

# Error and Flow Control

**Required reading:**  
**Garcia 5.2**

**CSE 3213, Fall 2010**  
**Instructor: N. Vlajic**

# Error Control

## Error Control Approaches

- (1) Forward Error Correction (FEC)
- (2) Error Detection + Automatic Retrans. Req. (ARQ)

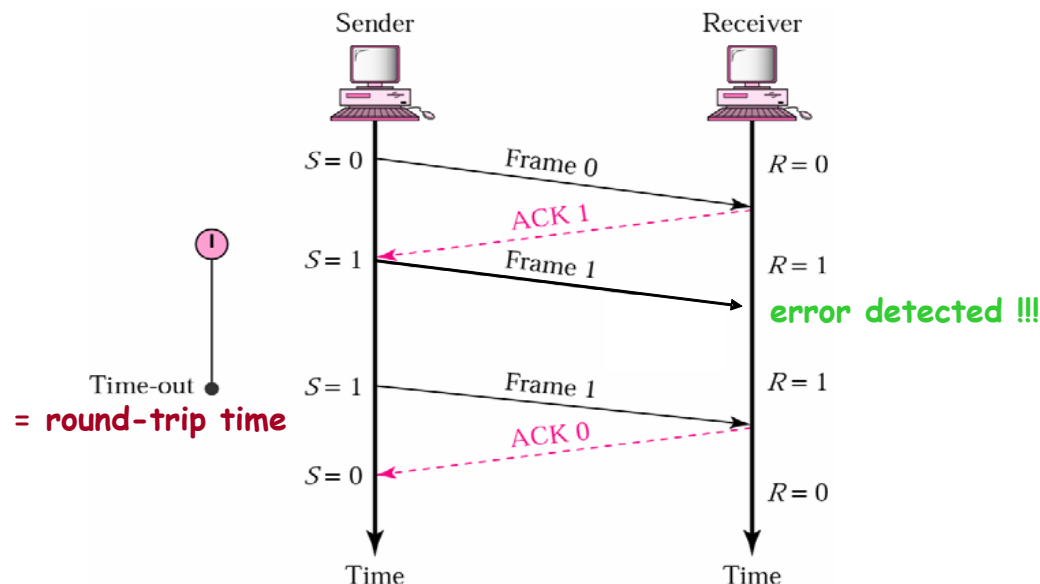
- not enough redundant info to enable error correction

case (a) receiver detects no errors

- an ACK packet is sent back to sender

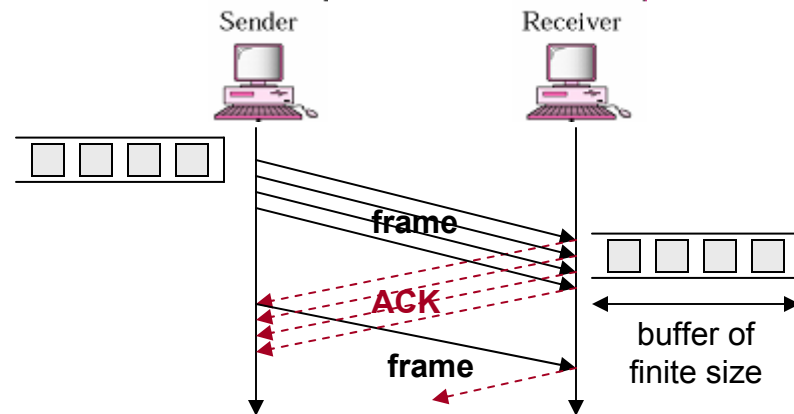
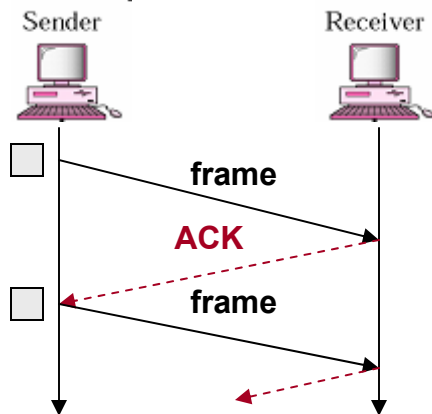
case (b) receiver detects errors

- no ACK sent back to sender
- sender retransmits frame after a 'time-out'



## Challenges of ARQ-based Error Control

- **send one frame at the time, wait for ACK**
  - easy to implement, but inefficient in terms of channel usage
- **send multiple frames at once**
  - better channel usage, but more complex to implement - sender must keep (all) sent but unACKed frame(s) in a buffer, as such frame(s) may have to be retransmitted



How many frames should be sent at any point in time?

How should frames be released from the sending buffer?

# Error and Flow Control

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**Flow Control** – set of procedures used to restrict the amount of data that sender can send while waiting for acknowledgment

- two main strategies
  - (1) **Stop-and-Wait**: sender waits until it receives ACK before sending next frame
  - (2) **Sliding Window**: sender can send W frames before waiting for ACKs

**Error + Flow Control Techniques**

- (1) Stop-and-Wait ARQ
- (2) Go-Back-N ARQ
- (3) Selective Repeat ARQ

**Error Detection + ARQ (error detection with retransmissions)**

must be combined with methods that intelligently limit the number of 'outstanding' (unACKed) frames.

