bar, and as such mercury is good from thermodynamic point of view. However, mercury i costly, toxic and poisonous. Water being easily available and harmless is universally adopted Mercury is used only as topping fluid in binary plants.

### 13.2.5. Thermal Refinement of Rankine Cycle

The power output and thermal efficiency of a steam plant can be improved by :

(*i*) *Superheating of steam* : Superheating increases the average temperature at which hea is supplied. Accordingly for the fixed condenser pressure (temperature) the cycl efficiency improves.

(*ii*) *Exhaust* (*condenser*) *pressure* : For fixed initial state, the enthalpy drop increases wit decrease in exhaust pressure. With increase in enthalpy drop more work is develope and so the efficiency improves.

The increase in efficiency could also be expected from the fact that as the condense pressure is decreased, the saturation temperature of steam also decreases and hence the heat rejection takes place at lower temperature.

(*iii*) *Boiler Pressure* : With increase in pressure of steam supplied to engine, the enthalpy drop increases. More work is then available at the shaft and so the efficiency improves.

The exhaust steam quality is, however, decreased by lowering the exhaust (condenser) pressure and by raising the boiler pressure. High moisture content causes undue pitting and erosion of steam turbine blades and that necessitates costly repairs. Moreover, efficiency of the plant decreases approximately 1.5% for every 1% of wetness of steam. Superheating, however, keeps the condition of exit steam within safe limits and also provides saving in cost of fuel. This is due to more heat content of the superheated steam and consequently less steam consumption for a given output.

### EXAMPLE 13.3

**A power generating plant uses steam as a working fluid and operates on Rankine cycle between a source temperature of 311.1°C (boiler pressure 100 bar) and a sink temperature of 32.9° (condenser pressure 0.05 bar). Determine the cycle efficiency and work ratio if all the processes are reversible. Also determine the rate of steam generation if the power output**

**of the plant is 1 MW. (*b*) How the cycle efficiency and work ratio would be affected if isentropic efficiency of turbine is 0.8 and the saturated liquid coming out of the condenser is compressed to the boiler pressure with an isentropic efficiency of 0.9.**

**Solution :** Refer Fig 13.8 for the *T-s* plot when both the expansion and compression processes are reversible.

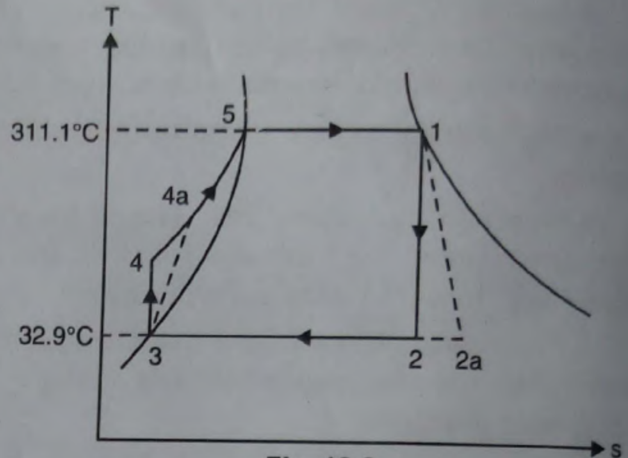

Fig. 13.8.

**At 311.1°C source temperature** (boiler pressure 100 bar)

$$h_1 = h_{g1} = 2727.7 \text{ kJ/kg}$$

$$s_1 = s_{g1} = 5.62 \text{ kJ/kg}$$

**At 32.9°c sink temperature** (condenser pressure 0.05 bar)

$$h_f = 137.8 \text{ kJ/kg},$$

$$h_{fg} = 2423.7 \text{ kJ/kg}$$

$$s_f = 0.476 \text{ kJ/kg K},$$

$$s_{fg} = 7.919 \text{ kJ/kg K}$$

Considering adiabatic expansion from state 1 to state 2

$$s_1 = s_2 \,; \quad s_1 = (s_f + x_2\, s_{fg})_{0.05 \text{ bar}}$$

$$5.62 = 0.476 + x_2 \times 7.919; \quad x_2 \approx 0.65$$

$$h_2 = (h_f + x_2\, h_{fg})_{0.05 \text{ bar}}$$

$$= 137.8 + 0.65 \times 2423.7 = 1713.2 \text{ kJ/kg}$$

$$h_3 = (h_f)_{0.05 \text{ bar}} = 137.8 \text{ kJ/kg}$$

$$\text{Pump work} = v_f\,(p_1 - p_2)$$

$$= 0.001005\,(100 - 0.05) \times 10^5$$

$$= 10045 \text{ J/kg} = 10.045 \text{ kJ/kg}$$

$$h_4 = h_3 + \text{pump work}$$

$$= 137.8 + 10.045 = 147.845 \text{ kJ/kg}$$

$$\text{Now, turbine work} = h_1 - h_2$$

$$= 2727.7 - 1713.2 = 1014.15 \text{ kJ/kg}$$

$$\text{Net work} = \text{turbine work} - \text{pump work}$$

$$= 1014.15 - 10.045 = 1004.1 \text{ kJ/kg}$$

$$\text{Heat input} = h_1 - h_4$$

$$= 2727.7 - 147.845 = 2579.86 \text{ kJ/kg}$$

$$\text{Work ratio} = \frac{\text{net work}}{\text{turbine work}} = \frac{1004.1}{1014.15} = 0.99$$

$$\text{Rankine efficiency} = \frac{\text{net work}}{\text{heat input}} = \frac{1004.1}{2579.86} = 0.389 \text{ or } 38.9\%$$

$$\text{Specific steam consumption} = \frac{3600}{1004.1} = 3.585 \text{ kg/kWh} = 3585 \text{ kg/MWh}$$

*i.e.*, for 1 MW of net power output, the required rate of steam production is 3585 kg/hr (*b*) From the isentropic efficiency relations

$$\eta_t = \frac{h_1 - h_{2a}}{h_1 - h_2} \text{ and } \eta_p = \frac{h_4 - h_3}{h_{4a} - h_3}$$

$$\text{Actual turbine work, } (h_1 - h_{2a}) = \eta_t \times (h_1 - h_2)$$

$$= 0.8 \times 1014.15 = 811.32 \text{ kJ/kg}$$

$$\text{Actual pump work, } (h_{4a} - h_3) = \frac{h_4 - h_3}{\eta_p} = \frac{10.045}{0.9} = 11.16 \text{ kJ/kg}$$

Then :

$$h_{4a} = h_3 + \text{acutal pump work}$$

$$= 137.8 + 11.16 = 148.96 \text{ kJ/kg}$$

$$\text{Actual net work} = 811.32 - 11.16 = 800.16 \text{ kJ/kg}$$

$$\text{Actual heat input} = h_1 - h_{4a}$$

$$= 2727.7 - 148.96 = 2578.74 \text{ kJ/kg}$$

∴ For the actual Rankine cycle

$$\eta = \frac{800.16}{2578.74} = 0.31 \text{ or } 31\%$$

$$r_w = \frac{800.16}{811.32} = 0.986$$

$$ssc = \frac{3600}{800.16} = 4.499 \text{ kg/kWh} = 4499 \text{ kg/MWh}$$

**Comments :** The effect of turbine and compressor efficiencies is to decrease the work ratio and thermal efficiency, and to increase the specific steam consumption.

## EXAMPLE 13.4

**A steam turbine working on Rankine cycle is supplied with dry saturated steam at 25 bar and the exhaust takes place at 0.2 bar. For a steam flow rate of 10 kg/s, determine : (*a*) quality steam at end of expansion, (*b*) turbine shaft work, (*c*) power required to drive the pump, (*d*) work ratio, (*e*) Rankine efficiency, and (*f*) heat flow in the condenser.**

**Solution :** Refer Fig. 13.9 for the *T-s* plot

From steam tables : At 25 bar;

$$h_1 = h_{g1} = 2800.9 \text{ kJ/kg}$$

$$s_1 = s_{g1} = 6.253 \text{ kJ/kg K}$$

At 0.2 bar

$$h_f = 251.5 \text{ kJ/kg;}$$

$$h_{fg} = 2358.4 \text{ kJ/kg}$$

$$s_f = 0.8321 \text{ kJ/kg K;}$$

$$s_{fg} = 7.077 \text{ kJ/kg K;}$$

$$v_f = 0.001017 \text{ m}^3\text{/kg}$$

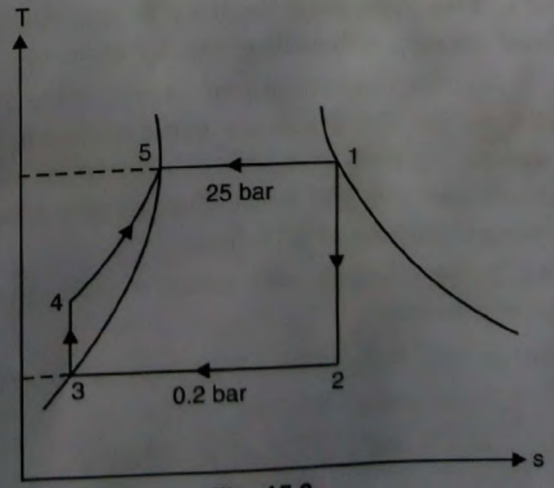

Fig. 15.9.

