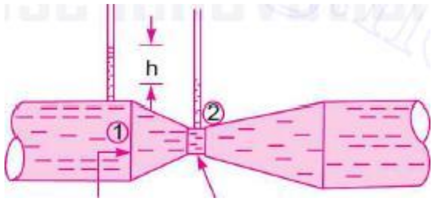


(ii) Venturimeter

Solution:

A venturimeter is a device used for measuring the rate of a flow of a fluid flowing through a pipe. It consists of three parts :

(i) A short converging part, *(ii)* Throat, and *(iii)* Diverging part. It is based on the Principle of Bernoulli's equation.



The discharge is calculated by the formula:

$$Q_{act} = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

d) Write and explain *Prandtl mixing length theory*. Find an expression for shear stress suggested by prandtl.

Answer:

According to Prandtl, the mixing length L , is that distance between two layers in the transverse direction such that the lumps of fluid particles from one layer could reach the other layer and the particles are mixed in the other layer in such a way that the momentum of the particles in the direction of x is same. He also assumed that the velocity fluctuation in the x-direction is related to the mixing length L as

$$u' = l\frac{du}{dy}$$

$$v' = l\frac{du}{dy}$$

Now $\overline{u' \times v'}$ becomes as $\overline{u'v'} = \left(l\frac{du}{dy}\right) \times \left(l\frac{du}{dy}\right) = l^2\left(\frac{du}{dy}\right)^2$

$$\bar{\tau} = \rho l^2 \left(\frac{du}{dy}\right)^2$$

Thus the total shear stress at any point in turbulent flow is the sum of shear stress due to viscous shear and turbulent shear and can be written as

$$\bar{\tau} = \mu\frac{du}{dy} + \rho l^2 \left(\frac{du}{dy}\right)^2$$

Section- C

Q-3. Attempt any one part of the following (1*5 = 5)

a) **Find the discharge through a trapezoidal notch which is 50 c.m. wide at the top and 10 c.m. at the bottom and is 30 c.m. in height. The head of water on the notch is 25 c.m. Assume $C_d = 0.6$ for both the portions (i.e. Rectangular and Triangular).**

Solution:

Base width $b = 10$ cm ; Top width = 50 cm ; Depth = 30 cm

From the geometry of the notch, $\tan\theta = \dfrac{(50-10)\times\frac{1}{2}}{30} = \dfrac{2}{3}$

Discharge over a trapezoidal notch is

= (discharge over the rectangular portion)

+ (discharge over the triangular portion)

$$= \frac{2}{3}\times C_{d1}\, b\sqrt{2g}\, H^{3/2} + \frac{8}{15} C_{d2}\sqrt{2g}\tan\theta\, H^{3/2}$$

$$= \frac{2}{3}\times 0.6\times 0.1\sqrt{2\times 9.81}\times(0.25)^{3/2} + \frac{8}{15}\times 0.6\times\sqrt{2\times 9.81}\times\frac{2}{3}(0.25)^{5/2}$$

$= 0.02214 + 0.02953 = 0.05167$ m^3/s = **51.67 litres/sec**

b) **Water is to be supplied to the inhabitants of a college campus through a supply main. The following data is given:**

Distance of the reservoir from the campus= 3000 m
Volume flow rate of fluid=0.0125 m^3/s
Loss of head due to friction=18 m
Coefficient of friction for the pipe, $f = 0.007$

Solution:

Using the relation :

$$h_f = \frac{4fLV^2}{D \times 2g}$$

where, $$V = \frac{Q}{A} = \frac{0.0125}{\frac{\pi}{4} \times D^2} = \frac{0.0159}{D^2}$$

$\therefore$ $$18 = \frac{4 \times 0.007 \times 3000 \times (0.159 / D^2)^2}{D \times 2 \times 9.81}$$

or, $$D^5 = \frac{4 \times 0.007 \times 3000 \times 0.0159^2}{18 \times 2 \times 9.81} = 6.013 \times 10^{-}$$

$\therefore$ $$D = 0{\cdot}143 \text{ m or } \mathbf{143 \text{ mm (Ans.)}}$$

Q-4. Attempt any one part of the following (1*5 = 5)

a) The frictional torque (T) of a disc of diameter (D) rotating at a speed (N) in a fluid of viscosity (μ) and density (ρ) in a turbulent flow is given by, $T = D^5 . N^2 . \rho . \phi\left[\frac{\mu}{D^2 N \rho}\right]$. Prove this by the method of dimensions.

