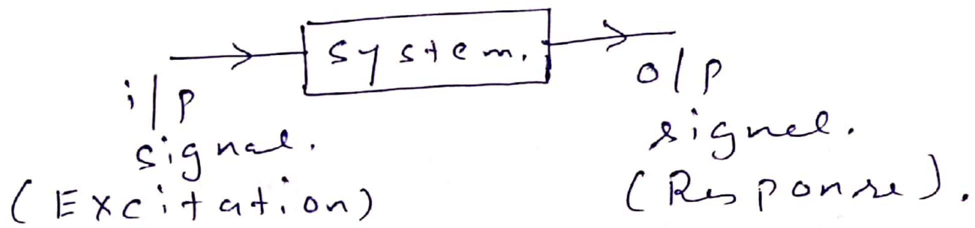
Subject :- Basic signal and system. (KEE-303).

Section A

1) a) Signal :- It is a function of one or more variable; which contain information on the nature of a physical phenomenon.

System :- It is well arrangment of element so that when a input signal is applied over it; extract information from given signal or producing a new signal.



b). Roc :- It is region where z-transform exsits. or it is range of value of z for which z-transform occur or converge.

c). Find value of $\int_{-\infty}^{\infty} (t-2)^2 \delta(t-2)$.

by the property of impluse function. i.e

$$\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & t > 0 \end{cases}$$

$\therefore$ it occur only at $t = 2$.

$$(t-2)^2 \Big|_{t=2} = (2-2)^2 = 0.$$

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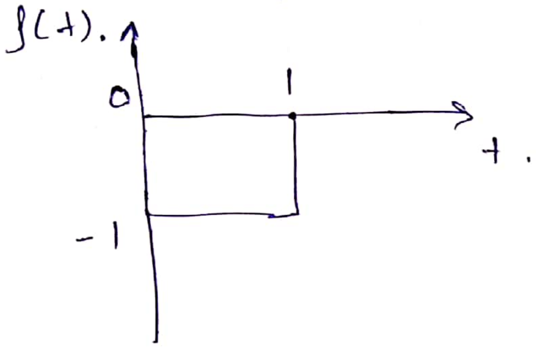
(iv). Sketch waveform.

$$u(t) - 2u(t) + u(t-1).$$

$$\Rightarrow \quad -u(t) + u(t-1).$$

$$\Rightarrow \quad -[u(t) - u(t-1)].$$

It is gate function between 0 to 1. and its magnitude is negative.



(v). Energy signal :- A signal which satisfie the condition. $0 < E < \infty$.

where $$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt.$$

power signal :- A power signal will satisfies the condition $0 < P < \infty$.

where
$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt.$$

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Section - B.

(a) $y(t) = t\,x(t)$, it is Linear; Time varying & causal system.

For linear system.

$$T[a_1 x(t) + a_2 x(t)] = a_1 T x(t) + a_2 T x_2(t).$$

For $y(t) = t\,x(t)$. : L.H.S.

$$t[a_1 x_1(t) + a_2 x_2(t)]$$

$$\Rightarrow t\,a_1 x_1(t) + t\,a_2 x_2(t)].$$

$$\Rightarrow a_1 t\,x_1(t) + a_2 t\,x_2(t); \quad -(1)$$

R.H.S.

$$= t\,x_1(t) + t\,x_2(t).$$

$$\Rightarrow a_1 t x_1(t) + a_2 t x_2(t). \quad -(2).$$

From (1) & (2) LHS = R.H.S i.e the.

$y(t) = t\,x(t)$ is linear.

(b). Time variant or invariant.

$$y(t,k) = y(t-k).$$

i.e time delay = time shifting.

$$y(t,k) = t\,x(t-k).$$

$$y(t-k) = (t-k)\,x(t-k).$$

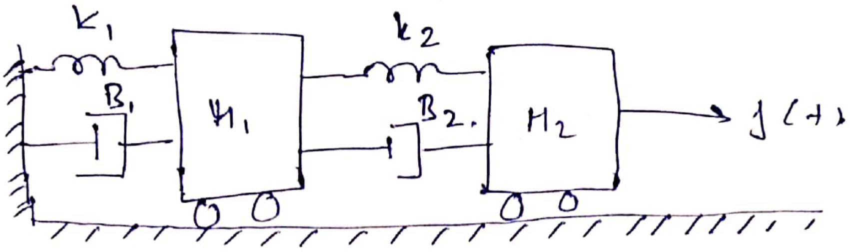
i.e $y(t,k) \neq y(t-k)$.