(*a*) Considering reversible adiabatic expansion from state 1 to state 2.

$$s_1 = s_2\,; \quad s_1 = (s_f + x_2\, s_{fg})_{0.2 \text{ bar}}$$

$$6.253 = 0.8321 + x_2 \times 7.077; \quad x_2 = 0.776$$

The quality of steam at the end of expansion is 77.6%

(*b*)
$$h_2 = (h_f + x_2\, h_{fg})_{0.2\ \text{bar}}$$
$$= 251.5 + 0.776 \times 2358.4 = 2061.62\ \text{kJ/kg}$$
$$\text{Turbine work} = (h_1 - h_2) = 2800.9 - 2061.62 = 739.28\ \text{kJ/kg}$$

∴ Turbine power = 10 kg/s × 739.28 kJ/kg ≈ **7393 kW**

(*c*)
$$\text{Pump work} = v_f\,(p_1 - p_2)$$
$$= 0.001017\ (25 - 0.2) \times 10^5$$
$$= 2522\ \text{J/kg} = 2.522\ \text{kJ/kg}$$

∴ Power required to drive the pump

$$= 10\ \text{kg/s} \times 2.552\ \text{kJ/kg} = \mathbf{25.52\ kW}$$

(*d*)
$$\text{Net work} = \text{turbine work} - \text{pump work}$$
$$= 739.28 - 2.522 = 736.758\ \text{kJ/kg}$$
$$\text{Work ratio} = \frac{\text{net work}}{\text{turbine work}} = \frac{736.758}{739.28} = \mathbf{0.996}$$

(*e*)
$$h_4 = h_3 + \text{pump work} = (h_f)_{0.2\ \text{bar}} + \text{pump work}$$
$$= 251.5 + 2.522 = 254.02\ \text{kJ/kg}$$
$$\text{Heat input} = h_1 - h_4 = 2800.9 - 254.02 = 2546.88\ \text{kJ/kg}$$
$$\text{Rankine efficiency} = \frac{\text{net work}}{\text{heat input}} = \frac{736.758}{2546.88} = 0.289 \text{ or } 28.9\%$$

(*f*)
$$\text{Condenser heat flow} = (h_2 - h_3)$$
$$= (2061.62 - 251.5) = 1810.12\ \text{kJ/kg}$$

With steam flow rate of 10 kg/s,

$$\text{Condenser heat flow} = 10 \times 1810.12 = 18100.12\ \text{kJ/s} \approx \mathbf{18100\ kW}$$

✓

### EXAMPLE 13.5

**In a thermal power plant which uses the Rankine cycle, superheated steam is produced at 1.5 MPa and 300°C and fed to the turbine where it expands to a condenser pressure of 80 kPa. The saturated liquid coming out of the condenser is pumped back to the boiler by a feed pump. Assuming ideal processes, determine :** ( assuming neglecting the pump work)

**(*a*) the condition of steam after isentropic expansion, (*b*) Rankine cycle efficiency,] (*c*) the mean effective pressure, (*d*) the ideal steam consumption per kW hour, and (*e*) the actual steam consumption per kW hour.**

**Take relative efficiency = 60% and neglect pump work.**

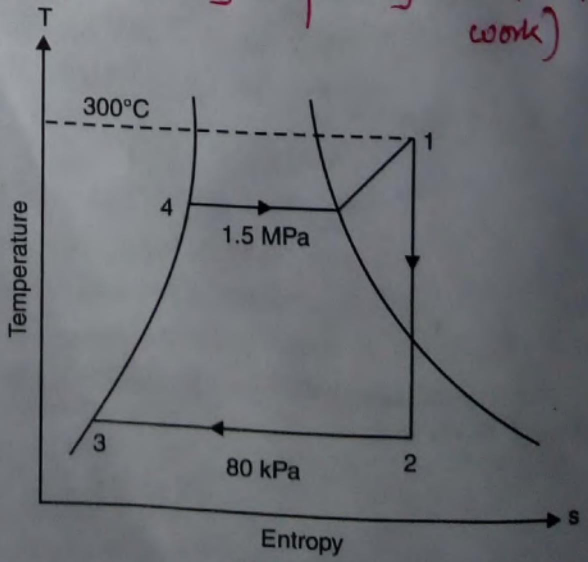

**Fig. 13.10.**

**Solution :** Refer Fig. 13.10 for the *T*-*s* plot

From steam tables :

**At 1.5 MPa (15 bar) and 300°C**

$$h_1 = 3037.6\ \text{kJ/kg and}$$
$$s_1 = 6.918\ \text{kJ/kg K}$$

**At 80 kPa (0.8 bar)**

$$h_f = 391.7\ \text{kJ/kg}$$

$$h_{fg} = 2274 \text{ kJ/kg}$$
$$s_f = 1.233 \text{ kJ/kg K}$$
$$s_{fg} = 6.202 \text{ kJ/kg K}$$
$$v_g = 2.087 \text{ m}^3\text{/kg}$$

(*a*) Considering adiabatic expansion from state 1 to state 2

$$s_1 = s_2 \,; \quad s_1 = (s_f + x_2\, s_{fg})_{0.8 \text{ bar}}$$
$$6.918 = 1.233 + x_2 \times 6.202 \,; \quad x_2 = 0.916$$

(*b*)
$$h_2 = (h_f + x_2\, h_{fg})_{0.8 \text{ bar}}$$
$$= 391.7 + 0.916 \times 2274 = 2474.7 \text{ kJ/kg}$$
$$h_3 = (h_f)_{0.8 \text{ bar}} = 391.7 \text{ kJ/kg}$$

$$\text{Rankine efficiency} = \frac{h_1 - h_2}{h_1 - h_3} \qquad \text{(neglecting pump work)}$$
$$= \frac{3037.6 - 2474.7}{3037.6 - 391.7} = 0.2127 \text{ or } 21.27\%$$

(*c*)
$$V_2 = x_2 \times (v_g)_{0.8 \text{ bar}} = 0.916 \times 2.087 = 1.917 \text{ m}^3\text{/kg}$$

$$\text{Mean effective pressure} = \frac{\text{work done}}{\text{stroke volume}} = \frac{h_1 - h_2}{V_2}$$
$$= \frac{3037.6 - 2474.7}{1.917} = \mathbf{293.63 \text{ kN/m}^2}$$

(*d*) Ideal specific steam consumption

$$= \frac{3600}{\text{work done}} = \frac{3600}{h_1 - h_2}$$
$$= \frac{3600}{3037.6 - 2474.7} = 6.39 \text{ kg/kWh}$$

(*e*) $$\text{Relative efficiency} = \frac{\text{Indicated thermal efficiency}}{\text{Rankine cycle efficiency}}$$

∴ Indicated thermal efficiency = 0.6 × 0.2127 = 0.1276

$$\text{Indicated work} = \text{indicated thermal efficiency} \times \text{heat supplied}$$
$$= \text{indicated thermal efficiency} \times (h_1 - h_3)$$
$$= 0.1276 \times (3037.6 - 391.7) = 337.62 \text{ kJ/kg}$$

$$\therefore \text{ Actual specific steam consumption} = \frac{3600}{337.62} = \mathbf{10.66 \text{ kg/kWh}}$$

### EXAMPLE 13.6

**In a thermal power plant operating on an ideal Rankine cycle, steam at 15 bar and 250°C enters a turbine which generates 40 kW indicated power. If the steam consumption is 300 kg/hr and condenser is maintained at 0.15 bar, determine the final condition of steam, Rankine efficiency and relative efficiency. Neglect pump work. Also determine the fuel to be supplied per hour if its calorific value is 41850 kJ/kg.**

**Solution :** Refer Fig. 13.11 for the *T-s* plot

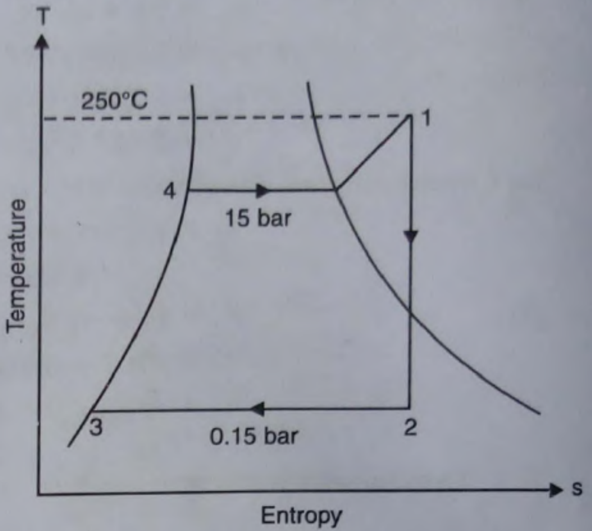

**Fig. 13.11.**

From steam tables :

**At 15 bar**, (steam is superheated)

$h_1 = 2923.5$ kJ/kg

$s_1 = 6.709$ kJ/kg K

**At 0.15 bar**

$h_f = 226$ kJ/kg

$h_{fg} = 2372.2$ kJ/kg

$s_f = 0.755$ kJ/kg K

$s_{fg} = 7.254$ kJ/kg K

(*a*) Considering adiabatic expansion from state 1 to state 2

$$s_1 = s_2 \,; \quad s_1 = (s_f + x_2\, s_{fg})_{0.15 \text{ bar}}$$

$$6.709 = 0.755 + x_2 \times 7.254$$

$$x_2 = \mathbf{0.821}$$

The dryness fraction of steam at exit from the turbine is 0.821.

