Electrical Measurements & Measuring Instruments

EEMI [NEE-302]

Unit – I

1.1 Philosophy of Measurement

Measurement:

Measurement of a given quantity is essentially an act of comparison between the quantity (whose magnitude is unknown) and a predefined standard. Since two quantities are compared, the result is expressed in numerical values.

Methods of Measurement:-

There are two methods of measurement:

1) Direct comparison with the standard, and

2) Indirect comparison with the standard.

1) **Direct Comparison with the Standard**

In the direct comparison method of measurement, we compare the quantity directly with the primary or secondary standard. Say for instance, if we have to measure the length of the bar, we will measure it with the help of the measuring tape or scale that acts as the secondary standard.

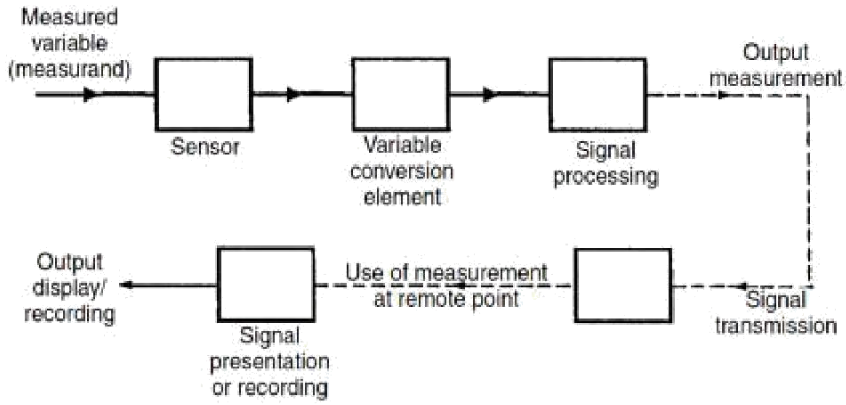
2) **Indirect Method of Measurement**

There are number of quantities that cannot be measured directly by using some instrument. For instance we cannot measure the strain in the bar due to applied force directly. We may have to record the temperature and pressure in the deep depths of the ground or in some far off remote places. In such cases indirect methods of measurements are used.

Instruments and Measurement System:

Measurement involves the use of instruments as a physical means of determining quantities and variables. In simple cases, an instrument consists of a single unit which gives an output reading or signal according to the unknown variable applied to it. In more complex measurement situations, however, a measuring instrument may consist of several separate elements. These elements may consist of transuding elements which convert the measured to the analogous form.

The analogous signal is then processed by some intermediate means and then fed to the end devices to present the results of the measurement for the purposes of display and or control.



Type of Instruments:

Mechanical Instruments:-

- Reliable for static and stable conditions
- Unable to respond rapidly to measurements of dynamic and transient conditions
- Have moving parts that are rigid, heavy and bulky and have large mass
- Virtually impossible to measure a 50 Hz voltage by using a mechanical instruments
- Potential source of noise and create noise pollution

Electrical Instruments:-

- Depends upon a mechanical meter movement as indicating device
- Mechanical movement has some inertia and therefore these instruments have a limited time response.

  Example:- Electrical recorders – 0.25s
  Industrial recorders – 0.5 to 24s

Electronic Instruments:-

- Necessity to step up response time and also the detection of dynamic changes in certain parameters, which requires monitoring time of the order of ms and many a times μs.
- Use of semiconductor devices
- Movement involved is that of electrons, response time is extremely small on account of very small inertia of electrons.

  e., g.: CRO – dynamic and transient changes – few ns (10^{-9}s)
- More reliable
- Very weak signals can be detected by using pre-amplifiers and amplifiers. Power amplification is very important from bio instrumentation point of view.
- Ability to obtain indication at a remote location which helps in monitoring inaccessible or hazardous locations.
- These are used in measurement of non electrical quantities by converting non electrical quantities to electrical quantities using transducers.
- Detection of electromagnetically produced signals such as radio, video and microwave.
- These instruments are light in weight, compact, have a high degree of reliability and their power consumption is low
- Particularly useful in intermediate signal modifying stages.

Classification of Electrical Instruments:-

Electrical measuring instruments may be classified into two groups:
(a) Absolute (or primary) instruments.
(b) Secondary instruments.

Absolute Instruments

These instruments give the value of the electrical quantity in terms of absolute quantities (or some constants) of the instruments and their deflections.

In this type of instruments no calibration or comparison with other instruments is necessary.

They are generally not used in laboratories and are seldom used in practice by electricians and engineers. They are mostly used as means of standard measurements and are maintained lay national laboratories and similar institutions.

Some of the examples of absolute instruments are:

- Tangent galvanometer
- Raleigh current balance
- Absolute electrometer.

Secondary Instruments

They are direct reading instruments. The quantity to be measured by these instruments can be determined from the deflection of the instruments.

They are often calibrated by comparing them with either some absolute instruments or with those which have already been calibrated.

The deflections obtained with secondary instruments will be meaningless until it is not calibrated.

These instruments are used in general for all laboratory purposes.

On the basis of types of indication

Deflecting Type Instruments:-

In these instruments, the deflection of the instrument provides a basis for determining the quantity under measurement. The measured quantity produces some physical effect which deflects or produces a mechanical displacement of the moving system of the instrument. An opposing effect is built in the instrument which tries to oppose the deflection or the mechanical displacement of the moving system. The opposing effect is so designed that its magnitude increases with increases of deflection or mechanical displacement of the moving system caused by the quantity under measurement. The balance is achieved when opposing effect equals to cause producing the deflection or mechanical movement. The value of the measured quantity can then be inferred from the deflection or mechanical displacement at the point of balance.

For example: - In PMMC ammeter the deflection of moving coil is proportional to I i.e. the quantity under measurement.

$$T_d = GI$$

Where T_d = torque acting on moving coil

G = constant dependent upon flux density, number of turns and area of moving coil.

The opposing effect whose torque T_c is proportional to deflection T_c = KI. Where, K is the spring constant based on the material and the dimensions of spring. (Under balance condition)

$$T_c = T_d$$

Therefore, $$I = (K/G)\theta$$

Null Type Instruments:-

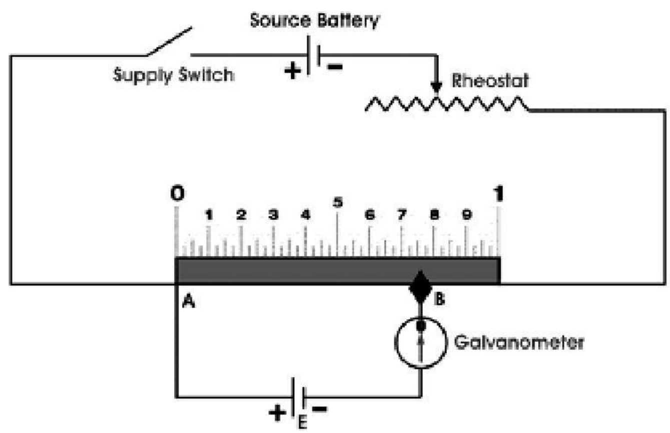
A zero or null indication leads to determination of the magnitude of measured quantity. For the operation of a null type of instrument the following are required:

- The effect produced by measured quantity
- The opposing effect, whose value is accurately known,
- A detector, which detects the null conditions i.e. a device which indicates zero deflection when the effect produced by the measured quantity is equal to the effect produced by the opposing quantity.

For example: D.C. Potentiometer

Unknown voltage is measured.

The slide wire has been calibrated in terms of emf with the help of a standard emf source. Null detector is the current galvanometer whose deflection is proportional to the difference between emf across portion of the slide wire and unknown voltage. As soon as two are equal there is no current through galvanometer and therefore it shows zero deflection thereby indicating null conditions. Therefore, the unknown emf is equal to the emf across portion of the slide wire which is directly indicated by the calibrated scale placed alongside the slide wire.



Comparison of deflection type and null type instrument:

In deflection type instruments, the value of the quantity being measured is displayed in terms of the amount movement of a pointer. The pressure-measuring device shown is an example of a deflection type instrument.

An alternative type of pressure gauge is the deadweight gauge which is a null-type instrument. Here, weights are put on top of the piston until the downward force balances the fluid pressure. Weights are added until the piston reaches a datum level, known as the null point. Pressure measurement is made in terms of the value of the weights needed to reach this null position.

The accuracy of the deflection type pressure measurement instrument depends on the linearity and calibration of the spring, whilst for the second it relies on the calibration of the weights.

As calibration of weights is much easier than careful choice and calibration of a linear-characteristic spring, this means that the second type of instrument will normally be the more accurate. This is in accordance with the general rule that null-type instruments are more accurate than deflection types.

In terms of usage, the deflection type instrument is clearly more convenient. It is far simpler to read the position of a pointer against a scale than to add and subtract weights until a null point is reached.

A deflection-type instrument is therefore the one that would normally be used in the workplace. However, for calibration duties, the null-type instrument is preferable because of its superior accuracy. The extra effort required to use such an instrument is perfectly acceptable in this case because of the infrequent nature of calibration operations.