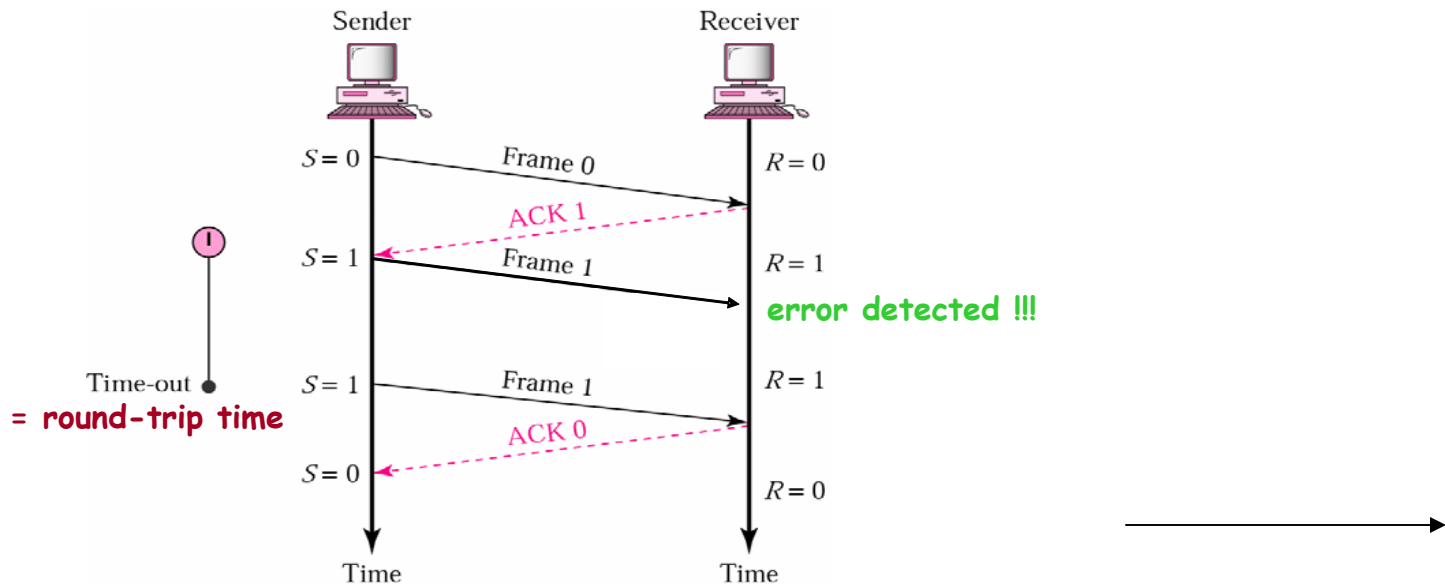
Fewer unACKed frames  $\Rightarrow$  fewer packets buffered at sender and receiver.

# **(1) Stop-and-Wait ARQ**

# Stop-and-Wait ARQ

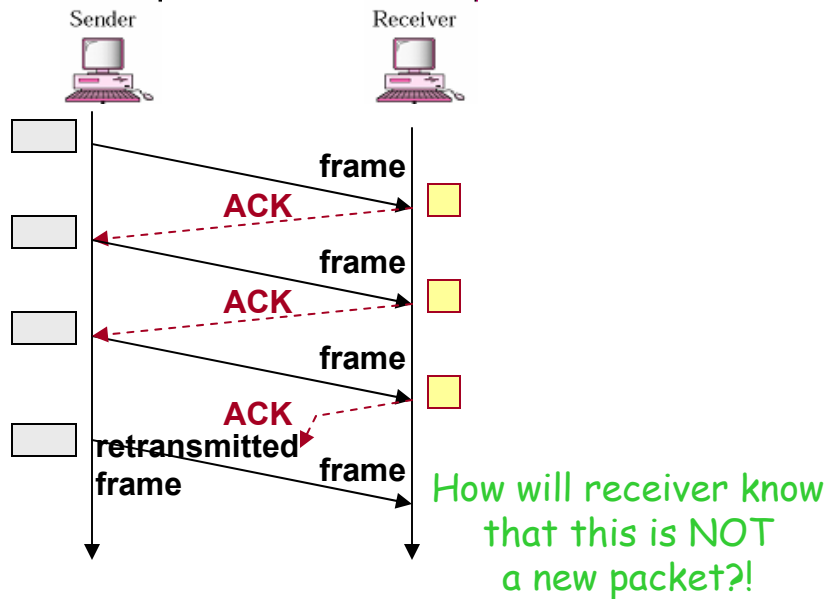
**Stop-and-Wait ARQ** – simplest flow and error control mechanism

- sender sends an information frame to receiver
- sender, then, stops and waits for an ACK
- if no ACK arrives within time-out, sender will resend the frame, and again stop and wait
  - **time-out period > roundtrip time**
- abnormalities (and how to fix them)
  - lost acknowledgment
  - delayed acknowledgment