(*b*)
$$h_2 = (h_f + x_2\, h_{fg})_{0.15 \text{ bar}}$$
$$= 226 + 0.821 \times 2372.2 = 2174.4 \text{ kJ/kg}$$
$$h_3 = (h_f)_{0.15 \text{ bar}} = 226 \text{ kJ/kg}$$

$$\text{Rankine efficiency} = \frac{h_1 - h_2}{h_1 - h_3}$$

$$= \frac{2923.5 - 2174.4}{2923.5 - 226} = \mathbf{0.278 \text{ or } 27.8\%}$$

(*c*)
$$\text{Indicated power} = 40 \text{ kW} = 40 \text{ kJ/s}$$
$$\text{Heat input} = (h_1 - h_3) = (2923.5 - 226) \text{ kJ/kg}$$
$$= \frac{300}{3600}(2923.5 - 226) = 224.79 \text{ kJ/s}$$

$$\text{Indicated thermal efficiency} = \frac{40}{224.79} = 0.178$$

$$\therefore \quad \text{Relative efficiency} = \frac{0.178}{0.278} = \mathbf{0.64 \text{ or } 64\%}$$

## EXAMPLE 13.7

**In a thermal power plant operating on an ideal Rankine cycle, superheated steam produced at 5 MPa and 500°C is fed to a turbine where it expands to the condenser pressure of 10 kPa. If the net power output of the plant is to be 20 MW, determine :**

**(*a*) heat added in the boiler per kg of water, (*b*) thermal efficiency of the cycle, (*c*) mass flow rate of steam in kg/s, (*d*) mass flow rate of cooling water in the condenser if the cooling water enters the condenser at 25°C and leaves at 35°C.**

**Solution :** Refer Fig. 13.12 for the *T-s* plot.

From steam tables :

**At 5 MPa (50 bar) and 500°C** the steam is superheated and

$$h_1 = 3433.8 \text{ kJ/kg}$$
$$s_1 = 6.976 \text{ kJ/kg K}$$

**At 10 kPa (0.1 bar)**

$$h_f = 191.8 \text{ kJ/kg}$$
$$h_{fg} = 2392.8 \text{ kJ/kg}$$
$$s_f = 0.649 \text{ kJ/kg K}$$
$$s_{fg} = 7.501 \text{ kJ/kg K}$$
$$v_f = 0.00101 \text{ m}^3\text{/kg}$$

Considering adiabatic expansion from state 1 to state 2

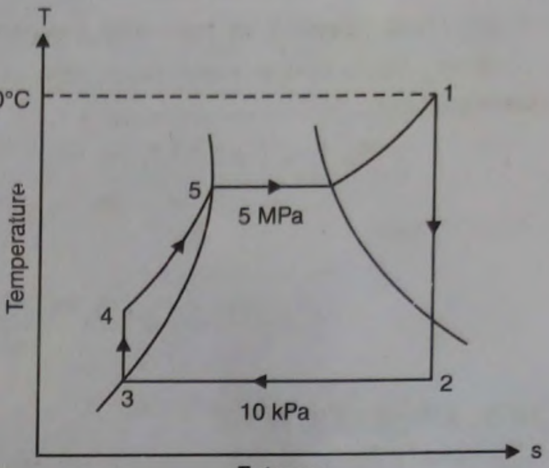

**Fig. 13.12.**

$$s_1 = s_2 \ ; \quad s_1 = (s_f + x_2\, s_{fg}) \text{ at 10 kPa}$$
$$6.976 = 0.649 + x_2 \times 7.501 \ ; \quad x_2 = 0.843$$
$$h_2 = (h_f + x_2\, h_{fg}) \text{ at 10 kPa}$$
$$= 191.8 + 0.843 \times 2392.8 = 2208.93 \text{ kJ/kg}$$
$$h_3 = (h_f) \text{ at 10 kPa} = 191.8 \text{ kJ/kg}$$
$$\text{Pump work} = (h_4 - h_3) = v_f\,(p_1 - p_2)$$
$$= 0.00101\,(50 - 0.1) \times 10^5$$
$$= 5040 \text{ J/kg} = 5.04 \text{ kJ/kg}$$
$$h_4 = h_3 + \text{pump work}$$
$$= 191.8 + 5.04 = 196.84 \text{ kJ/kg}$$

(*a*) Heat added in the boiler,

$$h_1 - h_4 = 3433.8 - 196.84 = \mathbf{3236.96 \text{ kJ/kg}}$$

(*b*)

$$\text{Turbine work} = (h_1 - h_2)$$
$$= 3433.8 - 2208.93 = 1224.87 \text{ kJ/kg}$$
$$\text{Net work} = \text{turbine work} - \text{pump work}$$
$$= 1224.87 - 5.04 = 1219.83 \text{ kJ/kg}$$
$$\text{Rankine efficiency} = \frac{\text{net work}}{\text{heat input}} = \frac{1219.83}{3236.96} = \mathbf{0.3768 \text{ or } 37.68\%}$$

(*c*) Specific steam consumption,

$$ssc = \frac{3600}{\text{net work out put}} = \frac{3600}{1219.83} = 2.951 \text{ kg/kWh}$$

*i.e.*, for 1 kW of net power output, the required rate of steam production is 2.951 kg/hr

Therefore, for 20 MW ($20 \times 10^3$ kW) of net power output, mass flow rate of steam

$$= 2.951 \times (20 \times 10^3) \text{ kg/hr}$$

$$m_s = \frac{2.951 \times (20 \times 10^3)}{3600} = 16.39 \text{ kg/s}$$

**process. If there is a transfer of 45 kJ of heat from the gas during the process, evaluate change in the internal energy of the gas.**

**Solution :** $Q_{1-2} = -45$ kJ (–ve because heat is lost by the gas)

$$W_{1-2} = \int p\,dV = p\,(V_2 - V_1) = 0.1 \times 10^6\,(0.4 - 0.2)$$

$$= 20 \times 10^3 \text{ Nm (or J)} = 20 \text{ kJ}$$

From non-flow energy equation, $Q_{1-2} = W_{1-2} + dU$

$$-45 = -20 + dU; \quad dU = \mathbf{-25\ kJ}$$

The internal energy of the gas decreases by 25 kJ during the compression process.

### EXAMPLE 5.13

**A closed system having a mass of 50 kg has an initial velocity of 10 m/s. Subsequently its velocity increases to 30 m/s and its elevation also rises by 40 m. During this process, the system receives 30000 J of heat and 4500 J of work. If the system delivers 0.002 kWh of electrical energy, make calculations for a change in the internal energy of the system.**

**Solution :** $\Delta PE = mg(h_2 - h_1) = 50 \times 9.81 \times 40 = 19620$ J

$$\Delta KE = \frac{1}{2}m\left(V_2^2 - V_1^2\right) = \frac{1}{2} \times 50 \times \left(30^2 - 10^2\right) = 20000 \text{ J}$$

$Q_{1-2} = +30000$ J (+ve sign because heat is received by the system)

$W_{1-2}$ = work delivered – work received

$= 0.002 \times 1000 \times 3600 - 4500 = 2700$ J

From non-flow energy equation: $Q_{1-2} = W_{1-2} + dE$

The energy $E$ includes the internal energy $U$, the kinetic energy and the potential energy.

$$\therefore \quad Q_{1-2} = W_{1-2} + dU + \Delta KE + \Delta PE$$

$$30000 = 2700 + dU + 20000 + 19620$$

Then, change in internal energy,

$$dU = 30000 - 2700 - 20000 - 19620 = \mathbf{-12320\ J}$$

The internal energy of the system decreases by 12320 J during the process undergone by the system.

### EXAMPLE 5.14

**A workpiece of 30 cm diameter is being turned on a lathe and the tangential force on the cutting tool is estimated to be 100 N. When the turning operation is executed for 10 minutes, the increase in the internal energy of the workpiece works out 125 kJ. Make calculations for the heat transfer from the workpiece if the workpiece turns at 180 rev/min.**

**Solution :** Revolutions made by the workpiece in 10 minutes,

$$N = 180 \times 10 = 1800$$

$$\text{Torque, } T = \text{Force} \times \text{radius} = 100 \times \frac{0.3}{2} = 15 \text{ Nm}$$

Work done on the work piece,

$$\delta W = 2\pi NT = 2\pi \times 1800 \times 15$$

$$= 169560 \text{ Nm (or J)} = 169.56 \text{ kJ}$$

This work is negative as it has been on the work piece (system)

Increase in internal energy, $dU = 150$ kJ

From first law of thermodynamics (non-flow energy equation)

$$\delta Q = \delta W + dU; \quad \delta Q = -169.56 + 125 = \mathbf{-44.56\ kJ}$$

Since this is negative, heat flows from the system (work piece).

### EXAMPLE 5.15

**The work output and heat input involved by a system in process A are 20 kJ and 15 kJ respectively. Another process B between the same end conditions involves a heat input of 10 kJ. Determine the change in internal energy involved and also the work done in process B. Show that if a cycle is formed using processes A and B, the given data is consistent with the first law.**

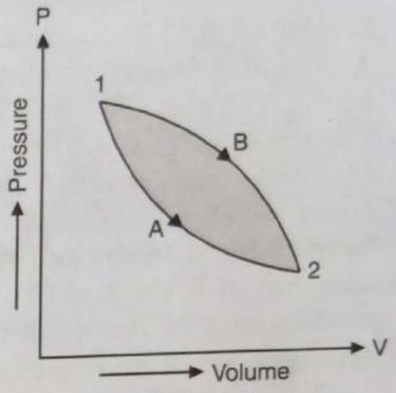

Fig. 5.5.

**Solution:** For process 1 – A – 2

$$\delta W = 20 \text{ kJ and } \delta Q = 15 \text{ kJ}$$

From non-flow energy equation

$$dU = U_2 - U_1 = \delta Q - \delta W = 15 - 20 = -5 \text{ kJ}$$

The negative sign implies that the internal energy of the system has decreased.