Characteristics of Measurement System Performance

Static Characteristics: Some applications involve the measurement of quantities that are either constant or varies slowly with time. Under these circumstances it is possible to define a set of criteria that gives a meaningful description of quality of measurement without interfering with dynamic descriptions that involve the use of differential equations. These criteria are called static characteristics.

Dynamic Characteristics: Many measurements are concerned with rapidly varying quantities and therefore, for such cases we must examine the dynamic relations which exist between the output and the input. This is normally done with the help of differential equations. Performance criteria based upon dynamic relations constitute the dynamic characteristics.

Static characteristics:

Accuracy:

The *accuracy* of an instrument is defined as the degree of closeness of the measured value to its true value.

Accuracy is usually expressed in term of percentage error with respect to full scale reading.

Precision/repeatability/reproducibility:

Precision is a term that describes an instrument's degree of freedom from random errors. If a large number of readings are taken of the same quantity by a high precision instrument, then the spread of readings will be very small.

Precision is often, though incorrectly, confused with accuracy. High precision does not imply anything about measurement accuracy. A high precision instrument may have a low accuracy. Low accuracy measurements from a high precision instrument are normally caused by a bias in the measurements, which is removable by recalibration.

The terms repeatability and reproducibility mean approximately the same but are applied in different contexts as given below. *Repeatability* describes the closeness of output readings when the same input is applied repetitively over a short period of time, with the same measurement conditions, same instrument and observer, same location and same conditions of use maintained throughout.

Tolerance:-

Tolerance is a term that is closely related to accuracy and defines the maximum error that is to be expected in some value. When used correctly, tolerance describes the maximum deviation of a manufactured component from some specified value. For instance, crankshafts are machined with a diameter tolerance quoted as so many microns (10^{-6}m), and electric circuit components such as resistors have tolerances of perhaps 5%. One resistor chosen at random from a batch having a nominal value 1000W and tolerance 5% might have an actual value anywhere between 950W and 1050 W.

Range or span:-

The *range* or *span* of an instrument defines the minimum and maximum values of a quantity that the instrument is designed to measure.

Linearity:-

It is normally desirable that the output reading of an instrument is linearly proportional to the quantity being measured. The Xs marked on Figure which show a plot of the typical output readings of an instrument when a sequence of input quantities is applied to it. Normal procedure is to draw a good fit straight line through the Xs, as shown in Figure given below. The non-linearity is then defined as the maximum deviation of any of the output readings marked X from this straight line. Non-linearity is usually expressed as a percentage of full-scale reading.

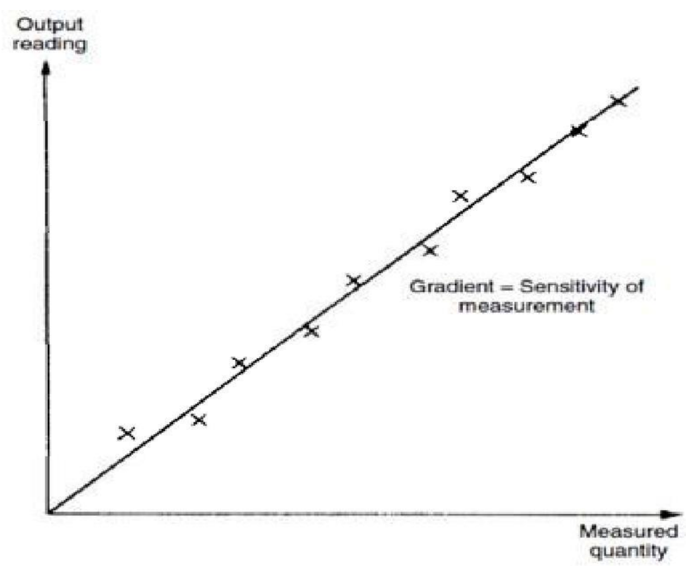


Figure: Instrument output characteristic

Sensitivity:

The sensitivity of measurement is a measure of the change in instrument output that occurs when the quantity being measured changes by a given amount. Thus, sensitivity is the ratio:

$$\frac{\text{Scale deflection}}{\text{Value of measured producing deflection}}$$

The sensitivity of measurement is therefore the slope of the straight line drawn on Figure shown above. If, for example, a pressure of 2 bar produces a deflection of 10 degrees in a pressure transducer, the sensitivity of the instrument is 5 degrees/bar (assuming that the deflection is zero with zero pressure applied).

Threshold:

If the input to an instrument is gradually increased from zero, the input will have to reach a certain minimum level before the change in the instrument output reading is of a large enough magnitude to be detectable. This minimum level of input is known as the *threshold* of the instrument. Manufacturers vary in the way that they specify threshold for instruments. Some quote absolute values, whereas others quote threshold as a percentage of full-scale readings. As an illustration, a car speedometer typically has a threshold of about 15 km/h. This means that, if the vehicle starts from rest and accelerates, no output reading is observed on the speedometer until the speed reaches 15 km/h.

Resolution:

When an instrument is showing a particular output reading, there is a lower limit on the magnitude of the change in the input measured quantity that produces an observable change in the instrument output. Like threshold, *resolution* is sometimes specified as an absolute value and sometimes as a percentage of f.s. deflection. One of the major factors influencing the resolution of an instrument is how finely its output scale is divided into subdivisions. Using a car speedometer as an example again, this has subdivisions of typically 20 km/h. This means that when the needle is between the scale markings, we cannot estimate speed more accurately than to the nearest 5 km/h. This figure of 5 km/h thus represents the resolution of the instrument.

Drift:

The drift is the gradual shift of the instrument indication, over an extended period during which the value of the input variable does not change. The Zero Drift is defined as the deviation in the instrument output with time, from its zero value, when the variable to be measured is constant. The whole instrument calibration may gradually shift by the same amount.

If there exists a proportional change in the indication, all along the upward scale then the drift from nominal characteristics is called sensitivity drift.

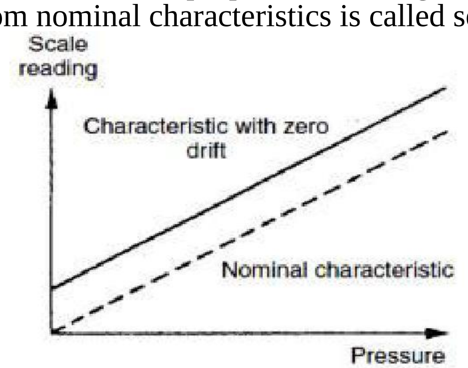


Figure: Zero Drift

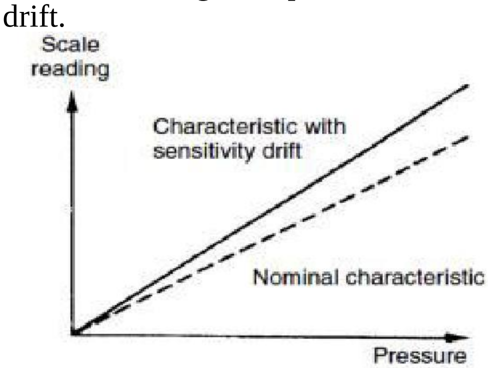


Figure: Sensitivity Drift

Dead Space:

In some instruments it is possible that till input increases beyond certain value, the output does not change. So for a certain range of input values there is no change in output. This range of input is called dead space.

Speed of response:

It is the rapidity with which the system responds to the changes in the quantity to be measured. It gives the information about how fast the system reacts to the changes in the input. It indicates activeness of the system.

Fidelity:

It indicates how much faithfully the system reproduces the changes in the input. It is the ability of an instrument to produce a wave shape identical to wave shape of input w.r.t time.

Dynamic Error:

It is the difference between the true value of the variable to be measured, changing with time, and the value indicated by the measurement system, assuming zero static error.

ERRORS IN MEASUREMENT

In order to understand the concept of **errors in measurement**, following two terms defines the error.

True Value

It is not possible to determine the true of quantity by experiment means. True value may be defined as the average value of an infinite number of measured values when average deviation due to various contributing factor will approach to zero.

Measured Value

It may be defined as the approximated value of true value. It can be found out by taking means of several measured readings during an experiment, by applying suitable approximations on physical conditions.

Limiting Errors or Guarantee Errors

The concept of guarantee errors can better clear following example. Suppose there is a manufacturer who manufacture an ammeter, now he should promises that the error in the ammeter he is selling not greater the limit he sets. This limit of error is known as limiting errors or guarantee error.

Relative Error or Fractional Error:

It is defined as the ratio of the error and the specified magnitude of the quantity. Mathematically we write as,

$$\text{Relative error} = \frac{dA}{A}$$

Where dA is the error and A is the magnitude (true value)

Types of Errors:

Basically there are three **types of errors** on the basis they may arise from the source.

Gross Errors

This category of errors includes all the human mistakes while reading, recording and the readings. Mistakes in calculating the errors also come under this category.

i. A proper care should be taken in reading, recording the data. Also calculation of error should be done accurately.
ii. By increasing the number of experimenters we can reduce the gross errors. If each experimenter takes different reading at different points, then by taking average of more readings we can reduce the gross errors.