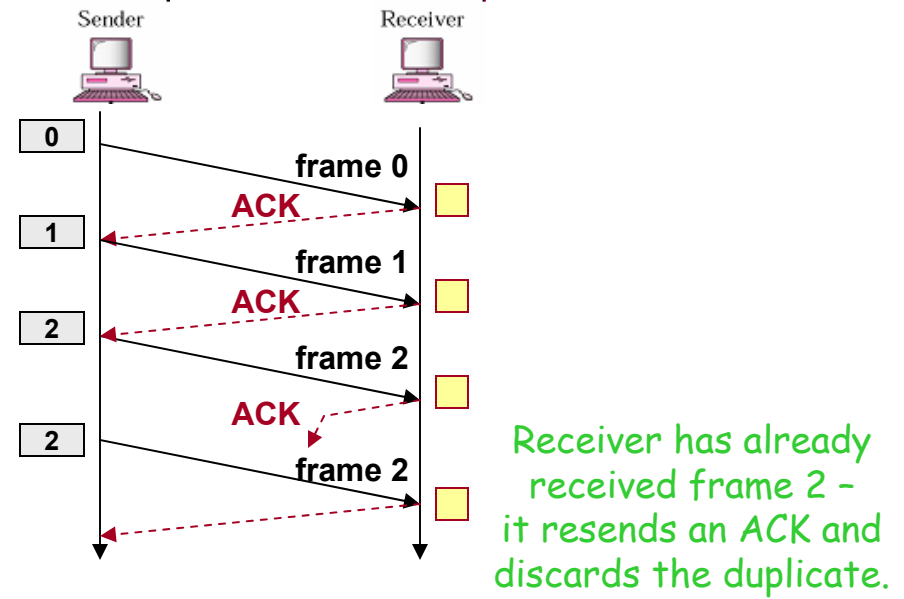


## Lost Acknowledgment

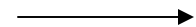
- frame received correctly, but ACK undergoes errors / loss
  - after time-out period, sender resends frame
  - receiver receives the same frame twice
- **frames must be numbered** so that receiver can recognize and discard duplicate frames
  - **sequence # are included in packet header**