For process 1 – B – 2 :

$$\delta Q = 10 \text{ kJ};$$

$$dU = U_2 - U_1 = -5 \text{ kJ as calculated above}$$

$$\therefore \quad \delta W = \delta Q - dU = 10 - (-5) = 15 \text{ kJ}$$

When the cycle is formed by the processes *A* and *B*

$$\oint \delta Q = Q_{1-A-2} + Q_{2-B-1} = 15 - 10 = 5 \text{ kJ}$$

$$\oint \delta W = W_{1-A-2} + W_{2-B-1} = 20 + (-15) = 5 \text{ kJ}$$

It is to be noted that : $Q_{2-B-1} = -Q_{1-B-2}$, and

$$W_{2-B-1} = -W_{1-B-2}$$

Since $\oint \delta Q = \oint \delta W$, the data confirms to the first law of thermodynamics.

### EXAMPLE 5.16

**A mixture of powdered coal and air at 10 bar pressure and 300 K temperature is contained in an insulated rigid vessel. When the mixture is ignited, heat energy is liberated which causes a rise in both pressure and temperature; the temperature attains a value of 550 K. Evaluate the increase in energy of the system comprising the vessel and the contents.**

**(*b*) Upon removal of insulation, a heat transfer of 50 kJ of heat causes the temperature to drop to the initial value of 300 K. What will be the decrease in energy of the system during the process?**

**(*c*) If the initial energy of the system is 30 kJ, write down the energy values after the process (*a*) and process (*b*).**

**Solution:** The vessel is rigid; there occurs no change in the volume of the system and therefore, $W_{1-2} = 0$. Further, the system is insulated and accordingly $Q_{1-2} = 0$. Then from non-flow energy equation,

$$Q_{1-2} = W_{1-2} + dU; \quad 0 = 0 + dU \quad \therefore \quad dU = 0$$

Even though there has been rise in both pressure and temperature of the system, there will be no change in energy. All that happens is that the chemical energy of the system gets changed to internal energy.

$$W_{1-2} = 0; \quad Q_{1-2} = -50 \text{ kJ}$$

(*b*) $$Q_{1-2} = W_{1-2} + dE; \quad -50 = 0 + dE \quad \text{or} \quad dE = -50 \text{ kJ}$$

∴ Decrease in energy = 50 kJ

(*c*) When insulation is provided; $dE = 0$. That is

$$\text{final energy} = \text{initial energy} = 30 \text{ kJ}$$

When insulation is removed; $dE = -50$ kJ. That is

$$E_2 - E_1 = -50; \; E_2 - 30 = -50$$

$$\therefore \; E_2 = -20 \text{ kJ}$$

## EXAMPLE 5.17

**A combustible mixture is enclosed in a 10 cm diameter vertical cylinder fitted with a frictionless piston. The piston exerts a constant pressure of 300 kPa upon the mixture and can move freely. The upper surface of the piston is exposed to the atmosphere. When the mixture is ignited, the combustion products make the piston move outwards in quasi-static manner and the heat flows to the surroundings. After completion of the reaction and with the contents having been reduced to the initial temperature of 300 K, it is found that the piston has moved a distance of 8 cm and the magnitude of heat transfer to the surroundings is 4 kJ. Make calculations for the work done and change in energy of the system.**

**Solution:** Work done under constant pressure conditions,

$$W_{1-2} = \int p \, dV = 300 \times 10^3 \times \left[\frac{\pi}{4}(0.1)^2 \times 0.08\right]$$

$$= 188.4 \text{ Nm (or J)} = 0.1884 \text{ kJ}$$

Since this is positive, work is done by the system.

From non-flow energy equation,

$$Q_{1-2} = W_{1-2} + dU; \quad -4 = 0.1884 + dU$$

Heat interaction has been taken negative as it flows out of the system.

$$dU = -4 - 0.1884 = -4.1884 \text{ kJ}$$

∴ Decrease in energy of the system = **4.1884 kJ**

## EXAMPLE 5.18

**The relationship between pressure and volume in a non-flow process is prescribed by the expression:**

$$p = \left(\frac{3}{V} + 2\right)$$ **where presssure $p$ is in bar and volume $V$ is in $m^3$.**

**During the process 1600 kJ of heat is added to the gas and the volume changes form 1.2 $m^3$ to $4m^3$. Determine the change in internal energy.**

**Solution:** Work done $$W_{12} = \int_1^2 p \, dV = \int_1^2 \left(\frac{3}{V} + 2\right) \times 10^5 \times dV$$

$$= 10^5 \left|3 \log_e v + 2V\right|_{1.2}^{4} = 10^5 \left[2 \log_e \frac{4}{2} + 2(4 - 1.2)\right]$$

$$= 10^5 \, [3 \times 1.203 + 2 \times 2.8]$$

$$= 9.2116 \times 10^5 \text{ J} = \mathbf{921.16 \text{ kJ}}$$

From the non-flow energy equation, $\delta Q = \delta W + dU$, we have change in internal energy,

$$dU = \delta Q - \delta W = 1600 - 921.16 = \mathbf{678.84} \text{ kJ}$$

## EXAMPLE 5.19

**A cylinder fitted with a movable piston contains 0.04 m$^3$ of air at 10 bar pressure and 400 K temperature. The air expands according to the law $p = \left[\frac{A}{V^2} - \frac{B}{V}\right]$ to a final pressure of 1 bar and 0.2 m$^3$.**

**Determine work done, change in internal energy and heat absorbed or rejected during the expansion process.**

**Take $c_v$ = 0.0718 kJ/kgK**

Solution: Substituting the data corresponding to initial and final conditions in the given expression, we may write

$$10 = \frac{A}{0.04^2} - \frac{B}{0.04} \text{ and } 1 = \frac{A}{0.2^2} - \frac{B}{0.2}$$

or

$$10 = 625\,A - 25\,B \text{ and } 1 = 25\,A - 5\,B$$

Solving for $A$ and $B$, we get $A = 0.01$ and $B = -\,0.15$

$$\text{work done } W_{1\text{-}2} = \int_1^2 p\,dV = \int_1^2 \left(\frac{A}{V^2} - \frac{B}{V}\right) dV$$

$$= A\left|-\frac{1}{V}\right|_1^2 - B\left|\log_e V\right|_1^2 = A\left[\frac{1}{V_1} - \frac{1}{V_2}\right] - B\log_e\frac{V_2}{V_1}$$

Substituting the values of $A$, $B$, $V_1$ and $V_2$, we obtain

$$W_{1\text{-}2} = \left[0.01\left[\left(\frac{1}{0.04} - \frac{1}{0.2}\right)\right] - (-0.15)\,\text{loge}\,\frac{0.2}{0.04}\right] \times 10^5$$

$$= (0.2 + 0.2414) \times 10^5$$

$$= 0.4414 \times 10^5 \text{ Nm (kJ)} = \mathbf{44.14} \text{ kJ}$$

(*b*) From ideal gas equation $pv = mRT$,

$$m = \frac{p_1V_1}{RT_1} = \frac{10 \times 10^5 \times 0.04}{287 \times 400} = 0.3484 \text{ kg}$$

$$T_2 = \frac{p_2V_2}{mR} = \frac{1 \times 10^5 \times 0.2}{0.3484 \times 287} = 200 \text{ K}$$

change in internal energy

$$dU = mc_v\,dT$$

$$= 0.3484 \times 0.718 \times (200 - 400) = \mathbf{-\,50 \text{ kJ}}$$

(*c*) From non-flow energy equation, $\delta Q = \delta W + dU$,

$$\text{Heat interaction } Q_{1\text{-}2} = 44.14 - 50 = \mathbf{-\,5.86} \text{ kJ}$$

The negative sign indicates loss or rejection of heat.

**EXAMPLE 5.20**

**A closed system undergoes a thermodynamic cycle consisting of four separate and distinct processes. The heat and work transferred in each process are as tabulated below :**

| Process | Heat transfer in kJ/min | Work transfer in kJ/min |
|---|---|---|
| 1–2 | 0 | – 10,500 |
| 2–3 | 30,000 | 0 |
| 3–4 | – 3,000 | 30,000 |
| 4–1 | – 9,000 | – 1,500 |

**Show that the data is consistent with the first law of thermodynamics and proceed to evaluate the net work output in kW, thermal efficiency of the cycle and the change in internal energy for each process.**

**Solution :**

$$\oint \delta Q = 0 + 30000 - 3000 - 9000 = 18000 \text{ kJ/min}$$

$$\oint \delta W = -10500 + 0 + 30000 - 1500 = 18000 \text{ kJ/min}$$

Since $\oint \delta Q = \oint \delta W$, the data is in accordance with the first law.

(*b*) Network output = 18000 kJ/min = $\frac{18000}{60}$ = 300 kJ/s = 300 kW

(*c*) The ratio of net work done by system to the heat supplied is known as thermal efficiency. Therefore,

$$\text{Thermal efficiency} = \frac{18000}{30000} = 0.6 \text{ or } 60\%.$$

(*d*) Changes in internal energy for each of the process can be evaluated by applying the non-flow energy equation,

$$\delta Q = \delta W + dU$$

*Process* 1–2 : $dU = \delta Q - \delta W = 0 - (-10500) = 10500$ kJ/min

*Process* 2–3 : $dU = 30000 - 0 = 30000$ kJ/min

*Process* 3–4 : $dU = -3000 - 30000 = -33000$ kJ/min

*Process* 4–1 : $dU = -9000 - (-1500) = -7500$ kJ/min

For the complete cycle $\oint dU = 10500 + 30000 - 33000 - 7500 = 0$

We see that for the complete cycle, the total change in internal energy is zero. This has to be so because in a cyclic process the system returns to the initial state and internal energy depends only on the state point and is independent of path.