Systematic Errors

In order to understand these kinds of errors, let us categorize the systematic errors as

(i) Instrumental Errors: These errors may be due to wrong construction, calibration of the measuring instruments. These types of error may be arises due to friction or may be due to hysteresis. These types of errors also include the loading effect and misuse of the instruments. Misuse of the instruments results in the failure to adjust the zero of Instruments. In order to minimize the gross errors in measurement various correction factors must be applied and in extreme condition instrument must be re-calibrated carefully.

(ii) Environmental Errors: This type of error arises due to conditions external to instrument. External condition includes temperature, pressure, humidity or it may include external magnetic field. Following are the steps that one must follow in order to minimize the environmental errors:

a) Maintain the temperature and humidity of the laboratory constant by making some arrangements.
b) Ensure that there should not be any external magnetic or electrostatic field around the instrument.

(iii) Observational Errors: As the name suggests these **types of errors** are due wrong observations. The wrong observations may be due to PARALLAX. In order to minimize the PARALLAX error highly accurate meters are required, provided with mirrored scales.

Random Errors: After calculating all systematic errors, it is found that there are still some errors in measurement are left. These errors are known as random errors. Some of the reasons of the appearance of these errors are known but still some reasons are unknown. Hence we cannot fully eliminate these kinds of error.

1.2 Analog Ammeters and Voltmeters

The essential requirements of a measuring instrument are:

(i) That its introduction into the circuit, where measurements are to be made, does not alter the circuit conditions.

(ii) The power consumed by them for their operation is small

Classification of Measuring Instruments:

(1) **Indicating instruments:** Indicating instruments indicate, generally the quantity to be measured by means of a pointer which moves on a scale. Examples are ammeter, voltmeter, wattmeter etc.

(2) **Recording instruments:** These instruments record continuously the variation of any electrical quantity with respect to time. In principle, these are indicating instruments but so arranged that a permanent continuous record of the indication is made on a chart or dial. The recording is generally made by a pen on a graph paper which is rotated on a dice or drum at a uniform speed. The amount of the quantity at any time (instant) may be read from the traced chart. Any variation in the quantity with time is recorded by these instruments.

(3) **Integrating instruments:** These instruments record the consumption of the total quantity of electricity, energy etc., during a particular period of time. That is, these instruments totalize events over a specified period of time. No indication of the rate or variation or the amount at a particular instant are available from them. Some widely used integrating instruments are: Ampere-hour meter: kilowatt-hour (kWh) meter, kilovolt-ampere-hour (kVARh) meter.

Essential Requirements of an instrument:

In an indicating instrument, it is essential that the moving system is acted upon by three distinct torque (or forces) for satisfactory working.

There torques are:

1. A deflecting or operating torque, Td
2. A controlling torque, Tc
3. A damping torque, Tv

Deflecting (Or the Operating) Torque:

The deflecting torque causes the moving system of the instrument to move from its zero position. It may be produced by utilizing any one of the effects of current or voltage in the instrument such as magnetic effect, electromagnetic induction effect, heating effect, electrostatic effect etc.

The actual method of producing a deflecting torque depends upon the type of the instruments. The deflecting torque has to supply the following torque-components presents in an instrument.

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a) The torque required to overcome the torque due to the inertia of the moving system, J ($d^2\theta/dt^2$), where J is the moment of inertia and θ is the movement (rotation in radians).
b) The torque required to overcome the controlling torque, $Tc \equiv (k_c\theta)$.
c) The torque required to overcome the damping torque, $Tv = k_v\, d\theta/dt$, where k_v is damping torque constant.
d) The torque required to overcome the frictional (coulomb) torque. This component is minimized by appropriate design considerations.

Controlling Torque

The controlling torque developed in an instrument has two functions:

(a) It limits the movement of the moving system and ensures that the magnitude of the deflections always remains the same for a given value of the quantity to be measured.

(b) It brings back the moving system to its zero position where the quantity being measured is removed or made zero.

The controlling torque is dependent on the magnitude of deflection produced. The moving system is deflected from zero to such a position that the controlling torque at that deflected position is equal to the deflecting torque. The controlling torque increases in magnitude with the deflection till it balances the deflecting torque.

That is,

For a steady deflection, Controlling torque, T_c = Deflection or operating torque, T_d

The controlling torque is entered in all commercial instruments by any one of the following three ways.

By means of one or two coiled springs. The corresponding instrument is termed spring controlled instruments (mostly used system).

By the action of gravity due to suitably placed weights on the moving system. Such instruments are known as gravity controlled instruments.

By means of a permanent magnet (magnetic control system).

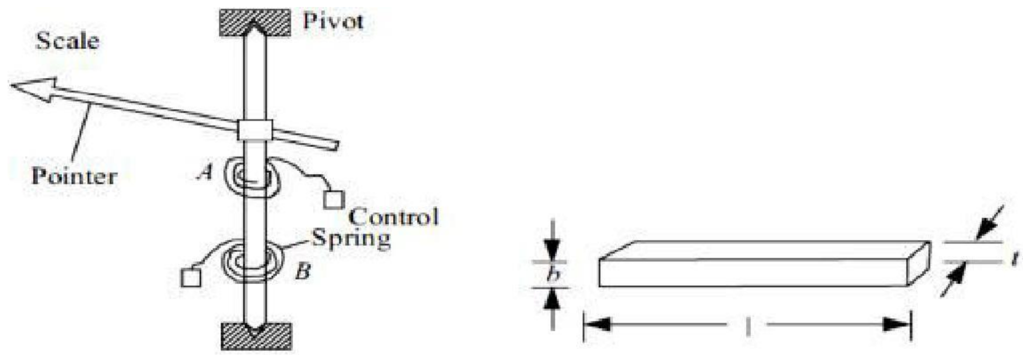
Spring control is now almost universal in indicating instruments. Gravity control is employed in a few cases, notably in special laboratory types, and magnetic control is applied to some galvanometers and certain moving iron instruments (the polarized form). We will discuss the first two methods of obtaining the controlling torque in a measuring instrument as given below.

Spring Control: The controlling torque developed in the spiral spring is given by

$$T_c = \frac{Ebr^2}{12l}\theta$$

$$T_C = k_s\,\theta$$

Where, *l* = Total length of spring strip
(*m*) *b* = depth of the strip (*m*)
t = thickness of the strip (*m*)
ks = spring constant



(a) (b)

Figure (a) Pivoted moving system with spring control (b) spring in strip form

Gravity Control: In gravity controlled instruments, as shown in figure below, a small adjustable weight is attached to the spindle of the moving system such that the deflecting torque produced by the instrument has to act against the action of gravity.

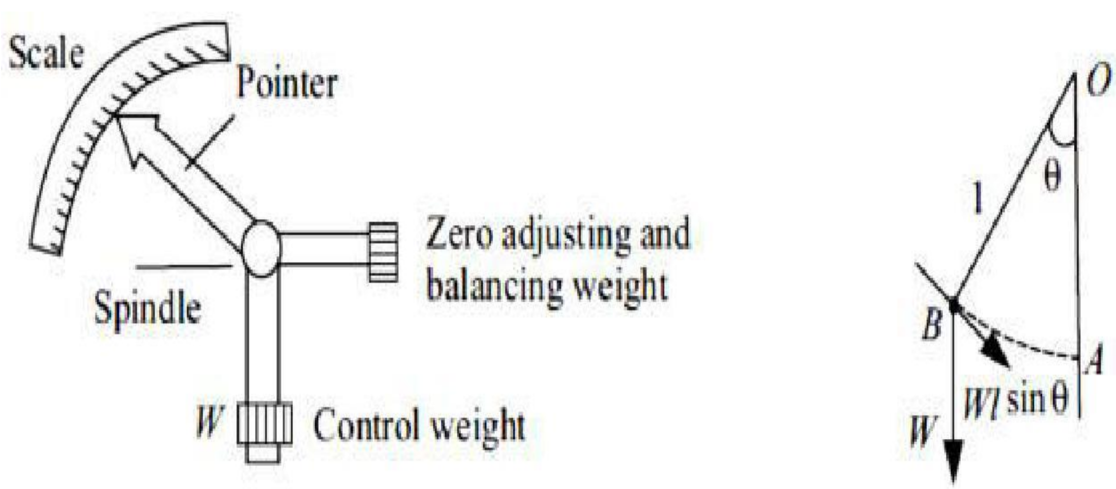
The controlling (or restoring) torque, T_c, is given by $T_c = Wl \sin \theta$.

Where, W is the control weight; l is the distance of the control weight from the axis of rotation of the moving system.

We have at the equilibrium position

$$T_d = T_c$$

$$\text{or } kI = Wl \sin \theta.$$



(a) (b)

Figure: (a) Gravity control moving system at zero position (b) moving system rotated by θ radian

Damping Torque

We have already seen that the moving system of the instrument will tend to move under the action of the deflecting torque. But on account of the control torque, it will try to occupy a position of rest when two torques are equal and opposite. However, due to inertia of the moving system, the pointer will not come to rest immediately but oscillate about its final deflected position as shown in Figure below and takes appreciable time to come to steady state.

To overcome this difficulty, a damping torque is to be developed by using a damping device attached to the moving system. The damping torque is proportional to the speed of rotation of the moving system, that is

$$T_v = k_v \, (d\theta/dt)$$

Where, k_v = damping torque constant.