without packet numbering



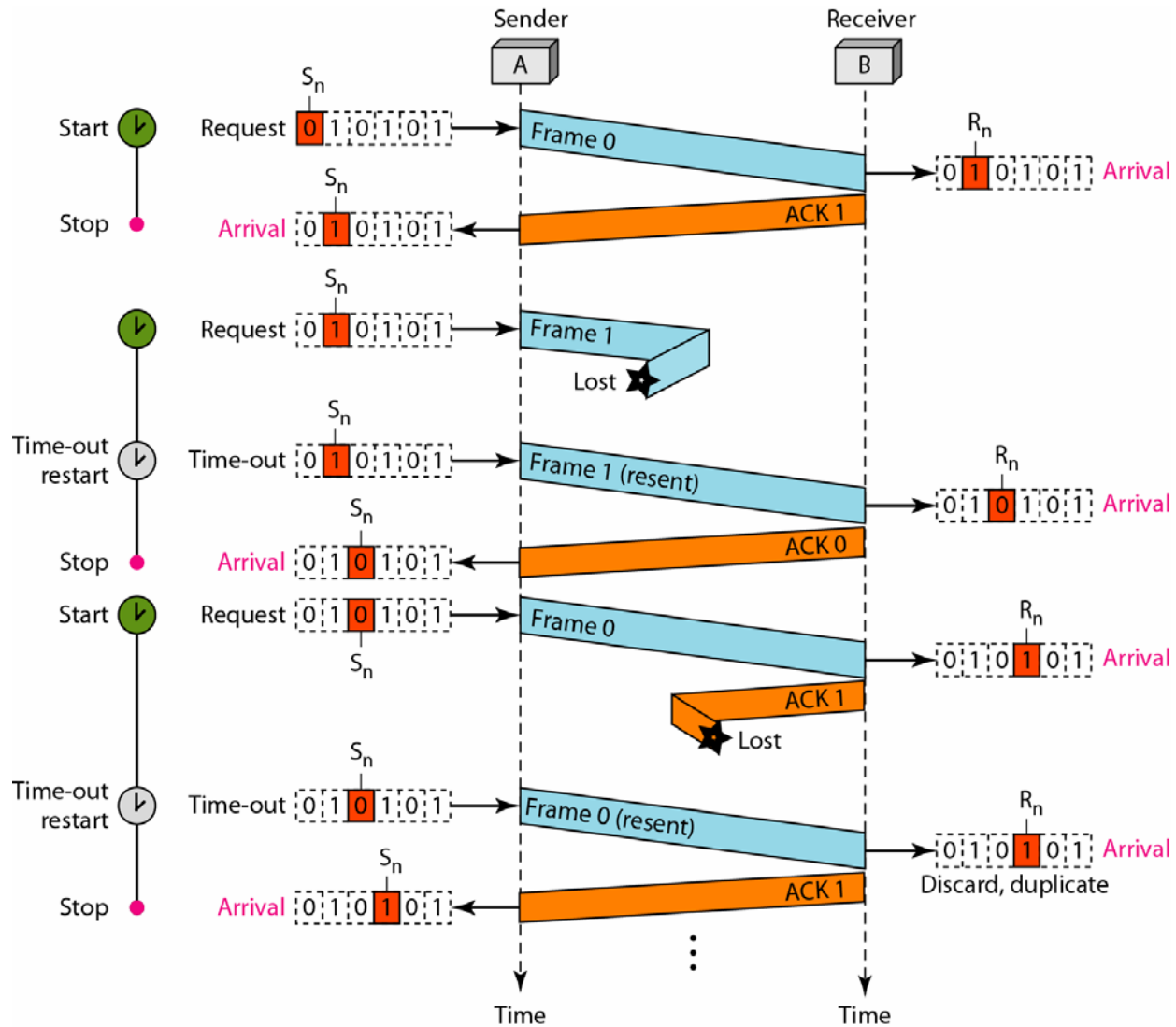
with packet numbering







# Stop-and-Wait ARQ (cont.)



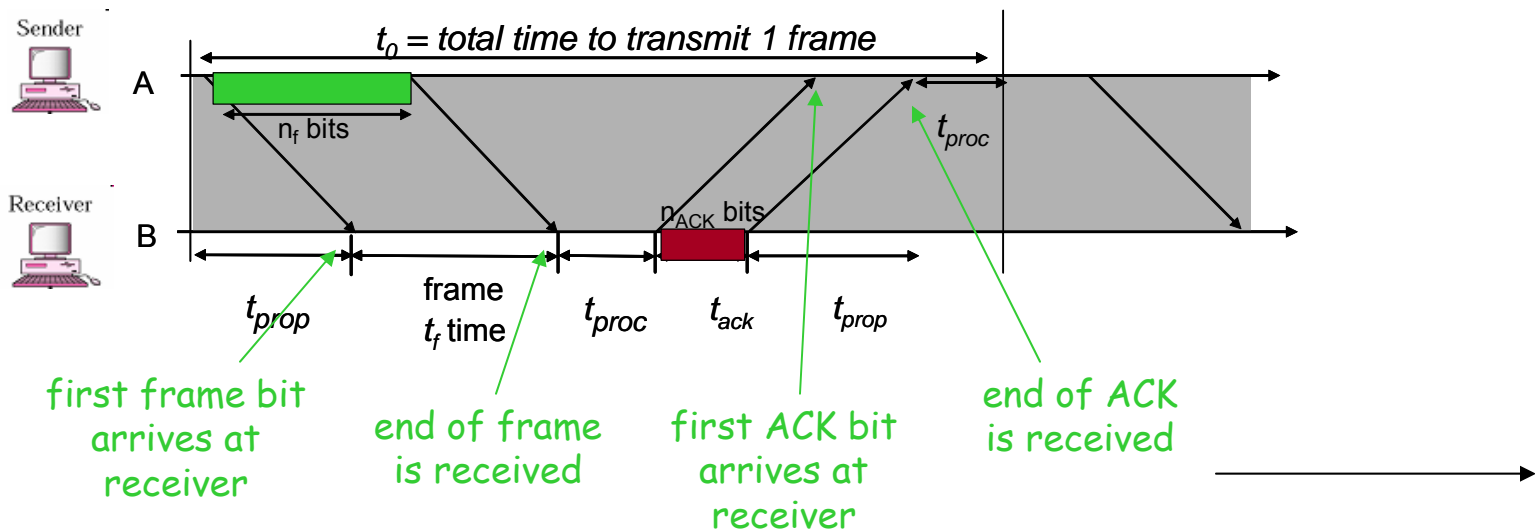
## Stop-and-Wait Efficiency

- $t_0$  = **basic Stop-and-Wait delay** – from time when frame is transmitted into channel until time when ACK arrives back to receiver, and another frame can be sent

$$t_0 = 2 \cdot t_{prop} + 2 \cdot t_{proc} + t_{frame} + t_{ACK} = 2 \cdot t_{prop} + 2 \cdot t_{proc} + \frac{n_f}{R} + \frac{n_{ACK}}{R}$$

- $R_{eff}$  = **effective transmission (data) rate:**

$$R_{eff} = \frac{\text{number of info bits delivered to destination}}{\text{total time required to deliver info bits}} = \frac{n_f - n_{header}}{t_0}$$



- $\eta_{sw}$  = **transmission efficiency**: ratio of actual and effective transmission (data) rate - **ideally,  $\eta_{sw} \approx 1$**
- **where do we lose channel efficiency**, and how can  $\eta_{sw} \rightarrow 1$  be achieved ?!

$$\eta_{sw} = \frac{R_{eff}}{R} = \frac{n_f - n_{header}}{t_0 R} = \frac{1 - \frac{n_{header}}{n_f}}{1 + \frac{n_{ACK}}{n_f} + \frac{2(t_{prop} + t_{proc})R}{n_f}}$$

should be as small as possible

(1)  $\frac{n_{header}}{n_f}$  - loss in efficiency due to (need for) header

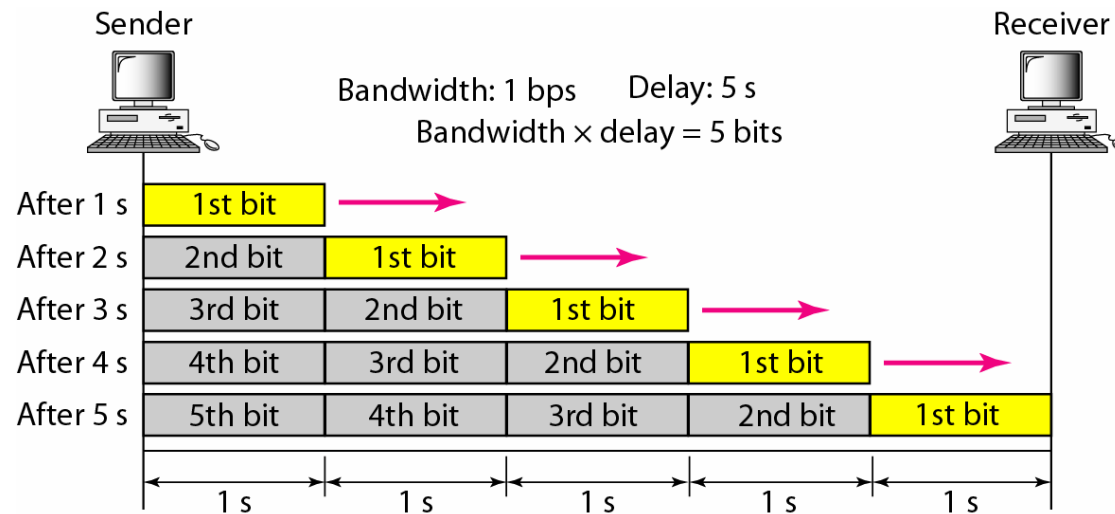
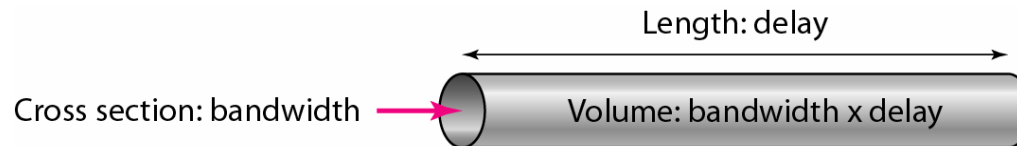
(2)  $\frac{n_{ACK}}{n_f}$  - loss in efficiency due to (need for) ACKs