**EXAMPLE 5.21**

**A piston and a cylinder machine contains a fluid system, which passes through a complete cycle of four processes. During a cycle, the sum of all heat transfer is – 150 kJ and the system completes 100 cycles per minute. Complete the following table, and calculate the net rate of work output**

| Process | Q (kJ/min) | W(kJ/min) | dU(kJ/min) |
|---|---|---|---|
| 1 – 2 | 0 | 2150 | .... |
| 2 – 3 | 20000 | 0 | .... |
| 3 – 4 | – 2000 | .... | – 36,000 |
| 4 – 1 | .... | .... | .... |

**Solution :**

*Process 1 – 2 :* $\delta Q = \delta W + dU$ ; $0 = 2\,150 + dU$

$\therefore \quad dU = -2\,150$ kJ/min

*Process 2 – 3 :* $\delta Q = \delta W + dU$ ; $20\,000 = 0 + dU$

$\therefore \quad dU = 20\,000$ kJ/min

*Process 3 – 4 :* $\delta Q = \delta W + dU$ ; $-2\,000 = \delta W + (-36\,000)$

$\therefore \quad \delta W = 34\,000$ kJ/min

*Process 4 – 1 :* $\oint \delta Q = -150$ kJ per cycle

The system completes 100 cycles per minute and therefore

$$Q_{12} + Q_{23} + Q_{34} + Q_{41} = -150 \times 100 = -15\,000$$

$$0 + 20\,000 - 2\,000 + Q_{41} = -15\,000 ; \quad Q_{41} = -33\,000 \text{ kJ/min}$$

The cyclic integral of any property is zero and accordingly

$$\oint dU = 0 ; \quad -2\,150 + 20\,000 - 36\,000 + dU_{41} = 0$$

$$\therefore \quad dU_{41} = 18\,150 \text{ kJ/min}$$

Again applying the non-flow energy equation to process 4 – 1,

$$Q_{41} = W_{41} + dU_{41} ; W_{41} = Q_{41} - dU_{41} = -33\,000 - 18\,150$$

$$= -51\,150/\text{kJ/min}$$

The complete table is then as shown below :

| *Process* | *Q* (kJ/min) | *W* (kJ/min) | *dU* (kJ/min) |
|---|---|---|---|
| 1 – 2 | 0 | 2 150 | – 2 150 |
| 2 – 3 | 20 000 | 0 | 20 000 |
| 3 – 4 | – 2 000 | 34 000 | – 36 000 |
| 4 – 1 | – 33 000 | – 51 150 | 18 150 |
| | $\Sigma Q = -15\,000$ kJ/min | $\Sigma W = -15\,000$ kJ/min | $\Sigma dU = 0$ |

(*b*) Work output $= -15\,000$ kJ/min $= \dfrac{-15000}{60}$ kJ/s $= -250$ kJ/s

The negative sign implies that work has been done by the surroundings on the given fluid system.

### EXAMPLE 5.22

**Consider a system taken from state 1 to state 2 (Fig. 5.6.) along the path 1 – a – 2; 100 kJ of heat flows into the system and the system does 40 kJ of work.**

**(*a*) Evaluate the heat that flows into the system along the path 1 – b – 2 if it is accompanied by 20 kJ of work transfer from the system.**

**(*b*) Calculate the work done and heat exchanged if the system returns to the initial state 1 along the straight path 2–1.**

**(*c*) Make calculations for the heat absorbed in the processes 1 – b and b – 2 if internal energy values at state point 1 and b are given to be 0 and 50 kJ respectively.**

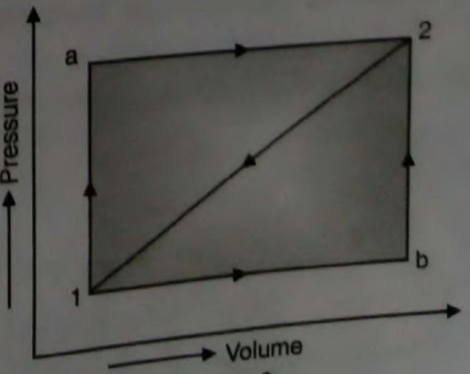

Fig. 5.6.

**Solution :** The example illustrates the application of non-flow energy equation,

$$\delta Q = \delta W + dU, \text{ applied to a process.}$$

**For path 1 - a - 2 :** $\delta Q = 100 \text{ kJ}$ and $\delta W = 40 \text{ kJ}$

$$\therefore\ U_2 - U_1 = Q_{1-a-2} - W_{1-a-2} = 100 - 40 = \mathbf{60\ kJ}$$

**For path 1 - b - 2 :** $\delta W = 20 \text{ kJ}$ and $U_2 - U_1 = 60 \text{ kJ}$

$$\therefore\ Q_{1-b-2} = W_{1-b-2} + (U_2 - U_1)$$
$$= 20 + 60 = \mathbf{80\ kJ}$$

(*b*) **Straight path 2—1 :** For path 1 - *a* - 2, no work is done during constant volume process 1 - *a* and the entire work of 40 kJ is done during the constant pressure path *a* - 2. Therefore,

area under line *a* - 2 = 40 kJ

Similarly, area under line 1 - *b* = 20 kJ

area of rectangle 1 - *a* - 2 - *b* = 40 - 20 = 20 kJ

$$\text{area of triangular portion } 2 - 1 - b = \frac{1}{2} \times 20 = 10 \text{ kJ}$$

area under straight line 2—1 (return path)

= area 2 - 1 - *b* + area under 1 - *b*

= 10 + 20 = 30 kJ

∴ Work done on the system,

$$W_{2-1} = -\,30 \text{ kJ}$$
$$Q_{2-1} = W_{2-1} + (U_1 - U_2)$$
$$= -\,30 - 60 = -\,90 \text{ kJ}$$

Since this is negative, heat flows out of the system

(*c*) For path 1 - *b* - 2

$$\text{Work done} = W_{1-b} + W_{b-2};\ \ 20 = W_{1-b} + 0;\ \ W_{1-b} = 20 \text{ kJ}$$
$$Q_{1-b} = W_{1-b} + (U_b - U_1) = 20 + (50 - 0) = \mathbf{70\ kJ}$$

Also
$$Q_{b-2} = Q_{1-b-2} - Q_{1-b} = 80 - 70 = \mathbf{10\ kJ}$$

### EXAMPLE 5.23

**When a closed system executes a certain non-flow process, the work and heat interactions per degree rise in temperature at each temperature attained are given by**

$$\frac{\delta W}{dT} = (4 - 0.08\,T) \text{ kJ/kg} \quad \text{and} \quad \frac{\delta Q}{dT} = 1.00 \text{ kJ/K}$$

**Make calculations for the increase or decrease in the internal energy of the system if it is to operate between the temperature limits of 200 °C and 400 °C.**

**Solution :**Work interaction,
$$W = \int (4 - 0.08\,T)\,dT = \left| 4T - \frac{0.08\,T^2}{2} \right|_{473}^{673}$$
$$= 4\,(673 - 473) - \frac{0.08}{2}\left(673^2 - 473^2\right) = -\,8368 \text{ kJ}$$

Since it is negative, work is done on the system.

$$\text{Heat interaction} = \int 1.00\,dT = \left|1.00\,T\right|_{473}^{673} = 1.00\ (673 - 473) = 200 \text{ kJ}$$

Since it is positive, heat is supplied to the system.

$$Q_{1-2} = W_{1-2} + dU = -748.7 + 430.5 = \mathbf{-318.2\ kJ}$$

The negative sign indicates that heat is rejected by the gas.

*Alternatively :*

$$Q_{1-2} = \frac{\gamma - n}{\gamma - 1} \times \text{polytropic work done}$$

$$= \frac{1.4 - 1.23}{1.4 - 1} \times (-748.7) = \mathbf{-318.2\ kJ}$$

## EXAMPLE 5.46

**2 kg of an ideal gas occupies a volume of 0.3 m$^3$ at 10 bar pressure and 500 K temperature. When this gas expands polytropically ($pV^{1.2} = C$) the internal energy decreases by 300 kJ. Presuming adiabatic exponent $\gamma = 1.4$, determine :**

**(*a*) specific gas constant,**

**(*b*) final temperature, pressure and volume of gas, and**

**(*c*) heat and work interactions across the system boundary.**

**Solution :** From characteristic gas equation, $pV = mRT$

$$\text{Gas constant, } R = \frac{pV}{mT} = \frac{10 \times 10^5 \times 0.3}{2 \times 500} = \mathbf{300\ J/kg\ K}$$

(*b*) From the relation, $(c_p - c_v) = R$ and $\frac{c_p}{c_v} = \gamma$

$$c_v = \frac{R}{\gamma - 1} = \frac{300}{1.4 - 1} = 750\ \text{J/kg K} = 0.75\ \text{kJ/kg K}$$

Change in internal energy, $dU = U_2 - U_1 = mc_v (T_2 - T_1)$

The decrease in internal energy implies that $dU$ is negative.

$$-300 = 2 \times 0.75 (T_2 - 500) ; \quad \therefore T_2 = \mathbf{300\ K}$$

From the relations : $$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \left(\frac{V_1}{V_2}\right)^{n-1}$$

$$p_2 = p_1\left(\frac{T_2}{T_1}\right)^{\frac{n}{n-1}} = 10 \times \left(\frac{300}{500}\right)^{\frac{1.2}{1.2-1}} = \mathbf{0.467\ bar}$$

$$V_2 = V_1\left(\frac{p_1}{p_2}\right)^{1/n} = \left(\frac{10}{0.467}\right)^{1/1.2} \times 0.3 = \mathbf{3.85\ m^3}$$