$d\theta/dt$ = speed of rotation of the moving system

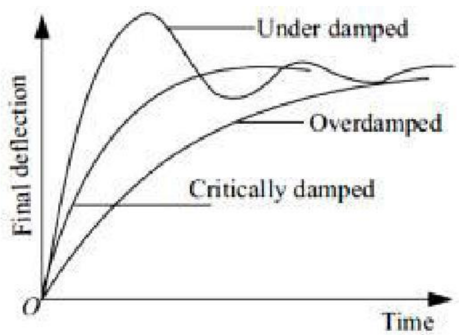


Figure: Dynamic Response of measuring instrument

Depending upon the degree of damping introduced in the moving system, the instrument may have any one of the following conditions as depicted in above figure.

a) **Under damped condition:** The response is oscillatory
b) **Over damped condition:** The response is sluggish and it rises very slowly from its zero position to final position.
c) C**ritically damped condition:** The response settles quickly without any oscillation.

In practice, the best response is obtained when the damping is slightly below the critical value *i.e.*, the instrument is slightly under damped.

The damping torque is produced by the following methods

(a) **Air Friction Damping**

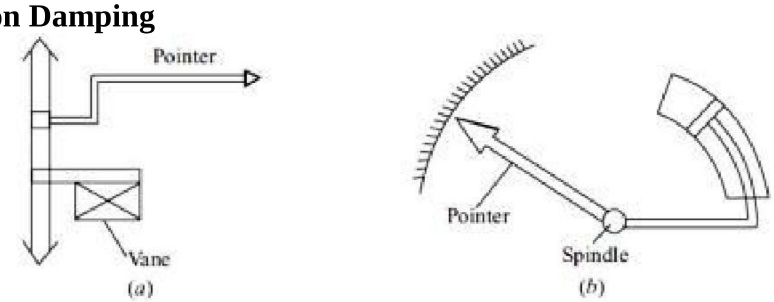


Figure: Air friction damping

The air friction damping is very simple and cheap. But care must be taken to ensure that the piston is not bent or twisted. This method is used in moving iron and hot wire instruments.

(b) Fluid Friction Damping

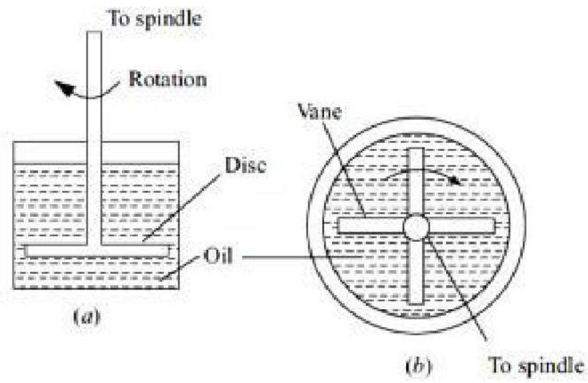
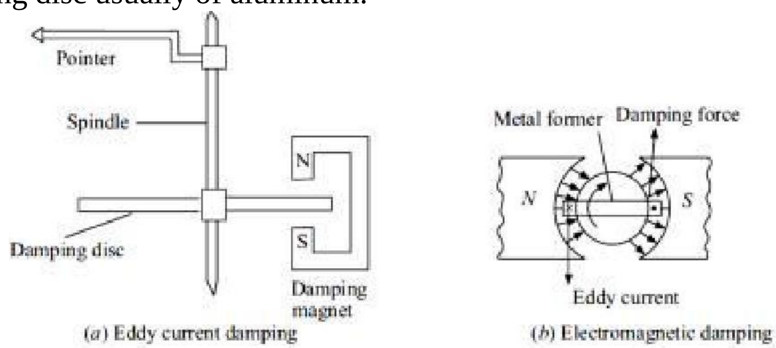


Figure: Fluid Friction damping

This arrangement provides greater damping torque.

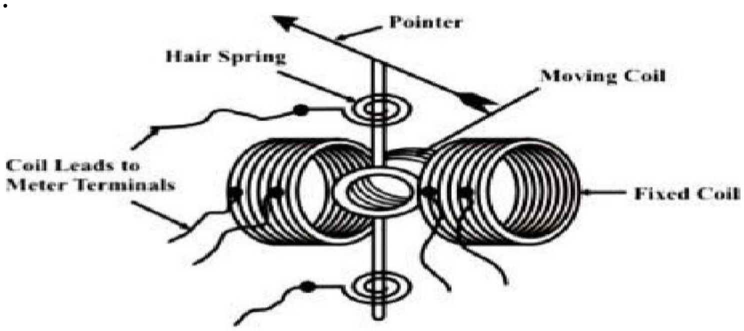
c) **Eddy Current Damping: E**ddy current damping is the most efficient form of damping. The essential components in this type of damping are a permanent magnet; and a light conducting disc usually of aluminum.



Eddy current damping and electromagnetic damping

ELECTRODYNAMOMETER TYPE INSTRUMENTS:-

Electrodynamics type instruments are similar to the PMMC-type elements except that the magnet is replaced by two serially connected fixed coils that produce the magnetic field when energized (see Fig. below).



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Construction of electro dynamic (or Dynamometer) type Instruments:

Fixed coil: The magnetic field is produced by the fixed coil which is divided into two sections to give more uniform field near the centre and to allow passage of the instrument shaft.

Moving coil: The moving coil is wound either as a self-sustaining coil or else on a non-magnetic former. A metallic former cannot be used, as eddy currents would be induced in it by alternating field. Light but rigid construction is used for the moving coil. It should be noted that both fixed and moving coils are air cored.

Springs: The controlling torque is provided by two control springs. These hairsprings also act as leads of current to the moving coil.

Dampers: Air friction damping is employed for these instruments and is provided by a pair of Aluminum-vanes attached to the spindle at the bottom. These vanes move in a sector shaped chamber.

Shielding: Since the magnetic field produced by fixed coils is weaker than that in other types of instruments, these meters need a special magnetic shielding. Electro-dynamic instruments are effectively shielded from the effects of external magnetic fields by enclosing the mechanism in a laminated iron hollow cylinder with closed ends.

Operating Principle:

Let us consider the currents in the fixed and moving coils are i_f and i_m respectively. The action of electrodynamic instrument depends upon the force exerted between fixed and moving coils carrying current. The flux density B (wb/m^2) produced by the fixed coil is proportional to i_f (fixed coil current). The force on the conductors of the moving coil, for a given strength field, will proportional to i_m (moving coil current) and the number of turns '*N*' of the moving coil. In case of ammeter and voltmeter fixed and moving coils are connected in series and the developed torque is due to the interaction of the magnetic fields produced by currents in the fixed and moving coils and thus it will be proportional to i^2 ($i_f = i_m = i$). Thus dynamic instruments can be used for both a.c. and d.c. measurement.

Expression for developed torque:

Torque expression based on energy concept

Let us assume that the fixed and moving coils having self-inductances Lf and Lm respectively. Further it is assumed that the mutual inductance between the fixed and movable coils is M.

Total energy stored in the magnetic field of the coils is given by

$$W = \frac{1}{2} L_f i_f^2 + \frac{1}{2} L_m i_m^2 + M i_f i_m$$

Where, i_f and i_m are the currents through the fixed and moving coils. The expression for torque developed can be written as

$$T_d = \frac{dW}{d\theta} = i_f i_m \frac{dM}{d\theta}$$

D.C operation: Expression for the developed torque is rewritten by setting $i_f = I_f$ (d.c) and $i_m = I_m$ (d.c)

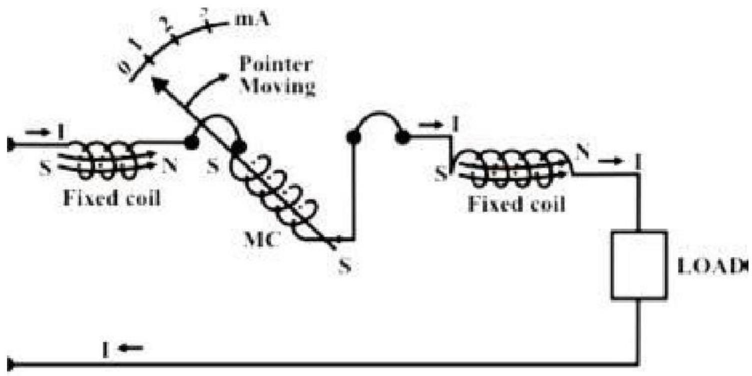
$$T_d = I_f\, I_m \frac{dM}{d\theta} = I_f\, I_m\, M_{max} \sin\theta$$

If the control is due to spiral springs, the controlling torque is proportional to the angle of deflection θ.

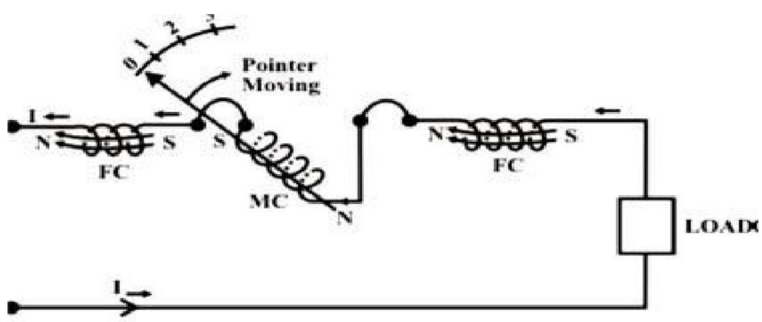
Controlling torque $T_c = k_s\theta$. Where, k_s is the spring constant.