It is Time varying.

Causal or non-causal.

$y(t) = t\,x(t)$ is only depend on current value / present value ∴ it is causal.

(b).



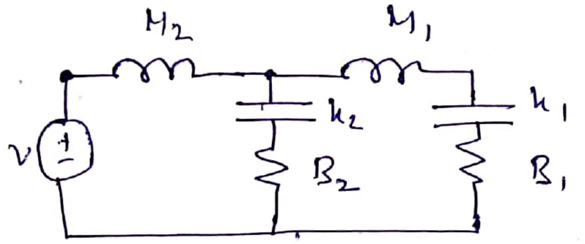
Force - voltage analogy.

Two Loop.

I^{st} Loop : $f(t)$; M_2 K_2 & B_2.

II^{nd} Loop : K_2 B_2 M_1 K_1 B_1.

F	V.
M	L
K	1/C
B	R.



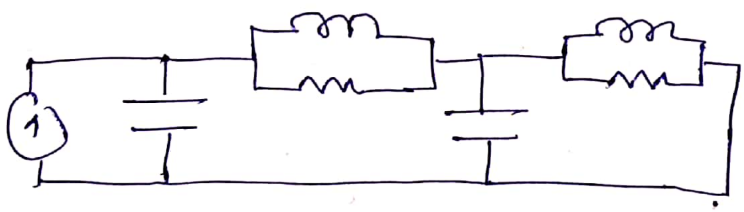
Force - current.

Two node.

1^{st} node : $f(t)$; M_2 K_2 B_2.

2^{nd} node : K_2 B_2 M_1 K_1 B_1

F	I
M	C
B	1/R.
K	1/L.



(c). (i)

$$x(t) = Ke^{-at}u(t).$$

$$E_\infty = \int_0^\infty |Ke^{-at}|^2 dt = K^2 \int_0^\infty e^{-2at} dt.$$

$$= K^2 \left. \frac{e^{-2at}}{-2a} \right|_0^\infty = \frac{K^2}{2a}.$$

i.e. $Ke^{-at}u(t)$ is energy signal.

So $P_\infty = 0$.

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c(ii) $x(t) = 4t.$

$$E_\infty = \int_{-\infty}^{\infty} |4t|^2 \, dt = 16 \int_{-\infty}^{\infty} t^2 \, dt = \frac{16}{3} t^3 \Big|_{-\infty}^{\infty}$$

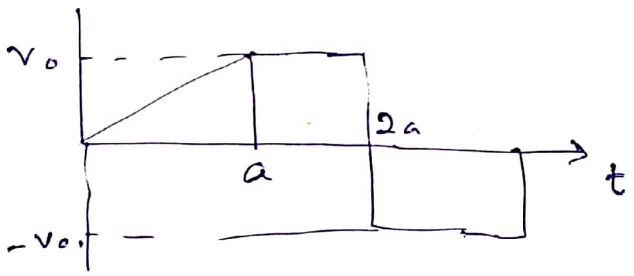
$$= \infty.$$

$$P_\infty = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |4t|^2 \, dt = \lim_{T \to \infty} \frac{1}{2T} \cdot \frac{16}{3} t^3 \Big|_{-T}^{T}.$$

$$= \lim_{T \to \infty} \frac{16}{6T} \left(T^6 + T^6 \right] = \frac{32}{6T} \cdot T^6 = \infty.$$

i.e neither energy nor power signal.

d).



$$g(t) = g_{0a} + g_{a,2a} + g_{2a,3a}$$

$$= \frac{v_0}{a} t \left[u(t) - u(t-a) \right] + v_0 \left[u(t-a) - u(t-2a) \right].$$

$$- v_0 \left[u(t-2a) - u(t-3a) \right].$$

$$= \frac{v_0}{a} \left[t u(t) - t u(t-a) + a u(t-a) \right].$$

$$- 2 v_0 u(t-2a) + v_0 u(t-3a)).$$

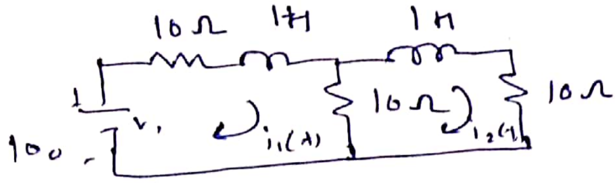
$$= \frac{v_0}{a} \left[t u(t) - (t-a) u(t-a) \right] - 2 v_0 u(t-2a) + v_0 u(t-3a)$$

$$= \frac{v_0}{a} \left[r(t) - r(t-a) \right] - 2 v_0 u(t-2a) + v_0 u(t-3a)$$

Section - C

— x —

(a)