(*c*) Work done, $$W_{1-2} = \frac{p_1V_1 - p_2V_2}{n-1} = \frac{mR(T_1 - T_2)}{n-1}$$

$$= \frac{2 \times 300 \times (500 - 300)}{1.2 - 1} = 600 \times 10^3 \text{ J} = \mathbf{600\ kJ}$$

From non-flow energy equation,

$$Q_{1\text{-}2} = W_{1\text{-}2} + (U_2 - U_1) = 600 + (-300) = \mathbf{300\ kJ}$$

*Alternatively* : $$Q_{1\text{-}2} = \frac{\gamma - n}{\gamma - 1} \times \text{polytropic work done}$$

$$= \frac{1.4 - 1.2}{1.4 - 1} \times 600 = \mathbf{300\ kJ}$$

## EXAMPLE 5.47

**One kg of air at a pressure of 7 bar and a temperature of 360 K undergoes a reversible polytropic process which may be represented by $pV^{1.1}$ = constant. If the final pressure is 1.4 bar, evaluate (*a*) final specific volume and temperature, (*b*) work done and heat transfer. How the work and heat interactions would be affected if the process is irreversible and 15 kJ of work is lost due to internal friction ?**

**Take $R$ = 287 J/kg K and $\gamma$ = 1.4**

**Solution:** Assuming air to be perfect gas, $p_1V_1 = mRT_1$

$$\therefore \quad V_1 = \frac{1 \times 287 \times 360}{7 \times 10^5} = 0.1476 \text{ m}^3/\text{kg}$$

From the given polytropic law,

$$V_2 = V_1\left(\frac{p_1}{p_2}\right)^{1/n} = 0.1476 \times \left(\frac{7}{1.4}\right)^{\frac{1}{1.1}} = \mathbf{0.637\ m^3/kg}$$

$$T_2 = \frac{p_2V_2}{mR} = \frac{1.4 \times 10^5 \times 0.637}{1 \times 287} = \mathbf{310.73\ K}$$

(*b*) Work done, $$W_{1\text{-}2} = \frac{p_1V_1 - p_2V_2}{n-1} = \frac{mR(T_1 - T_2)}{n-1}$$

$$= \frac{1 \times 287 \times (360 - 310.73)}{1.1 - 1} = 141405 \text{ Nm (or J)} = 141.405 \text{ kJ}$$

$$\text{Now, } c_v = \frac{R}{\gamma - 1} = \frac{287}{1.4 - 1} = 717.5 \text{ J/kg K} = 0.7175 \text{ kJ/kg K}$$

*Change in internal energy,* $dU = mc_v (T_2 - T_1)$

$$= 1 \times 0.7175 (310.73 - 360) = -35.35 \text{ kJ/kg}$$

From non-flow energy equation, $\delta Q = \delta W + dU$

$$Q_{1\text{-}2} = W_{1\text{-}2} + dU = 141.405 + (-35.35) = \mathbf{106.055 \text{ kJ/kg}}$$

*Alternatively :* $Q_{1\text{-}2} = \frac{\gamma - n}{\gamma - 1} \times$ polytropic work

$$= \frac{1.4 - 1.1}{1.4 - 1} \times 141.405 = \mathbf{106.053 \text{ kJ/kg}}$$

(*c*) The end states are the same, but the process is irreversible with work dissipation in friction.

$\therefore$ $dU = -35.35$ kJ/kg since the end states are the same and internal energy is the property or point function.

Net work done = displacement work – work dissipated in friction

= 141.405 – 15 = 126.405 kJ

and net heat transfer is given by

$Q_{net}$ = $dU$ + (displacement work – work dissipated in friction)

= – 35.35 + (141.405 – 15) = **91.055 kJ/kg**

### EXAMPLE 5.48

**A gaseous mixture of mean molecular mass 44 occupies a volume of 0.1 m$^3$ at 1 bar pressure and 340 K temperature. The gas is compressed polytropically according to the law $pV^n$ to 0.015 m$^3$ volume and 10 bar pressure. Evaluate the amount of work done, change in internal energy and heat added or subtracted during the compression process. For the gas, mean molar specific heat at constant volume = 25 kJ/kg mole K.**

**Solution :** (*i*) From the polytropic law,

$$p_1 V_1^n = p_2 V_2^n$$

$$n = \frac{\log_e \frac{p_2}{p_1}}{\log_e \frac{V_1}{V_2}} = \frac{\log_e \left(\frac{10}{1}\right)}{\log_e \left(\frac{0.1}{0.015}\right)} = \frac{2.302}{1.897} = 1.214$$

(*ii*) Work done, $W_{1\text{-}2} = \frac{p_1 V_1 - p_2 V_2}{n - 1} = \frac{1 \times 10^5 \times 0.1 - 10 \times 10^5 \times 0.015}{1.214 - 1}$

$$= -23365 \text{ Nm (or J)}$$

$$= \mathbf{-23.365 \text{ kJ}}$$

Negative sign implies work supplied to the system.

(*iii*) Change in internal energy,

$$dU = U_2 - U_1 = mc_v\,(T_2 - T_1)$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1}; \quad T_2 = 340\left(\frac{0.1}{0.015}\right)^{1.214-1} = 510.26 \text{ K}$$

$$c_v = \frac{\text{molar specific heat}}{\text{molecular mass}} = \frac{25}{44} = 0.5682 \text{ kJ/kg K}$$

$$R = \frac{R_{mole}}{\text{molecular mass}} = \frac{8314}{44} = 188.95 \text{ J/kg K}$$

$$\text{Mass of gas, } m = \frac{p_1V_1}{RT_1} = \frac{1\times 10^5 \times 0.1}{188.95 \times 340} = 0.1556 \text{ kg}$$

$$\therefore \quad dU = 0.1556 \times 0.5682\,(510.26 - 340) = 15.053 \text{ kJ/kg}$$

(*iv*) Heat interaction $Q_{1-2} = W_{1-2} + dU$

$$= -23.365 + 15.053 = \mathbf{-8.317\ kJ}$$

Negative sign implies that heat has been rejected by the system.

### EXAMPLE 5.49

**During a certain expansion process of a perfect gas, the variation in presure is estimated to conform to the linear relation $p = aV + b$ where $a$ and $b$ are constants and $V$ represents the volume in m$^3$. The gas has a mass of 0.75 kg, the initial and final pressures are 4 bar and 2 bar, and the corresponding volumes are 0.1 m$^3$ and 0.2 m$^3$. Make calculations for the work and heat interactions during the process. Comment on the nature of process.**

**Solution**: The pressure variation for the given expansion process is of the form $p = aV + b$. The values of constants $a$ and $b$ can be determined from the values of pressure and temperature at the initial and final state points.

$$4 = 0.1a + b \quad \text{and} \quad 2 = 0.2a + b$$

From these expressions: $a = -20$ and $b = 6$

The law of expansion becomes: $p = (-20V + 6)$ bar $= (-20V + 6) \times 10^5$ N/m$^2$

(*a*) Work done $W_{12} = \int_1^2 p\,dV = \int_1^2 (-20V+6)\times 10^5\,dV = 10^5\left|\frac{-20V^2}{2} + 6V\right|_{0.1}^{0.2}$

$$= 10^5\left[-10(0.2^2 - 0.1^2) + 6(0.2 - 0.1)\right] = 30000 \text{ Nm} = \mathbf{30\ kJ}$$

(*b*) The given data shows that

$$p_1V_1 = 4 \times 0.1 = 0.4\,; \quad p_2V_2 = 2 \times 0.2 = 0.4$$

Thus $p_1V_1 = p_2V_2$ and consequently $T_1 = T_2$ and $dU = 0$

Then from non-flow energy equation $\delta Q = \delta W + dU$, we have

$$\delta Q = \delta W = \mathbf{30\ kJ}$$

(*c*) Even though $T_1 = T_2$, the given process cannot be treated as isothermal. The relation is true only for the initial and final conditions. Further, the given relation is a straight line path whereas isothermal path is a parabolic curve.

### EXAMPLE 5.50

**A perfect gas undergoes the following three separate and distinct processes to execute a cycle:**

**(*i*) constant volume process during which 80 kJ of heat is supplied to the gas,**

**(*ii*) constant pressure process during which 85 kJ of heat is lost to the surroundings and 20 kJ of work is done on it.**

**(*iii*) adiabatic process which restores the gas back to its initial state.**

**Evaluate the work done during adiabatic process and the value of internal energy at all the state points if initially its value is 95 kJ.**

**Solution :** This example illustrates the application of non-flow energy equation and the 1st law applied to a cyclic process.

*Process* 1—2 :

$$Q_{1\text{-}2} = 80 \text{ kJ} \; ; \; W_{1\text{-}2} = 0$$

$$U_2 - U_1 = Q_{1\text{-}2} - W_{1\text{-}2}$$

$$= 80 - 0 = 80 \text{ kJ}$$

$$\therefore \; U_2 = 80 + 95 = \mathbf{175 \; kJ}$$

*Process* 2—3 :

$$Q_{2\text{-}3} = -\,85 \text{ kJ}; \; W_{2\text{-}3} = -\,20 \text{ kJ}$$

$$U_3 - U_2 = Q_{2\text{-}3} - W_{2\text{-}3} = -\,85 - (-\,20) = -\,65 \text{ kJ}$$

$$\therefore \; U_3 = -\,65 + 175 = \mathbf{110 \; kJ}$$

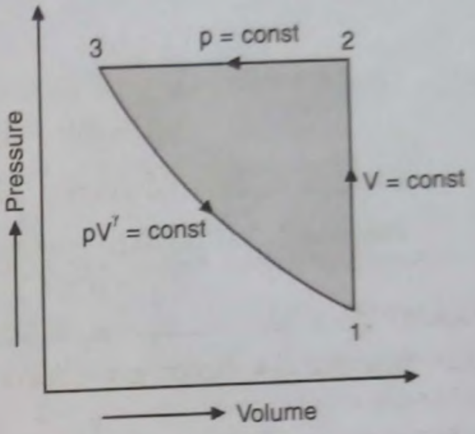

Fig. 5.12.