Ammeters

The figure below shows that fixed coils and moving coil of a dynamometer instrument connected in series and assumed the current through moving coil does not exceed a certain the upper limit depending on its construction.



Torque produces positive deflection



Current flowing from right to left produces positive deflection again

Connections for ammeter & voltmeter

Ammeter

When ammeters for ranges above about 250 mA, the moving coil cannot be connected in series with the fixed coil (note the control spring is unsuitable for currents above about 250 mA). Therefore, the moving coil must be connected in parallel with the fixed coils as shown in Fig 16.

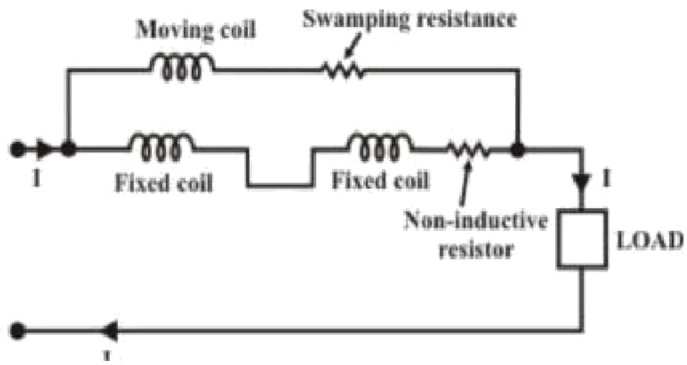


Figure: Electrodynamometer ammeter connection

Here the moving coil current is kept within 200 mA and the rest of current is passed through the fixed coil. Moving coil carries a small fraction of measured current through the moving coil. For extreme accuracy the connection shown in Fig. 43.4 must fulfill the following conditions.

- The resistance/reactance ratio must have the same value (i.e time constant of moving coil = time constant of fixed coil) for each branch.
- The percentage change of resistance with temperature must be the same for the two.

Voltmeters:

The connection for use as a voltmeter is shown in Fig.17, in which fixed and oving coils are connected in series with a high series resistance having "zero resistivity coefficients". branches.

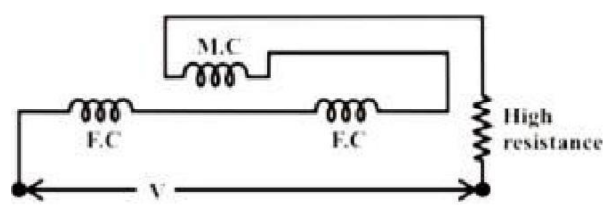


Figure: Electrodynamometer voltmeter connection

This combination is connected across the voltage source or across the load terminals whose voltage is to be measured. The deflecting torque is given by

$$T_d = i_f\, i_m \frac{dM}{d\theta} = \frac{V}{Z}\frac{V}{Z}\frac{dM}{d\theta}$$

where Z is the magnitude of total impedance of the voltmeter circuit. At steady state condition of deflection

$$T_d = T_c$$

$$\therefore \quad k_s\theta = \frac{V^2}{Z^2}\frac{dM}{d\theta}$$

$$\therefore \quad \theta = \frac{V^2}{Z^2 k_s}\frac{dM}{d\theta}$$

This implies that deflecting torque is directly proportional to V^2 if dM / dθ is kept nearly constant. This is possible if θ varies from 45º to 135º over the range of instrument scale.

Electrostatic Type Instruments:-

As the name suggests the **electrostatic type instrument** use static electrical field to produce the deflecting torque. These types of instrument are generally used for the measurement of high voltages but in some cases they can be used in measuring the lower voltages and powers of a given circuit. Now there are two possible ways in which the electrostatic force can act. The two possible conditions are written below,

Construction of Electrostatic Type Instruments

(a) When one of the plates is fixed and other plate is free to move, plates are oppositely charged in order to have attractive force between them. Now due this attractive force movable plate will move towards the stationary or fixed plate till the moving plate stored maximum electrostatic energy.

(b) In other arrangement there may be force of attraction or repulsion or both, due to some rotary of plate.

Force & Torque Equation of Electrostatic Type Instrument

Let us consider two plates as shown in the diagram given below.

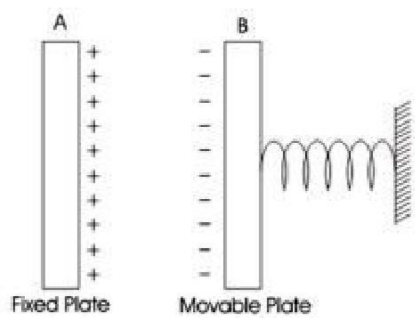


Plate A is positively charged and plate B is negatively charged. The plate A is fixed and plate B is free to move. Let us assume there exist some force F between the two plates at equilibrium when electrostatic force becomes equal to spring force. Based on energy conservation principle,

$$V^2dC + CVdV = \frac{1}{2}V^2dC + CVdV + Fdx$$

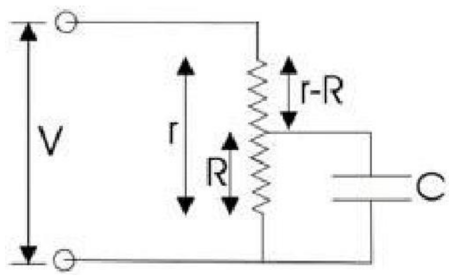
From the above equation the force can be calculated as

$$F = \frac{1}{2} V^2 \, dC/dx$$

Quadrant electrometer: This instrument is generally used in measuring the voltage ranging from 100V to 20 kilo volts.

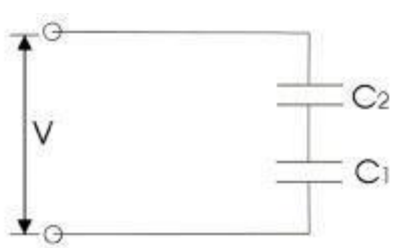
Again the deflecting torque obtained in the Quadrant electrometer is directly proportional to the square of the applied voltage; one advantage of this is that this instrument can used to measure both the ac and dc voltages. One advantage of using the electrostatic type instruments as voltmeters is that we can extend the range of voltage to be measured. Now there are two ways of extending the range of this instrument. We will discuss them one by one.

(a) By using resistance potential dividers: Given below is the circuit diagram of this type of configuration.



(b) The voltage which we want to measure is applied across the total resistance r and the electrostatic capacitor is connected across the portion of the total resistance which is marked as r. Now suppose the applied voltage is dc, then we should make one assumption that the capacitor which is connected is having infinite leakage resistance. In this case the multiplying factor is given by the ratio of electrical resistance r/R. The ac operation on this circuit can also be analyzed easily again in case of ac operation we multiplying factor equal to r/R.

(c) By using capacitor multiplier technique: We can increase the range of voltage to be measured by placing a series of capacitors as shown in the given circuit.



Let us derive the expression for multiplying factor for the circuit diagram 1. Let us mark the capacitance of the voltmeter be C_1 and series capacitor be C_2 as shown in the given circuit diagram. Now the series combination of these capacitor be equal to

$$C = \frac{C_1 C_2}{C_1 + C_2}$$

which is the total capacitance of the circuit. Now the impedance of the voltmeter is equal to $Z_1 = 1/j\omega C_1$ and thus total impedance will be equal to

$$Z = \frac{C_1 C_2}{J\omega C_1 C_2}$$

Now, the multiplying factor can be defined as the ratio of Z/Z_1 which is equal to $1 + C_2 / C_1$. Similarly the multiplying factor can also be calculated. Hence by this way we can increase the range of voltage to be measure.

Advantages of Electrostatic Type Instruments:

(a) The first and the most important advantage is that we can measure both ac and dc voltage and the reason are very obvious the deflecting torque is directly proportional to the square of the voltage.

(b) Power consumption is quite low in these types of instruments as the electric current drawn by these instruments is quite low.

(c) We can measure high value of voltage.

Disadvantages of Electrostatic Type Instruments

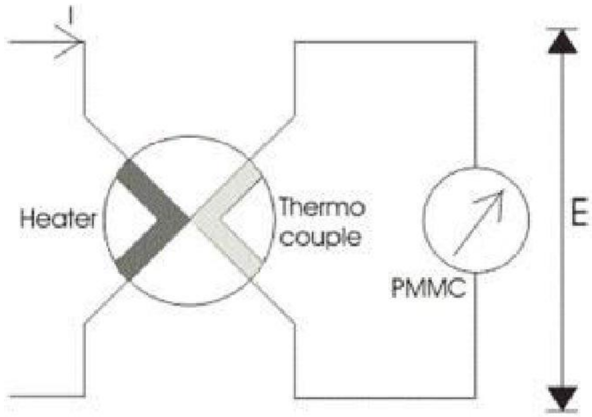
Instead of various advantages, electrostatic instruments posses few **disadvantages** and these are written below.

a) These are quite costly as compared to other instruments and also these have large size.
b) The scale is not uniform.
c) The various operating forces involved are small in magnitude.