Solution. Given : $T = f(D, N, \mu, \rho)$ or $f_1(T, D, N, \mu, \rho) = 0$ ...(*i*)

$\therefore$ Total number of variables, $n = 5$

Dimensions of each variable are expressed as

$$T = ML^2T^{-2},\ D = L,\ N = T^{-1},\ \mu = ML^{-1}T^{-1},\ \rho = ML^{-3}$$

$\therefore$ Number of fundamental dimensions, $m = 3$

Number of π-terms $= n - m = 5 - 3 = 2$

Hence equation (*i*) can be written as $f_1(\pi_1, \pi_2) = 0$ ...(*ii*)

Each π-term contains $m + 1$ variable, *i.e.*, $3 + 1 = 4$ variables. Three variables are repeating variables.

Choosing D, N, ρ as repeating variables, the π-terms are

$$\pi_1 = D^{a_1} . N^{b_1} . \rho^{c_1} . T$$
$$\pi_2 = D^{a_2} . N^{b_2} . \rho^{c_2} . \mu$$

Dimensional Analysis of π_1

$$\pi_1 = D^{a_1} . N^{b_1} . \rho^{c_1} . T$$

Substituting dimensions on both sides,

$$M^0L^0T^0 = L^{a_1} . (T^{-1})^{b_1} . (ML^{-3})^{c_1} . ML^2T^{-2}.$$

Equating the powers of M, L, T on both sides,

Power of M, $\quad 0 = c_1 + 1, \quad \therefore \quad c_1 = -1$

Power of L, $\quad 0 = a_1 - 3c_1 + 2, \quad \therefore \quad a_1 = 3c_1 - 2 = -3 - 2 = -5$

Power of T, $\quad 0 = -b_1 - 2, \quad \therefore \quad b_1 = -2$

Substituting the values of a_1, b_1, c_1 in π,

$$\pi_1 = D^{-5} . N^{-2} . \rho^{-1} . T = \frac{T}{D^5N^2\rho}.$$

Dimensional Analysis of π_2

$$\pi_2 = D^{a_2} . N^{b_2} . \rho^{c_2} . \mu$$

Substituting dimensions on both sides,

$$M^0L^0T^0 = L^{a_2} . (T^{-1})^{b_2} . (ML^{-3})^{c_2} . ML^{-1}T^{-1}.$$

Equating the powers of M, L, T on both sides,

Power of M, $\quad 0 = c_2 + 1, \quad \therefore \quad c_2 = -1$

Power of L, $\quad 0 = a_2 - 3c_2 - 1, \quad \therefore \quad a_2 = 3c_2 + 1 = -3 + 1 = -2$

Power of T, $\quad 0 = -b_2 - 1, \quad \therefore \quad b_2 = -1$

Substituting the values of a_2, b_2 and c_2 in π_2,

$$\pi_2 = D^{-2}N^{-1}\rho^{-1} . \mu = \frac{\mu}{D^2N\rho}.$$

Substituting the values of π_1 and π_2 in equation (*ii*),

$$f_1\left(\frac{T}{D^5N^2\rho}, \frac{\mu}{D^2N\rho}\right) = 0 \quad \text{or} \quad \frac{T}{D^5N^2\rho} = \phi\left(\frac{\mu}{D^2N\rho}\right)$$

or

$$T = \mathbf{D^5N^2}\ \rho\ \phi\left[\frac{\mu}{\mathbf{D^2N\rho}}\right]. \textbf{ Ans.}$$

b) The stream function for a two-dimensional flow is given by, ψ = 2xy, calculate the velocity at the point P (2, 3). Find also the velocity potential function at point P.

Solution. Given : $\psi = 2xy$

The velocity components u and v in terms of ψ are

$$u = -\frac{\partial \psi}{\partial y} = -\frac{\partial}{\partial y}(2xy) = -2x$$

$$v = \frac{\partial \psi}{\partial x} = \frac{\partial}{\partial x}(2xy) = 2y.$$

At the point P (2, 3), we get $u = -2 \times 2 = -4$ units/sec

$v = 2 \times 3 = 6$ units/sec

$\therefore$ Resultant velocity at $P = \sqrt{u^2 + v^2} = \sqrt{4^2 + 6^2} = \sqrt{16 + 36} = \sqrt{52} = 7.21$ units/sec.

Velocity Potential Function ϕ

We know $$\frac{\partial \phi}{\partial x} = -u = -(-2x) = 2x \quad \text{...}(i)$$

$$\frac{\partial \phi}{\partial y} = -v = -2y \quad \text{...}(ii)$$

Integrating equation (i), we get

$$\int d\phi = \int 2x dx$$

or $$\phi = \frac{2x^2}{2} + C = x^2 + C \quad \text{...}(iii)$$

where C is a constant which is independent of x but can be a function of y.