(3)  $2(t_{prop} + t_{proc})R$  - **bandwidth-delay product**

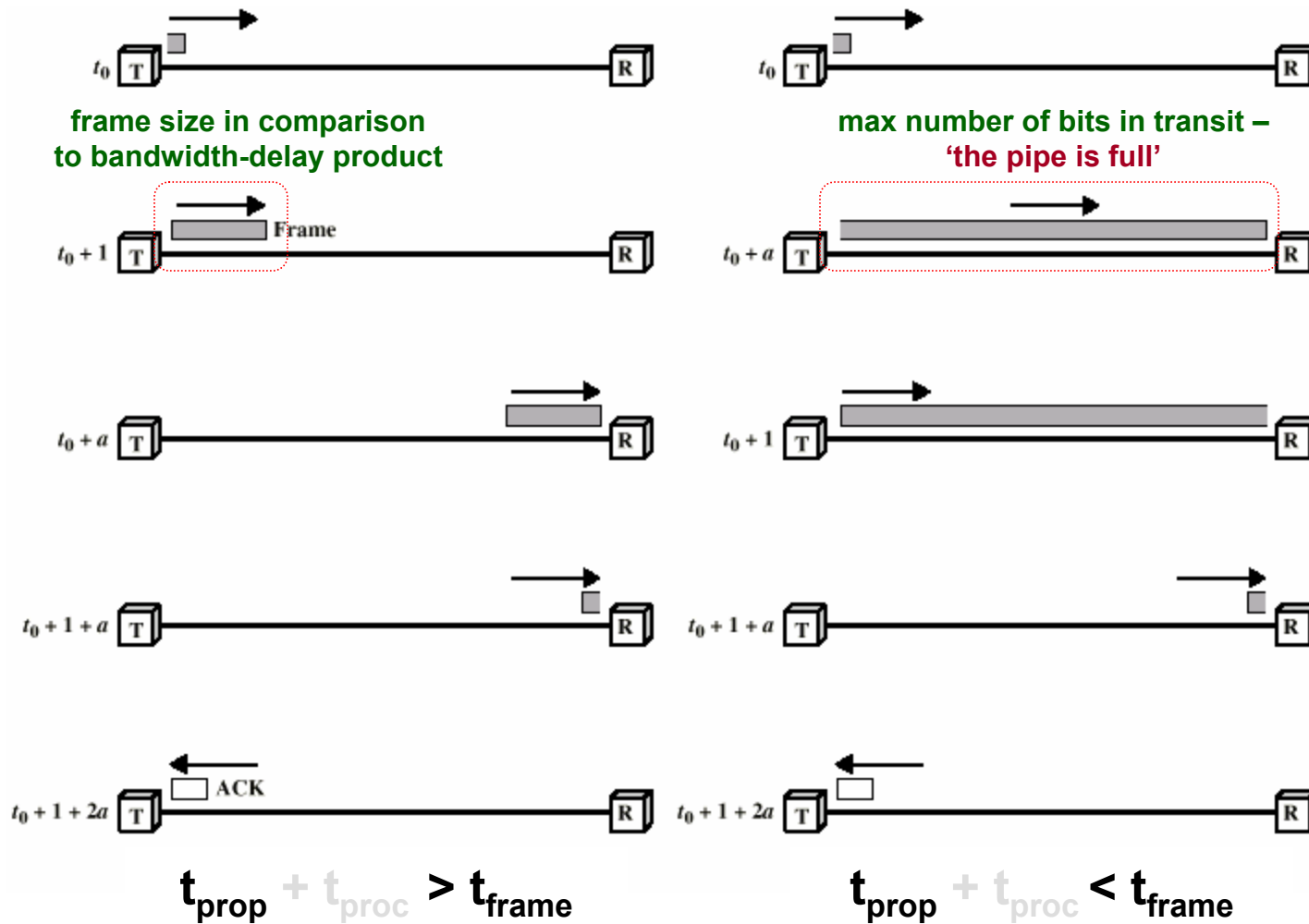
- max number of bits in transit at any given time
- in Stop-and-Wait ARQ delay-bandwidth product is a measure of lost opportunity in terms of transmitted bits

# Stop-and-Wait ARQ (cont.)

**Bandwidth-delay product =  $2 \cdot (t_{prop} + t_{proc}) \cdot R =$**   
**= capacity of the transmission pipe from the sender to the receiver and back.**



# Stop-and-Wait ARQ (cont.)



Stop-and-Wait ARQ becomes inadequate when data is fragmented into small frames, such that  $n_f / R = t_{frame}$  is small relative to  $t_{prop}$ .