Thermocouple type Instruments

Construction Principle of Operation:

Basically thermocouple consists of two different metals which are placed in contact with each other as shown in the diagram.



Thermocouple

First part is called the heater element because when the electric current will flow through this, a heat is produced and thus the temperature will increased at the junction. At this junction an emf is produced which is approximately proportional to the temperature difference of hot and cold junctions. The emf produced is a dc voltage which is directly proportional to root mean square value of electric current. A permanent magnet moving coil instrument is connected with the second part to read the electric current passing through the heater. One question must be arise in our mind that why we have used only a permanent magnet coil instrument? Answer to this question is very easy it is because PMMC instrument has greater accuracy and sensitivity towards the measurement of dc value. The **thermocouple type instruments** employ thermocouple in their construction. Thermocouple type instruments can be used for both ac and dc applications. Also thermocouple type of instruments has greater accuracy in measuring the electric current and voltages at very high frequency accurately.

Now we will look how the temperature difference is mathematically related to generated emf at the junction in thermocouple type of instruments. Let us consider temperature of the heater element be T_a and the temperature of cold metal be T_b. Now it is found that the generated emf at the junction is related to temperature difference as:

Now it is found that the generated emf at the junction is related to temperature difference

$$e = a(T_a - T_b) + (T_a - T_b)^2$$

Where, a and b are constant whose values completely depends upon the type of metal we are using. The above equation represents parabolic function.

The approximated value of a is from 40 to 50 micro volts or more per degree Celsius rise in temperature and value of constant b is very small and can be neglected if the air gap field of permanent magnet moving coil is uniform.

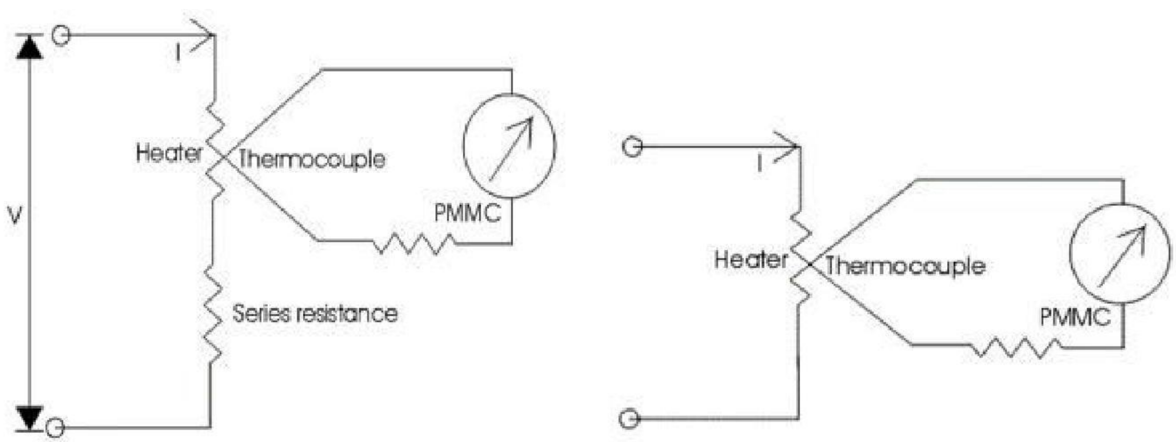
Thus, we can approximate the above temperature emf relation as $e = a\,(T_a - T_b)$, here we have assume b = 0. The electric current flowing through the heater coil produces heat as I^2R where I is the root mean square value of current, if we assume the temperature of cold junction is maintained at room temperature then the rise in the temperature of the hot junction will be equal to temperature rise at the junction.

Hence we can write (Ta-Tb) is directly proportional to I^2R.

Construction of Thermocouple Type Instrument:

Now let us look at the construction of Thermocouple type Instruments. Broadly speaking the thermocouple type of instruments consists of two major parts which are written below:

i. **Thermo electric elements:** The thermocouple type of instruments consists of thermo electric elements which can be of four types:
    i. Contact type: It has a separate heater which is shown in the diagram below.

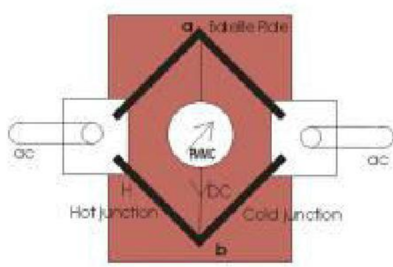


The action of thermocouple type instruments can be explained briefly as,

At the junction the electrical energy is being converted to thermal energy in the heater element. A portion of the heat is transferred to the hot junction while most of the heat energy is dissipated away.

The heat energy which is transferred to hot junction is again converted to electrical due to Seebeck effect. Only a portion of electrical energy is converted into mechanical energy which is used to produce a deflecting torque. The overall efficiency of the system is low thus the instrument consumes high power. So there is a requirement of highly accurate and sensitive dc instrument.

ii. Non contact type: In non contact type there is insulation between the heating element and the thermocouple i.e. there no direct contact between two. Due to this these instruments are not much sensitive as compared contact type.
iii. Vacuum thermo-elements: These types of instruments are mostly employed for the measurement of electric current at very high frequency of the order of 100 Mega hertz or more as these instruments retain their accuracy even at such high frequency.
iv. Bridge type: These bridges are manufactured on the ac ratings usually from 100 mili amperes to 1 ampere. Two thermocouple are connected to form a bridge which is shown in the figure given below:



There is no requirement of heating element, the electric current which directly passing through the thermocouple raises the temperature which is directly proportional to the I^2R losses. The bridge works on balanced condition at which there will be no electric current in the arm ab. The connected meter will show the potential difference between the junctions a and b.

Advantages of Thermocouple Type Instruments

i. The thermocouple type of instruments accurately indicates the root mean square value of electric current and voltages irrespective of the waveform. There is a wide varieties of range of thermocouple instruments are available in the market.
ii. Thermocouple type of instruments give very accurate reading even at high frequency, thus these types of instruments are completely free from frequency errors.
iii. The measurement of quantity under these instruments is not affected by stray magnetic fields.
iv. These instruments are known for their high sensitivity.

v. Usually for measuring the low value of electric current bridge type of arrangement is used i.e. ranging from 0.5 amperes to 20 amperes while for measuring the higher value of electric current heater element is required to retain accuracy.

Disadvantages of Thermocouple Type Instruments

Instead of many advantages these type of instruments posses one disadvantage,

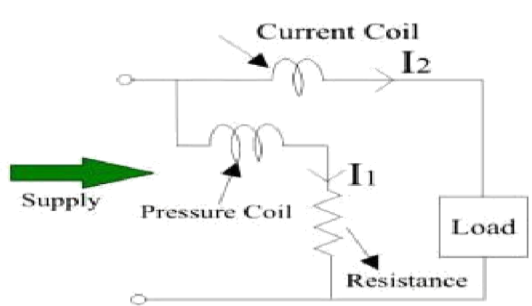
The over load capacity of thermocouple type of instrument is small, even fuse is not able to protect the heater wire because heater wire may burn out before the fuse blows out.

Construction and Working Principle of Electrodynamometer Type Wattmeter:

It consists of following parts. There are two types of coils present in the electrodynamometer.

a) Moving coil: Moving coil moves the pointer with the help of spring control. A limited amount of electric current flows through the moving coil so as to avoid heating. So in order to limit the electric current we have connect the high value resistor in series with the moving coil. The moving is air cored and is mounted on a pivoted spindle and can moves freely. In electrodynamometer type wattmeter, moving coil works as pressure coil. Hence moving coil is connected across the voltage and thus the electric current owing through this coil is always proportional to the voltage.
b) Fixed coil: The fixed coil is divided into two equal parts and these are connected in series with the load, therefore the load current will flow through these coils. Now the reason is very obvious of using two fixed coils instead of one, so that it can be constructed to carry considerable amount of electric current. These coils are called the electric current coils of electrodynamometer type wattmeter. Earlier these fixed coils are designed to carry the electric current of about 100 amperes but now the modern wattmeter are designed to carry electric current of about 20 amperes in order to save power.
c) Control system: Out of two controlling systems i.e.
    i. Gravity control
    ii. Spring control, only spring controlled systems are used in these types of wattmeter. Gravity controlled system cannot be employed because they will appreciable amount of errors.
d) Damping system: Air friction damping is used, as eddy current damping will distort the weak operating magnetic field and thus it may leads to error.
e) Scale: There is uniform scale is used in these types of instrument as moving coil moves linearly over a range of 40 degrees to 50 degrees on either sides.

Expressions for the deflecting torque: Let us consider the circuit diagram given



We know that instantaneous torque in Electrodynamics type instruments is directly proportional to product of instantaneous values of currents flowing through both the coils and the rate of change of flux linked with the circuit.

Let I_1 and I_2 be the instantaneous values of currents in pressure and electric current coils respectively. So the expression for the torque can be written as:

$$T = i_1 i_2 \frac{dM}{d\Theta}$$

Where, θ is the angle of deflection.
Now let the applied value of voltage across the pressure coil be

$$v = \sqrt{2} V \sin\omega t$$

Assuming the electrical resistance of the pressure coil to be very high hence we can neglect reactance with respect to its resistance. In this the impedance is equal to its electrical resistance therefore it is purely resistive.