Differentiating equation (iii) w.r.t. 'y', we get $\frac{\partial \phi}{\partial y} = \frac{\partial C}{\partial y}$

But from (ii), $$\frac{\partial \phi}{\partial y} = -2y$$

$\therefore$ $$\frac{\partial C}{\partial y} = -2y$$

Integrating this equation, we get $C = \int -2y\, dy = -\frac{2y^2}{2} = -y^2$

Substituting this value of C in equation (iii), we get $\phi = \mathbf{x^2 - y^2}$. **Ans.**

Q-5. Attempt any one part of the following **(1*5 = 5)**

a) What are the minor losses and major losses in a pipe flow? Derive the chazy's formula for the loss of head due to friction.

Answer:

When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of the energy of fluid is lost. This loss of energy is classified as :

Energy Losses

1. Major Energy Losses

This is due to friction and it is calculated by the following formulae :

(*a*) Darcy-Weisbach Formula

(*b*) Chezy's Formula

2. Minor Energy Losses

This is due to

(*a*) Sudden expansion of pipe

(*b*) Sudden contraction of pipe

(*c*) Bend in pipe

(*d*) Pipe fittings etc.

(*e*) An obstruction in pipe.

Chezy's Formula for loss of head due to friction in pipes. Refer to chapter 10 article 10.3.1 in which expression for loss of head due to friction in pipes is derived. Equation (*iii*) of article 10.3.1, is

$$h_f = \frac{f'}{\rho g} \times \frac{P}{A} \times L \times V^2 \qquad ...(11.2)$$

where h_f = loss of head due to friction, P = wetted perimeter of pipe,
A = area of cross-section of pipe, L = length of pipe,
and V = mean velocity of flow.

Now the ratio of $\frac{A}{P}\left(=\frac{\text{Area of flow}}{\text{Perimeter (wetted)}}\right)$ is called hydraulic mean depth or hydraulic radius and is denoted by m.

$$\therefore \text{ Hydraulic mean depth, } \quad m = \frac{A}{P} = \frac{\frac{\pi}{4}d^2}{\pi d} = \frac{d}{4}$$

Substituting $\frac{A}{P} = m$ or $\frac{P}{A} = \frac{1}{m}$ in equation (11.2), we get

$$h_f = \frac{f'}{\rho g} \times L \times V^2 \times \frac{1}{m} \text{ or } V^2 = h_f \times \frac{\rho g}{f'} \times m \times \frac{1}{L} = \frac{\rho g}{f'} \times m \times \frac{h_f}{L}$$

$$\therefore \quad V = \sqrt{\frac{\rho g}{f'} \times m \times \frac{h_f}{L}} = \sqrt{\frac{\rho g}{f'}}\sqrt{m\frac{h_f}{L}} \qquad ...(11.3)$$

Let $\sqrt{\frac{\rho g}{f'}} = C$, where C is a constant known as Chezy's constant and $\frac{h_f}{L} = i$, where i is loss of head per unit length of pipe.

Substituting the values of $\sqrt{\frac{\rho g}{f'}}$ and $\sqrt{\frac{h_f}{L}}$ in equation (11.3), we get

$$V = C\sqrt{mi} \qquad ...(11.4)$$

Equation (11.4) is known as Chezy's formula. Thus the loss of head due to friction in pipe from Chezy's formula can be obtained if the velocity of flow through pipe and also the value of C is known. The value of m for pipe is always equal to $d/4$.

b) State and prove Bernoulli's theorem. Mention the assumptions made. How is it modified while applying in real problem?

Solution. Statement of Bernoulli's Theorem. It states that in a steady, ideal flow of an incompressible fluid, the total energy at any point of the fluid is constant. The total energy consists of pressure energy, kinetic energy and potential energy or datum energy. These energies per unit weight of the fluid are :

$$\text{Pressure energy} = \frac{p}{\rho g}$$

$$\text{Kinetic energy} = \frac{v^2}{2g}$$

$$\text{Datum energy} = z$$

Thus mathematically, Bernoulli's theorem is written as

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{Constant.}$$

The following are the assumptions made in the derivation of Bernoulli's equation :

(*i*) The fluid is ideal, *i.e.,* viscosity is zero
(*ii*) The flow is steady
(*iii*) The flow is incompressible
(*iv*) The flow is irrotational.

Bernoulli's equation is obtained by integrating the Euler's equation of motion (6.3) as

$$\int \frac{dp}{\rho} + \int g dz + \int v dv = \text{constant}$$

If flow is incompressible, ρ is constant and

$$\therefore \quad \frac{p}{\rho} + gz + \frac{v^2}{2} = \text{constant}$$

or $$\frac{p}{\rho g} + z + \frac{v^2}{2g} = \text{constant}$$

or $$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant} \quad \text{...(6.4)}$$

Equation (6.4) is a Bernoulli's equation in which

$\frac{p}{\rho g}$ = pressure energy per unit weight of fluid or pressure head.

$v^2/2g$ = kinetic energy per unit weight or kinetic head.

z = potential energy per unit weight or potential head.