# Stop-and-Wait ARQ (cont.)

## Example [ impact of delay-bandwidth product ]

$$\left. \begin{array}{l} n_f = 1250 \text{ bytes} = 10000 \text{ bits} \\ n_{ACK} = n_{header} = 25 \text{ bytes} = 200 \text{ bits} \end{array} \right\} \Rightarrow \frac{n_{ACK}}{n_f} = \frac{n_{header}}{n_f} = 0.02$$

$$\eta_{SW} = \frac{R_{eff}}{R} = \frac{1 - \frac{n_{header}}{n_f}}{1 + \frac{n_{ACK}}{n_f} + \frac{2 \cdot (t_{prop} + t_{proc})R}{n_f}} = \frac{0.98}{1.02 + \frac{2 \cdot (t_{prop} + t_{proc})R}{n_f}}$$

Efficiency	200 km ( $t_{prop} = 1 \text{ ms}$ )	2000 km ( $t_{prop} = 10 \text{ ms}$ )	20000 km ( $t_{prop} = 100 \text{ ms}$ )	200000 km ( $t_{prop} = 1 \text{ sec}$ )
1 Mbps	$10^3$ 88%	$10^4$ 49%	$10^5$ 9%	$10^6$ 1%
1 Gbps	$10^6$ 1%	$10^7$ 0.1%	$10^8$ 0.01%	$10^9$ 0.001%

Stop-and-Wait does NOT work well for very high speeds or long propagation delays.

## Stop-and-Wait Efficiency in Channel with Errors

- $P_f$  = probability that transmitted frame has errors and need to be retransmitted

- $(1-P_f)$  – probability of successful transmission

- $\frac{1}{1-P_f}$  – average # of (re)transmission until first correct arrival

and including

- total delay per frame:

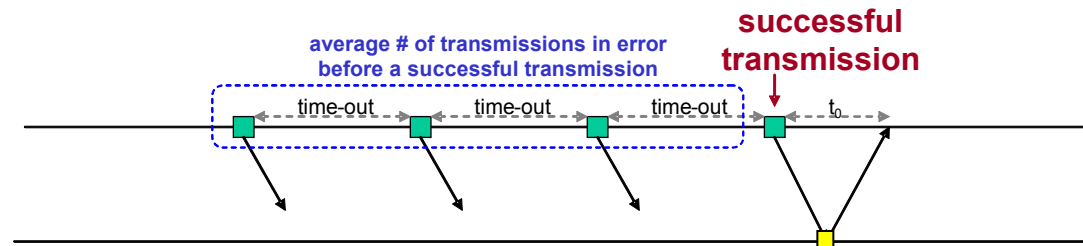
$$t_0 \cdot (\text{average \# of retrans.}) = t_0 \cdot \frac{1}{1-P_f}$$

$$\eta_{SW\_error} = \frac{R_{eff\_error}}{R} = \frac{\frac{n_f - n_{header}}{t_0} (1-P_f)}{R} = (1-P_f) \cdot \frac{1 - \frac{n_{header}}{n_f}}{1 + \frac{n_{ACK}}{n_f} + \frac{2(t_{prop} + t_{proc})R}{n_f}} \quad (*)$$

$$\eta_{SW\_error} = (1-P_f) \cdot \eta_0$$

$P_f$  increases  $\Rightarrow \eta_{sw}$  decreases

# Stop-and-Wait ARQ (cont.)



**Probability that  $i$  transmission are needed to deliver frame successfully**  
 (  $i-1$  transmission in error and the  $i^{\text{th}}$  transmission is error free ):

$$P[\text{\# of trans. in error} = i-1] = (1-P_f) P_f^{i-1}$$

$$\begin{aligned} E[\text{\# of transmissions in error}] &= \sum_{i=1}^{\infty} (i-1) \cdot P[n_{\text{trans in error}} = i-1] = \sum_{i=1}^{\infty} (i-1) \cdot (1-P_f) P_f^{i-1} = \\ &= (1-P_f) \cdot \sum_{i=1}^{\infty} (i-1) \cdot P_f^{i-1} = (1-P_f) \cdot \sum_{n=1}^{\infty} n \cdot P_f^n = \\ &= (1-P_f) \cdot P_f \cdot \sum_{n=1}^{\infty} n \cdot P_f^{n-1} = (1-P_f) \cdot P_f \cdot \frac{1}{(1-P_f)^2} = \\ &= \frac{P_f}{1-P_f} \end{aligned}$$

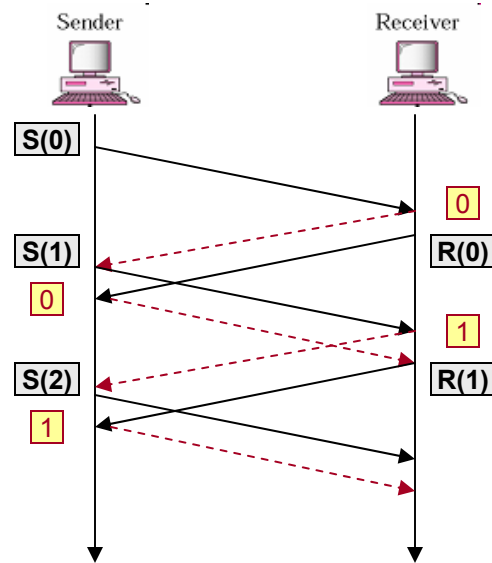
**Total average delay per frame:**

$$t_0 + \text{time - out} \cdot E[\text{\# of transmiss in error}] = t_0 + \text{time - out} \cdot \frac{P_f}{1-P_f} \approx \frac{1}{1-P_f} t_0$$