Loop ①

$$100 = 10\, i_1(t) + 1\, \frac{di_1(t)}{dt} + 10\,[i_1(t) - i_2(t)].$$

Apply Laplace.

$$\frac{100}{s} = 20\, I_1(s) + s\, I_1(s) - 10\, I_2(s) \quad — ①.$$

$$\frac{100}{s} = (s+20)\, I_1(s) - 10\, I_2(s).$$

Loop ②

$$\frac{di_2(t)}{dt} + 10\, i_2(t) + 10\,[i_2(t) - i_1(t)] = 0.$$

Apply Laplace.

$$s\, I_2(s) + 20\, I_2(s) - 10\, I_1(s) = 0.$$

$$I_2(s) = \frac{10\, I_1(s)}{s+20} \quad — ②.$$

Put the value of $I_2(s)$ in eq ①.

$$\frac{100}{s} = (s+20)\, I_1(s) - 10 \times \frac{10\, I_1(s)}{s+20}.$$

$$\frac{100}{s} = \left[\frac{(s+20)^2 - 100}{(s+20)}\right] I_1(s).$$

$$\therefore\ I_1(s) = \frac{100\,(s+20)}{s\,(s^2 + 40s + 300)}.$$

$$I_1(s) = \frac{100}{s(s+10)(s+30)} = \frac{20/3}{s} - \frac{5}{s+10} - \frac{5/3}{s+30}.$$

Apply Laplace inverse.

$$i_1(t) = \frac{20}{3} - 5e^{-10t} - \frac{5}{3}e^{-30t}. \quad A.$$

Now put $I_1(s)$ in eq ②.

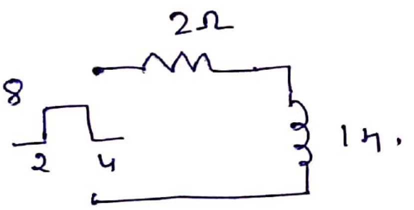
$$I_2(s) = \frac{1000}{s(s+10)(s+30)}.$$

$$= \frac{10/3}{s} - \frac{5}{s+10} + \frac{5/3}{s+30}.$$

Apply Laplace inverse.

$$i_2(t) = \frac{10}{3} - 5e^{-10t} + \frac{5}{3}e^{-30t} \quad A.$$

(b)



$$V_i = 8[u(t-2) - u(t-4)].$$

$$= \frac{8e^{-2s}}{s} - \frac{8e^{-4s}}{s}.$$

Apply KVL.

$$V_i = 2i(t) + 1\frac{di(t)}{dt}.$$

Apply Laplace

$$8\left[\frac{e^{-2s}}{s} - \frac{e^{-4s}}{s}\right] = I(s)[s+2].$$

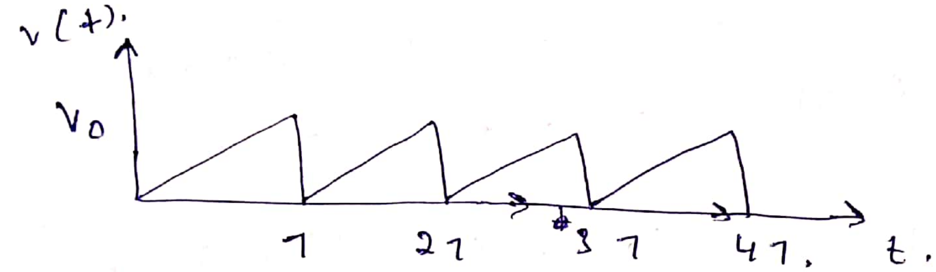
$$I(s) = \frac{8\left[e^{-2s} - e^{-4s}\right]}{s(s+2)}.$$

$$I(s) = e^{-2s}\left[\frac{4}{s} - \frac{4}{s+2}\right] - e^{-4s}\left[\frac{4}{s} - \frac{4}{s+2}\right].$$

Apply Laplace inverse

$$i(t) = 4\left[1 - e^{-2(t-2)}\right]u(t-2) - 4\left(1 - e^{-2(t-4)}\right]u(t-4) \text{ A}.$$

4). (a).