For the complete cycle : $\oint \delta Q = \oint \delta W$ . That is

$$Q_{1\text{-}2} + Q_{2\text{-}3} + Q_{3\text{-}1} = W_{1\text{-}2} + W_{2\text{-}3} + W_{3\text{-}1}$$

$$80 - 85 + 0 = 0 + (-\,20) + W_{3\text{-}1}$$

$\therefore$ Work done during adiabatic process, $W_{3\text{-}1}$ = **15 kJ**

### EXAMPLE 5.51

**One kg of air at 1 bar and 300 K is compressed adiabatically till its pressure becomes 5 times the original pressure. Subsequently it is expanded at constant pressure and finally cooled at constant volume to return to its original state. Calculate the heat and work interactions, and change in internal energy for each process and for the cycle.**

**Solution:** Refer fig. 5.13 which represents the given processes on *p-V* coordinates.

Let it be presumed that for air:

$$c_p = 1.005 \text{ kJ/kg K}$$

$$c_v = 0.718 \text{ kJ/kg K}$$

Then

$$\gamma = \frac{c_p}{c_v} = \frac{1.005}{0.718} = 1.4$$

$$R = c_p - c_v = 1.005 - 0.718$$

$$= 0.287 \text{ kJ/kg K} = 287 \text{ J/K}$$

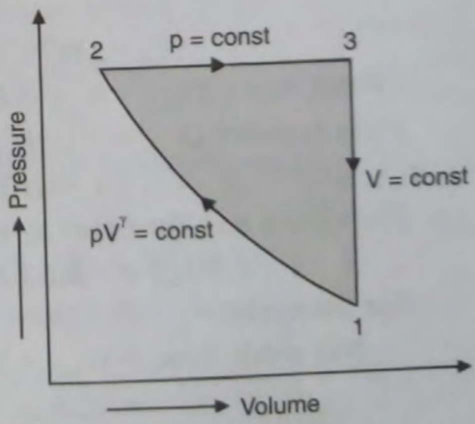

Fig. 5.13.

**Process 1-2** (adiabatic)

$$V_1 = \frac{mR\,T_1}{p_1} = \frac{1 \times 287 \times 300}{1 \times 10^5} = 0.861 \text{ m}^3$$

$$V_2 = \left(\frac{p_1}{p_2}\right)^{1/\gamma} \times V_1 = \left(\frac{1}{5}\right)^{1/1.4} \times 0.861 = 0.273 \text{ m}^3$$

$$T_2 = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \times T_1 = (5)^{\frac{1.4-1}{1.4}} \times 300 = 475.36 \text{ K}$$

$$\text{Work done, } W_{1\text{-}2} = \frac{p_1 V_1 - p_2 V_2}{\gamma - 1} = \frac{mR\,(T_1 - T_2)}{\gamma - 1}$$

$$= \frac{1 \times 287\,(300 - 475.36)}{1.4 - 1} = -125281 \text{ J} = -125.28 \text{ kJ}$$

Heat transfer, $Q_{1\text{-}2} = 0$ as the process is adiabatic. Then from non-flow energy equa

$$\delta Q = \delta W + dU$$

Change in internal energy, $dU_{1\text{-}2} = -\delta W = 125.28$ kJ.

**Process 2–3** (Constant pressure)

$$V_2 = 0.273 \text{ m}^3\,; \quad V_3 = 0.861 \text{ m}^3$$

$$p_3 = p_2 = 5 \text{ bar} \quad \text{and} \quad T_2 = 475.36 \text{ K}$$

Charles law holds good for a constant pressure process and therefore,

$$T_3 = \frac{V_3}{V_2} \times T_2 = \frac{0.861}{0.273} \times 475.36 = 1499.2 \text{ K}$$

$$\text{Work done, } W_{2\text{-}3} = p(V_3 - V_2) = 5 \times 10^5\,(0.861 - 0.273)$$

$$= 294000 \text{ J} = 294 \text{ kJ}$$

$$\text{Heat transfer, } Q_{2\text{-}3} = m\,c_p\,(T_3 - T_2)$$

$$= 1 \times 1.005\,(1499.2 - 475.36) = 1028.96 \text{ kJ}$$

Then from non-flow energy equation, $\delta Q = \delta W + dU$

Change in internal energy, $dU_{2\text{-}3} = 1028.96 - 294 = 734.96$ kJ

**Process 3–1** (Constant volume)

$$p_3 = 5 \text{ bar}; \quad V_3 = V_1 = 0.861 \text{ m}^3$$

$$T_3 = 1499.2 \text{ K} \quad \text{and} \quad T_1 = 300 \text{ K}$$

Work done, $W_{3\text{-}1} = 0$ as volume remains constant

$$\text{Heat transfer } Q_{3\text{-}1} = m\,c_v\,(T_1 - T_3)$$

$$= 1 \times 0.718\,(300 - 1499.2) = -861.02 \text{ kJ}$$

Then from non-flow energy equation, $\delta Q = \delta W + dU$

$$dU_{3\text{-}1} = -861.02 \text{ kJ}$$

For the cycle:

$$\text{Net work done} = W_{1\text{-}2} + W_{2\text{-}3} + W_{3\text{-}1}$$

$$= -125.28 + 294 + 0 = 168.72 \text{ kJ}$$

$$\text{Net heat transfer} = 0 + 1028.96 - 861.02 = 167.94 \text{ kJ}$$

Obviously for a cyclic process $\oint \delta Q = \oint \delta W$

$$\oint dU = 125.28 + 734.96 - 861.02 = 0$$

This has to be so because internal energy is a property of the system and the cyclic integral of a property is zero.

### EXAMPLE 5.52

**0.2 m$^3$ of air at a pressure of 400 kPa and 430 K is contained inside a thermally insulated cylinder. A movable piston fitted inside the cylinder is pushed outwards to a position where the pressure is 100 kPa. During this operation, the piston assembly does not exchange any heat with the surroundings. The air is then supplied heat at constant pressure till its enthalpy increases by 60 kJ. Evaluate the net work done by the gaseous system. What should be the index of expansion of a single reversible polytropic process if that is to produce the same work whilst operating between the same initial and final states ?**

**Solution:** *Process* 1 – 2 : Since the piston-cylinder assembly does not exchange heat with the surroundings, the expansion process 1 – 2 is isentropic (reversible adiabatic) and for such a process

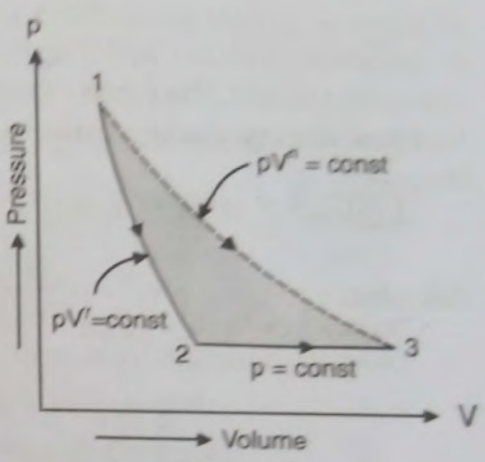

Fig. 5.14.

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$= 430 \times \left(\frac{100}{400}\right)^{\frac{1.4-1}{1.4}} = 289.25 \text{ K}$$

$$V_2 = V_1(p_1/p_2)^{1/\gamma} = 0.2 \times \left(\frac{400}{100}\right)^{\frac{1}{1.4}} = 0.538 \text{ m}^3$$

$$\text{Mass of gas, } m = \frac{p_1V_1}{RT_1} = \frac{400 \times 10^3 \times 0.2}{287 \times 430} = 0.6482 \text{ kg}$$

$$\text{Work done, } W_{1\text{-}2} = \frac{p_1V_1 - p_2V_2}{\gamma - 1} = \frac{mR(T_1 - T_2)}{\gamma - 1}$$

$$= \frac{0.6482 \times 287 \times (430 - 289.25)}{1.4 - 1}$$

$$= 65460 \text{ Nm (or J)} = 65.46 \text{ kJ}$$

*Process* 2 – 3 : The heat supplied at constant pressure equals the change in enthalpy

$$\therefore \quad Q_{2\text{-}3} = (H_3 - H_2) = mc_p\,(T_3 - T_2)$$

$$60 = 0.6482 \times 1.005 \times (T_3 - 289.25); \quad T_3 = 381.35 \text{ K}$$

Also, Charles law hold good for a constant pressure process and therefore,

$$V_3 = V_2\frac{T_3}{T_2} = 0.538 \times \frac{381.35}{289.25} = 0.7093 \text{ m}^3$$

$$\text{Work done, } W_{2\text{-}3} = p_2\,(V_3 - V_2) = 100 \times 10^3\,(0.7093 - 0.538)$$

$$= 17130 \text{ Nm (or J)} = 17.13 \text{ kJ}$$

Total work done by both the processes,

$$W_{1-2} + W_{2-3} = 65.46 + 17.13 = 82.59 \text{ kJ}$$

According to the given condition this work must equal the work done by a hypothetical polytropic process between the same initial and final states. That is

$$82.59 = \frac{p_1V_1 - p_3V_3}{n-1} = \frac{m\,R(T_1 - T_3)}{n-1}$$

$$= \frac{0.6482 \times 287 \times (430 - 381.35)}{n-1} \times 10^{-3}$$

Solution gives : $n$ = **1.1096**

**EXAMPLE 5.53**

**60 litres of an idle gas at 290 K and 1 bar is compressed adiabatically to 10 bar. It is then cooled at constant volume and further expanded isothermally so as to reach the condition from where it started. Evaluate : (*a*) pressure at the end of constant volume cooling, (*b*) change in internal energy during constant volume process, (*c*) net work done and heat transfer during the cycle.**