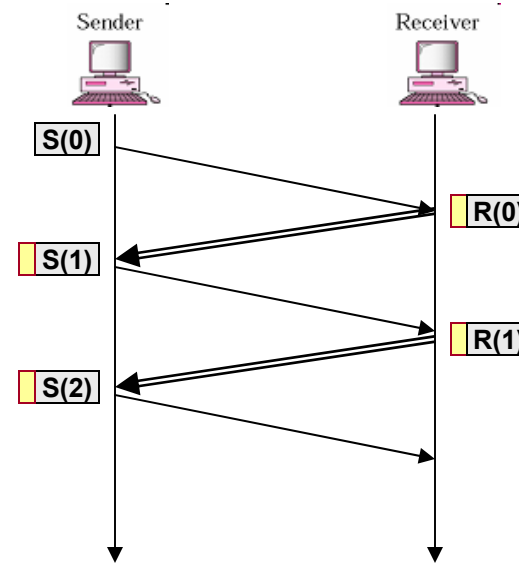


## Piggybacking

- Stop-and-Wait discussed so far was ‘unidirectional’
- in ‘**bidirectional**’ communications, both parties send & acknowledge data, i.e. **both parties implement flow control**
- **piggybacking method: outstanding ACKs are placed in the header of information frames**
- piggybacking can save bandwidth since the overhead from a data frame and an ACK frame (addresses, CRC, etc) can be combined into just one frame



without piggybacking



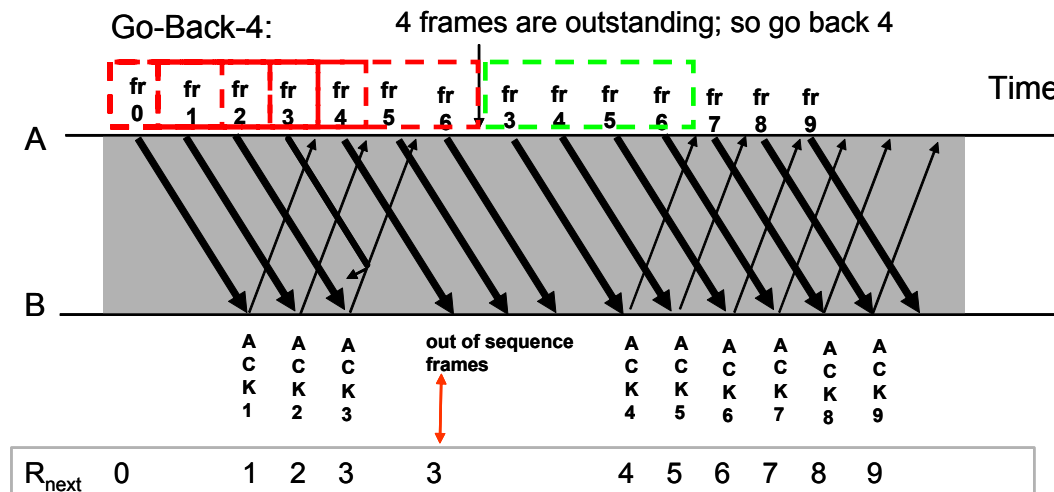
with piggybacking

## **(2) Go-Back-N ARQ**

# Go-Back-N ARQ

**Go-Back-N ARQ** – overcomes inefficiency of Stop-and-Wait ARQ – sender continues sending enough frames to keep channel busy while waiting for ACKs

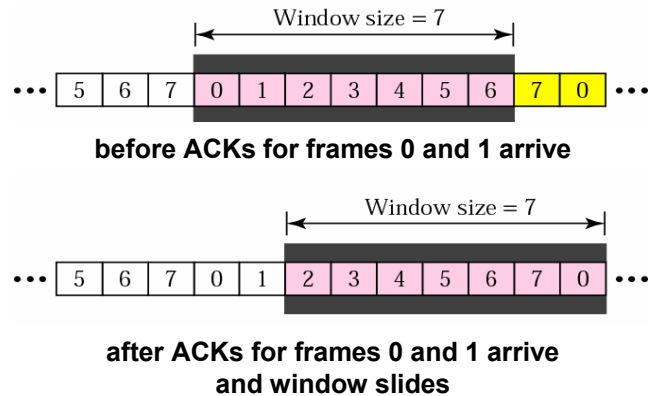
- a window of  $W_s$  outstanding frames is allowed
- **m-bit sequence numbers** are used for both - frames and ACKs, and  $W_s = 2^m - 1$



Assume:  $W_s = 4$

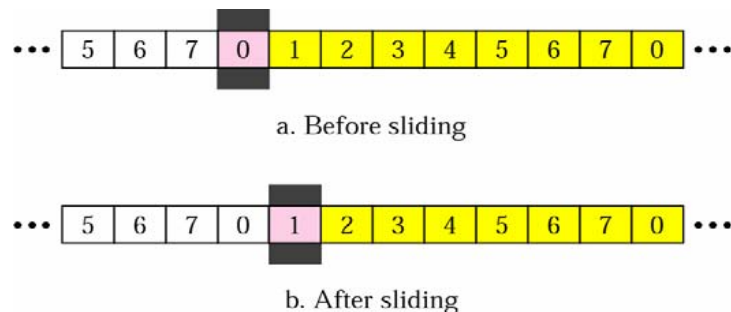
- 1) **sender** sends frames one by one
- 2) frame 3 undergoes transmission error – **receiver** ignores frame 3 and all subsequent frames
- 3) **sender** eventually reaches max number of outstanding frames, and takes following action:
  - go back  $N=W_s$  frames and retransmit all frames from 3 onwards