For periodic function.

$$V(s) = \frac{V_1(s)}{1 - e^{-Ts}}.$$

$$V_1(t) = \frac{V_0}{T} \cdot t\left[u(t) - u(t-T)\right].$$

$$= \frac{V_0 t}{T} u(t) - \frac{V_0}{T}(t-T)u(t-T) - V_0 u(t-T).$$

$$V_1(s) = \frac{V_0}{T}\left[\frac{1}{s^2} - \frac{e^{-Ts}}{s^2}\right] - V_0\frac{e^{-sT}}{s}.$$

$$V(s) = \frac{V_0}{Ts^2} - \frac{V_0 e^{-Ts}}{s(1 - e^{-Ts})}.$$

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(b). If $f_1(t)$ & $f_2(t)$ are 2 function of time which are zero $t<0$ and their laplace are $F_1(s)$ & $F_2(s)$ then convolution theorem state that the laplace transform of convolution of $f_1(t) * f_2(t) = F_1(s) \cdot F_2(s)$,

$$f_1(t) * f_2(t) = \int_0^t f_1(\tau)\, f_2(t-\tau)\, d\tau.$$

$$L\{f_1(t) * f_2(t)\} = L\left[\int_0^t f_1(t-\tau)\, f_2(\tau)\, d\tau\right].$$

$$= \int_0^\infty \left[\int_0^t f_1(t-\tau)\, f_2(\tau)\, d\tau\right] e^{-st}\, dt.$$

$$= \int_0^\infty \left[\int_0^\infty f_1(t-\tau)\, u(t-\tau)\, f_2(\tau)\, d\tau\right] e^{-st}\, dt.$$

put $t-\tau = x$, $dt = dx$.

$t = 0$; $x = -\tau$

$t = \infty$; $x = \infty$.

$$= \int_0^\infty \left[\int_{-\tau}^\infty f_1(x)\, u(x)\, f_2(\tau)\, d\tau\right) e^{-s(\tau+x)}\, dx.$$

$$= \int_{-\tau}^\infty f_1(x)\, u(x)\, e^{-sx}\, dx \int_0^\infty f_2(\tau)\, e^{-s\tau}\, d\tau,$$

$$= \int_0^\infty f_1(x)\, e^{-sx}\, dx \cdot \int_0^\infty f_2(\tau)\, e^{-s\tau}\, d\tau.$$

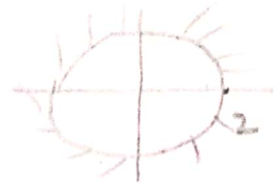
$$= F_1(s) \cdot F_2(s)$$

A5) a). $f(n) = [3(2^n) - 4(3^n)]u[n]$.

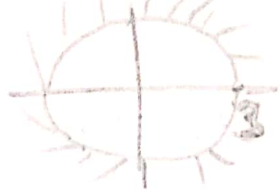
$f_1(n) = 3(2^n)$. $\quad f_2(n) = 4(3^n)$.

Apply z-transform.

$$F_1(z) = \frac{1}{1-2z^{-1}} \text{ ; ROC } |z|>2.$$



$$F_2(z) = \frac{1}{1-3z^{-1}} \text{ ; ROC } |z|>3.$$



$$F_d(z) = \frac{3}{1-2z^{-1}} - \frac{4}{1-3z^{-1}}.$$

$$F(z) = \frac{-(1+z^{-1})}{1-5z^{-1}+6z^{-2}}, \quad \text{ROC } |z|>3.$$

(b).

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = 5u(t)$$

Apply Laplace.

$$s^2Y(s) - sy(0^+) - y'(0^+) + 3sY(s) - 3Y(0^+) + 2Y(s) = \frac{5}{s}.$$

$$s^2Y(s) + s - 2 + 3sY(s) + 3 + 2Y(s) = 5/s.$$

$$Y(s) = \frac{-s^2 - s + 5}{s(s+1)(s+5)} = \frac{5/2}{s} - \frac{5}{s+1} + \frac{3/2}{s+2}.$$

$$y(t) = \frac{5}{2} - 5e^{-t} + 3/2\, e^{-2t} \quad t \geq 0.$$