The expression for instantaneous electric current through pressure coil can be written as $i_2 = v/R_p$ where R_p is the resistance pressure coil.

$$I_2 = \sqrt{2}\ \frac{V \sin wt}{R_p}$$

If there is phase difference between voltage and electric current, then expression for instantaneous electric current through electric current coil can be written as

$$I_1 = \sqrt{2} I \sin(wt - \emptyset)$$

As electric current through the pressure coil is very small compare to electric current through current coil, hence electric current through the current coil can be considered as equal to total load current.

Hence the instantaneous value of torque can be written as

$$T_i = \sqrt{2} \frac{V \sin wt}{Rp} \times \sqrt{2} I \sin(wt - \emptyset) \times \frac{dM}{d\Theta}$$

Average value of deflecting torque can be obtained by integrating the instantaneous torque from limit 0 to T, where, T is the time period of the cycle.

Controlling torque is given by T = Kθ where K is spring constant and θ is final steady state value of deflection.

Advantages of Electrodynamometer Type Wattmeter

Following are the advantages of electrodynamometer type wattmeter

a) Scale is uniform up to certain limit.
b) They can be used for both to measure ac as well dc quantities as scale is calibrated for both.

Errors in Electrodynamometer Type Wattmeter

Following are the errors in the electrodynamometer type wattmeter:

1. ***Pressure coil inductance:***

In an ideal dynamo-meter type watt meter the current in pressure coil in phase with the applied voltage. But in practically the pressure coil of watt meter has an inductance and current in it will lag behind the applied voltage. If there is no inductance the current in pressure coil will be in phase with the applied voltage. In the absence of inductance in pressure coil of wattmeter, it will read correctly in all power factors and frequency. The wattmeter will read high when the load power factor is lagging ,as in that case the effect of pressure coil inductance is to reduce the phase angle between load current and pressure coil current . Hence the wattmeter will read high. This is very serious error.

The wattmeter will read low when the load power factor is leading as in that case the effect of pressure coil inductance is to increase the phase angle between load current and pressure coil current. Hence the wattmeter will read low.

Compensation for inductance of pressure coil.

Inductance of pressure coil can be reduced by means of **capacitor** connected in parallel with a portion of **multiplier** (series resistance).

2. **Pressure coil capacitance.**

The pressure coil circuit may have capacitance in addition with inductance. This capacitance mainly due to the inter turn capacitance of the series resistance. The effect of capacitance is opposite to that due to inductance. Therefore the wattmeter will read high when the load power factor is leading.

The inductance in pressure coil circuit will always more than inductance, hence the error caused by capacitance will be nullified by that due to inductance.

3. Error due to mutual inductance.

Errors may occur due to the mutual inductance between the current and pressure coils of the watt meter. These errors are quite low at power frequencies. But they increased with increase in frequencies.

The effect of mutual inductance can be avoided by arranging the coil system in such a way that they have no mutual inductance. So we can eliminate the errors due to mutual inductance. The Dry dale Torsion head wattmeter is an example for such type.

4. **Eddy Current errors.**

Eddy currents are induced in the solid metal parts and within the thick conductors by the alternating magnetic field produced by the current coil. This eddy currents produce their own magnetic field and it will alter that produced by the main current in the current coil and thus error occurred.

This error can be minimized by avoiding solid metal parts as much as possible and by using stranded conductors for high current applications.

5. **Stray Magnetic field Errors.**

The electrodynamometer type wattmeter has a weak operating field and therefore it is affected by stray magnetic fields it will result in serious errors. Hence these instruments should be shielded against stray magnetic field.

6. Errors caused by vibration of moving system.

The torque on the moving system varies with frequency which is twice that of voltage. If the parts of the moving system have a natural frequency which is resonance with the frequency of torque pulsation, the moving system would vibrate with considerable amplitude. These vibrations will cause errors. This error can be reduced by design.

7. Temperature Error.

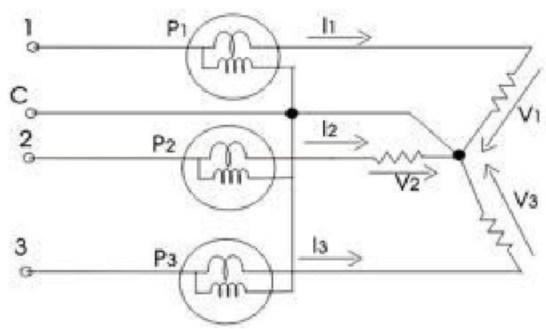
The change in room temperature may affect the indication of wattmeter. This is because of change in temperature will change in resistance of pressure coil and stiffness of springs which provide controlling torque. This effect are opposite in nature and cancel each other. The use of material of having negligible temperature coefficient of resistance will reduce change in resistance the pressure coils with change in temperature.

Measurement of Three Phase Power:

Various methods are used **measurement of three phase power** in three phase circuits on the basis of number of wattmeter used. We have three methods

i. Three wattmeter method.
ii. Two wattmeter method.
iii. Single 3-phase wattmeter method.

i. The circuit diagram is shown below



Here, it is applied to three phase four wire systems, electric current coil of all the three wattmeter marked as one, two and three are connected to respective phases marked as one, two and three.

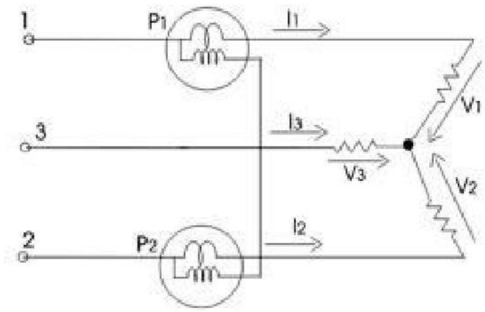
$$P = P_1 + P_2 + P_3 = V_1 I_1 + V_2 I_2 + V_3 I_3$$

Pressure coils of all the three wattmeter are connected to common point at neutral line. Clearly each wattmeter will give reading as product of phase electric current and line voltage which is phase power. The resultant sum of all the readings of wattmeter will give the total power of the circuit. Mathematically we can

$$P= P_1 + P_2 = I_1(V_1+V_2) + I_2(V_2\text{-}V3)$$

ii. Measurement of Three Phase Power by Two Wattmeter Method: There are two types of connections
   a. Star connection of loads
   b. Delta connection of loads

a. When the star connected load, the diagram is shown in below-

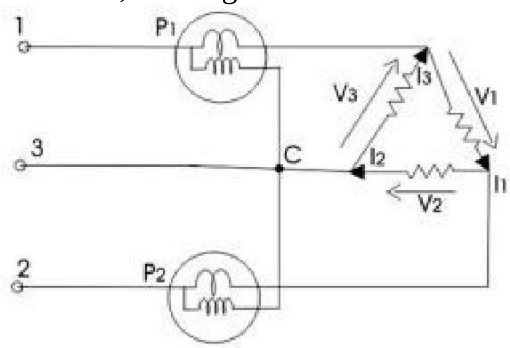


For star connected load clearly the reading of wattmeter one is product phase electric current and voltage difference (V_2-V_3). Similarly the reading of wattmeter two is the product of phase electric current and the voltage difference (V_2-V_3). Thus the total power of the circuit is sum of the reading of both the wattmeter. Mathematically we can write

$$P = P_1 + + P_2 = I_1(V_1 + V_2) + I_2(V_2 - V_3)$$

But, we have $I_1+I_2+I_3=0$, hence putting the value of $I_1+I_2=-I_3$. We get total power as $V_1I_1+V_2I_2+V_3I_3$.

b. When delta connected load, the diagram is shown in below



The reading of wattmeter one can be written as

$$P_1 = -V_3(I_1 - I_3)$$

And reading of wattmeter two is

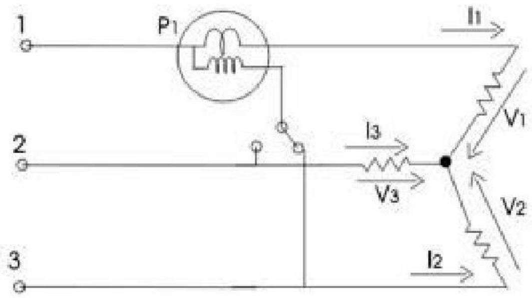
$$P_2 = -V_2(I_2 - I_1)$$

$$Total\ power\ is\ P = P_1 + P_2 = V_2I_2 + V_3I_3 - I_1(V_2 + V_3)$$

But $V_1+V_2+V_3=0$, hence expression for total power will reduce to V_1I1+V_2I_2+V_3I_3.