**Assume $c_p$ = 14.25 kJ/kg K and $c_v$ = 10.15 kJ/kg**

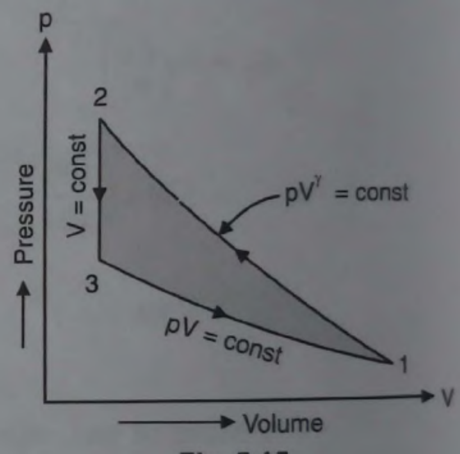

**Fig. 5.15.**

**Solution :** $$\gamma = \frac{c_p}{c_v} = \frac{14.25}{10.15} = 1.404$$

Characteristic gas constant,

$$R = c_p - c_v = 14.25 - 10.15 = 4.10 \text{ kJ/kg K}$$

$$V_1 = 60 \text{ litres} = 0.060 \text{ m}^3$$

Considering adiabatic process 1—2,

$$V_2 = \left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}} V_1$$

$$= \left(\frac{1}{10}\right)^{\frac{1}{1.404}} \times 0.060 = 0.01164 \text{ m}^3$$

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = 290 \times \left(\frac{10}{1}\right)^{\frac{1.404-1}{1.404}} = 562.47 \text{ K}$$

Considering isothermal process, 3—1,

$$p_3 = p_1\frac{V_1}{V_3} = \frac{1 \times 0.06}{0.01164} \qquad (\because V_3 = V_2)$$

$$= \mathbf{5.155 \text{ bar}}$$

(*b*) Mass of gas, $m = \dfrac{p_1V_1}{RT_1}$

$$= \frac{1 \times 10^5 \times 0.06}{4100 \times 290} = 5.046 \times 10^{-3} \text{ kg}$$

Change in internal energy during constant volume process 2–3,

$$U_3 - U_2 = mc_v\,(T_3 - T_2)$$

$$= 5.046 \times 10^{-3} \times 10.15\,(290 - 560.47) = -\,13.85 \text{ kJ}$$

During constant volume cooling process, temperature and hence internal energy is reduced. This decrease in internal energy equals the heat flow to surroundings since the work done is zero.

(c)

$$W_{1\text{-}2} = \frac{p_1V_1 - p_2V_2}{\gamma - 1} = \frac{mR(T_1 - T_2)}{\gamma - 1}$$

$$= \frac{5.046 \times 10^{-3} \times 4100\,(290 - 562.46)}{1.404 - 1}$$

$$= -\,13952 \text{ Nm (or J)} = -\,13.952 \text{ kJ}$$

$$W_{2\text{-}3} = 0 \quad \text{(as volume remains constant)}$$

$$W_{3\text{-}1} = p_3V_3 \log_e V_1/V_3 = p_1V_1 \log_e p_3/p_1$$

$$= 1 \times 10^5 \times 0.06 \log_e \frac{5.155}{1} = 9840 \text{ Nm (or J)} = 9.84 \text{ kJ}$$

Net work done, $W_n = W_{1\text{-}2} + W_{2\text{-}3} + W_{3\text{-}1} = -\,13.952 + 0 + 9.84 = -\,4.112$ kJ

Negative sign means that work has been done on the system.

For a cyclic process $\oint \delta Q = \oint \delta W$

∴ Heat transfer during the complete cycle = – 4.112 kJ

Negative heat transfer implies that heat has been lost from the system.

### EXAMPLE 5.54

**0.2 m³ of an ideal gas at a pressure of 2 MPa and 600 K is expanded isothermally to 5 times the initial volume. It is then cooled to 300 K at constant volume and then compressed back polytropically to its initial state. Determine the net work done and heat transfer during the cycle.**

**Solution :** Considering process 1–2 which is isothermal, *i.e.,* $T_1 = T_2 = 600$ K

$$p_2 = \frac{p_1V_1}{V_2} = \frac{2 \times 10^6 \times 0.2}{(5 \times 0.2)}$$

$$= 400 \times 10^3 \text{ N/m}^2 = 0.4 \text{ MPa}$$

Work done, $W_{1\text{-}2} = p_1V_1 \log_e \dfrac{V_2}{V_1}$

$$= 2 \times 10^6 \times 0.2 \log_e 5 = 643775 \text{ J}$$

$$= 643.775 \text{ kJ}$$

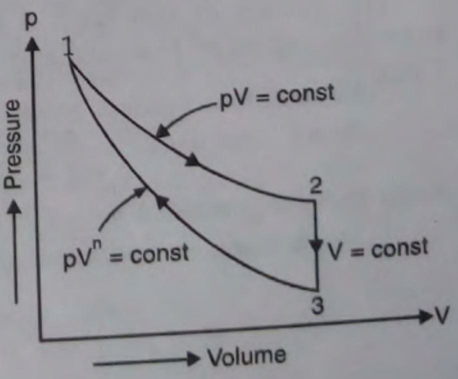

Fig. 5.16.

Considering process 2–3 which is at constant volume,

*i.e.,*

$$V_2 = V_3 = 5 \times 0.2 = 1\text{m}^3$$

$$p_3 = p_2 \times \frac{T_3}{T_2} = 400 \times 10^3 \times \frac{300}{600}$$

$$= 200 \times 10^3 \text{ N/m}^2 = 0.2 \text{ MPa}$$

$W_{2-3} = 0$ as volume remains constant.

For the polytropic process 3 – 1; $p_3V_3^n = p_1V_1^n$

$$n = \frac{\log_e \frac{p_1}{p_3}}{\log_e \frac{V_3}{V_1}} = \frac{\log_e \frac{2}{0.2}}{\log_e \frac{1}{0.2}} = \frac{2.3026}{1.6094} = 1.433$$

$$W_{3-1} = \frac{p_3V_3 - p_1V_1}{n-1} = \frac{0.2 \times 10^6 \times 1 - 2 \times 10^6 \times 0.2}{1.433 - 1}$$

$$= -\ 461894 \text{ J} = -\ 461.894 \text{ kJ}$$

Net work done, $W_n = W_{1-2} + W_{2-3} + W_{3-1}$

$$= 643.774 + 0 + (-\ 461.894) \ = \mathbf{181.88\ kJ}$$

For a cyclic process $\oint \delta Q = \oint \delta W$

∴ Heat transfer during the complete cycle = **181.88 kJ**

**EXAMPLE 5.55**

**A system consisting of 1 kg of an ideal gas at 6 bar pressure and 0.01 m$^3$ volume executes a cyclic process comprising the following three distinct operations :**

**(*i*) reversible expansion to 0.05 m$^3$ volume and 2 bar pressure, presuming pressure to be a linear function of volume ($p = a + bV$)**

**(*ii*) reversible cooling at constant pressure, and**

**(*iii*) reversible hyperbolic compression according to law $pV$ = const.**

**This brings the gas back to the initial conditions.**

**Sketch the cycle on $p-V$ diagram. Calculate the work done in each process stating whether it is done on or by the system and evaluate the net cyclic work and heat transfer.**

**Solution :** *Process* 1 – 2 (*Linear law*)

Let the linear law for the reversible expansion process 1 – 2 be of the form

$$p = a + bV$$

The values of constants $a$ and $b$ can be determined from the values of pressure and volume at the state points 1 and 2.

$$6 = a + 0.01\, b$$
$$2 = a + 0.05\, b$$

From these expressions $b = -\ 100$ and $a = 7$

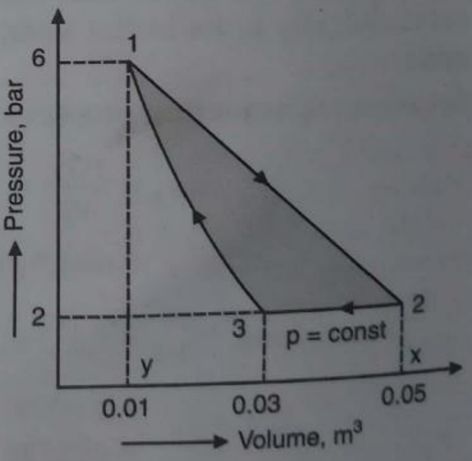

Fig. 5.17.

∴ Work done during expansion process, 1 – 2,

$$W_{1-2} = \int_1^2 p\,dV = \int_1^2 (a + bV)\,dV$$

$$= 10^5 \int_{0.01}^{0.05} (7 - 100\,V)\,dV = 10^5 \left| 7V - 100\frac{V^2}{2} \right|_{0.01}^{0.05} = 16000 \text{ Nm}$$