Measurement of Three Phase Power by One Wattmeter Method

Limitation of this method is that it cannot be applied on unbalanced load. So under this condition we have $I_1=I_2=I_3=I$ and $V_1=V_2=V_3=V$.
Diagram is shown below:



Two switches are given which are marked as 1-3 and 1-2, by closing the switch 1-3 we get reading of wattmeter as

$$P_1 = V_{13} I_1 \cos(30 - \phi) = \sqrt{3} \times VI \cos(30 - \phi)$$

Similarly the reading of wattmeter when switch 1-2 is closed is

$$P_2 = V_{12} I_1 \cos(30 + \phi) = \sqrt{3} \times VI \cos(30 + \phi)$$

$$Total\ power\ is\ P_1 + P_2 = 3VI \cos\phi$$

Energy Meter:

An instrument that is used to measure either quantity of electricity or energy, over a period of time is known as energy meter or watt-hour meter. In other words, energy is the total power delivered or consumed over an interval of time t may be expressed as:

If (v_t) is expressed in volts, (i_t) in amperes and t in seconds, the unit of energy is joule or watt second. The commercial unit of electrical energy is kilowatt hour (KWh). For measurement of energy in a.c. circuit, the meter used is based on "electro-magnetic induction" principle. They are known as induction type instruments. The measurement of energy is based on the induction principle is particularly suitable for industrial or domestic meters on the account of lightness and robustness of the rotating element. Moreover, because of smallness of the variations of voltage and frequency in supply voltage, the accuracy of the induction meter is unaffected by such variations. If the waveform of the supply is badly distorted, the accuracy, however, is affected. Basically, the induction energy meter may be derived from the induction watt-meter by substituting for the spring control and pointer an eddy current brake and a counting train, respectively. For the meter to read correctly, the speed of the moving system must be proportional to the power in the circuit in which the meter is connected.

Construction of induction type energy meter:
Induction type energy meter essentially consists of following components (a) Driving system (b) Moving system (c) Braking system and (d) Registering system.

Driving system: The construction of the electro magnet system is shown in Fig given below and it consists of two electromagnets, called "shunt" magnet and "series" magnet, of laminated construction.

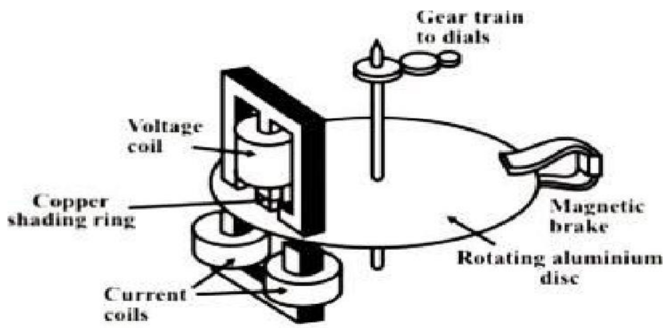


Figure: Watt Hour Meter

A coil having large number of turns of fine wire is wound on the middle limb of the shunt magnet. This coil is known as "pressure or voltage" coil and is connected across the supply mains. This voltage coil has many turns and is arranged to be as highly inductive as possible. In other words, the voltage coil produces a high ratio of inductance to resistance. This causes the current, and therefore the flux, to lag the supply voltage by nearly 90°. Adjustable copper shading rings are provided on the central limb of the shunt magnet to make the phase angle displacement between magnetic field set up by shunt magnet and supply voltage is approximately090. The copper shading bands are also called the power factor compensator or compensating loop. The series electromagnet is energized by a coil, known as "current" coil which is connected in series with the load so that it carry the load current. The flux produced by this magnet is proportional to, and in phase with the load current.

Moving system: The moving system essentially consists of a light rotating aluminum disk mounted on a vertical spindle or shaft. The shaft that supports the aluminum disk is connected by a gear arrangement to the clock mechanism on the front of the meter to provide information that consumed energy by the load. The time varying (sinusoidal) fluxes produced by shunt and series magnet induce eddy currents in the aluminum disc. The interaction between these two magnetic fields and eddy currents set up a driving torque in the disc. The number of rotations of the disk is therefore proportional to the energy consumed by the load in a certain time interval and is commonly measured in kilowatt-hours (Kwh).

Braking system: Damping of the disk is provided by a small permanent magnet, located diametrically opposite to the a.c magnets. The disk passes between the magnet gaps. The movement of rotating disc through the magnetic field crossing the air gap sets up eddy currents in the disc that reacts with the magnetic field and exerts a braking torque. By changing the position of the brake magnet or diverting some of the flux there form, the speed of the rotating disc can be controlled.

Registering or counting system: The registering or counting system essentially consists of gear train, driven either by worm or pinion gear on the disc shaft, which turns pointers that indicate on dials the number of times the disc has turned. The energy meter thus determines and adds together or integrates all the instantaneous power values so that total energy used over a period is thus known. Therefore, this type of meter is also called an "integrating" meter.

Basic operation:

Induction instruments operate in alternating-current circuits and they are useful only when the frequency and the supply voltage are approximately constant. The most commonly used technique is the shaded pole induction watt-hour meter, shown in figure given below.

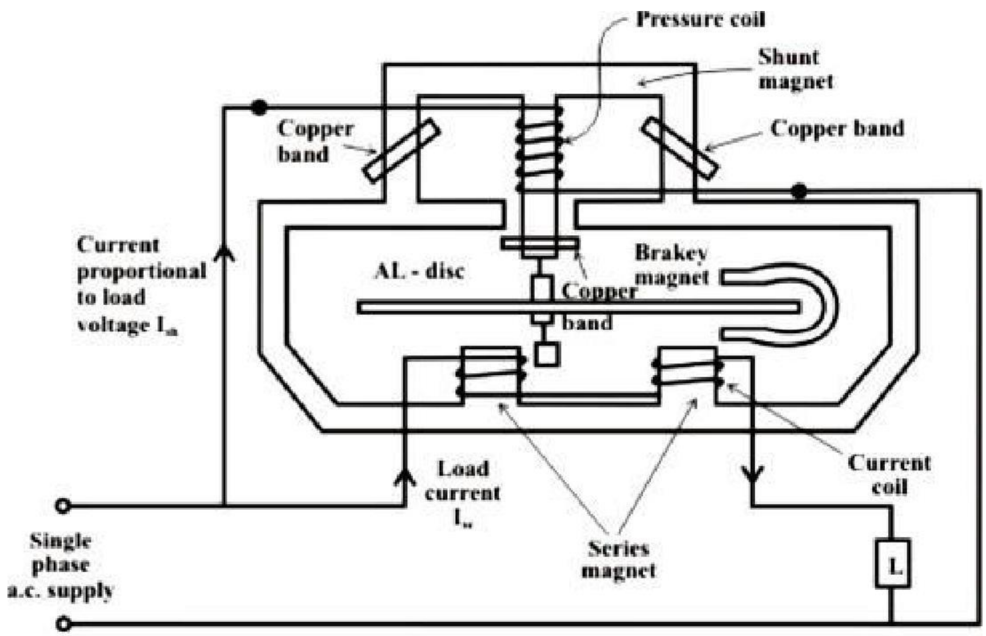
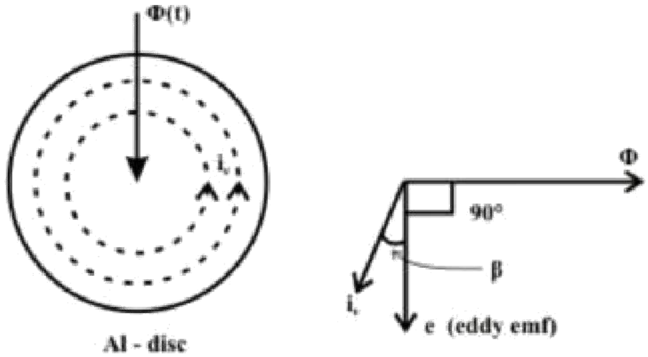


Figure: shaded pole induction watt-hour meter

The rotating element is an aluminum disc, and the torque is produced by the interaction of eddy currents generated in the disc with the imposed magnetic fields that are produced by the voltage and current coils of the energy meter.

Let us consider a sinusoidal flux (φ_t) is acting perpendicularly to the plane of the aluminum disc, the direction of eddy current by Lenz's law is indicated in figure. It is now quite important to investigate whether any torque will develop in aluminum disc by interaction of a sinusoid ally varying flux (φ_t) and the eddy currents i_e induced by it.



Where φ and I_e are expressed in r.m.s and β = 0 (because the reactance of the aluminum disc is nearly equal to zero). Therefore, the interaction of a sinusoid ally varying flux (φ_t) and its own eddy current i_e (induced) cannot produce torque any on the disc.

So in all induction instruments we have two fluxes produce by currents flowing in the windings of the instrument. These fluxes are alternating in nature and so they induce emfs in an aluminum disc or a drum provided for the purpose. These emfs in turn circulate eddy currents in the disc.

As in an energy meter instrument, we have two fluxes and two eddy currents and therefore, two torques are produced by

i. Flux (φ_1) interacting with the eddy currents (I_{e2}) generated by the second flux (φ_2), and
ii. Flux (φ_2) interacting with the eddy currents (I_{e1}) induced by the first flux (φ_1).

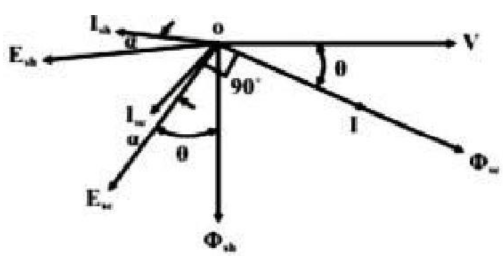


Figure: phasor of fluxes and eddy currents in energy meter