## Sender Sliding Window



- all frames are stored in a buffer, outstanding frames are enclosed in a window
  - frames to the left of the window are already ACKed and can be purged
  - frames to the right of the window cannot be sent until the window slides over them
  - whenever a new ACK arrives, the window slides to include new unsent frames
  - once the window gets full (max # of outstanding frames is reached), entire window gets resent

## Receiver Sliding Window

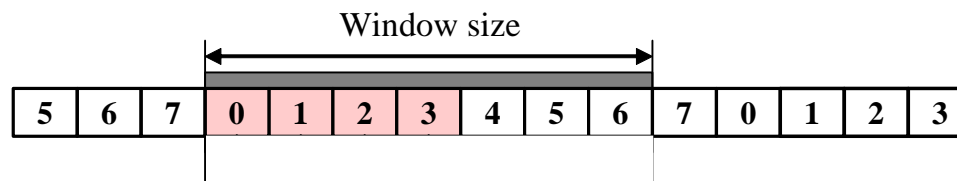


- the size of receiver window is always 1
  - receiver is always looking for a specific frame to arrive in a specific order
  - any frame arriving out of order is discarded and needs to be resent

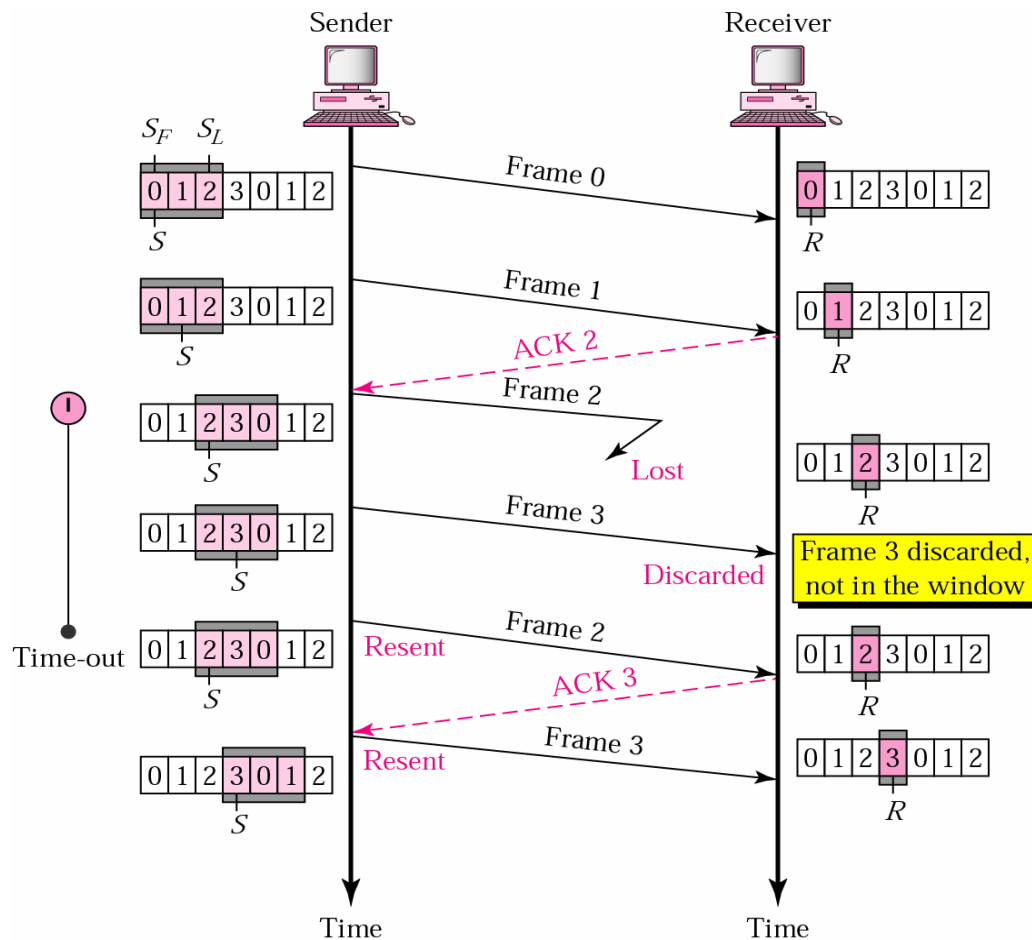
The complexity of the receiver in Go-Back-N is the same as that of Stop-and-Wait!!!  
 Only the complexity of the transmitter increases.

## Problems with Go-Back-N (Go-Back-N with Timeout)

- Go-Back-N works correctly (retransmission of damaged frames gets triggered) as long as the sender has an unlimited supply of packets that need to be transmitted
  - but, in case when **packets arrive sporadically**, there may not be  $W_s - 1$  subsequent transmissions  $\Rightarrow$  window will not be exhausted, retransmissions will not be triggered
  - this problem can be resolved by modifying Go-Back-N such that:
    - 1) set a timer for each sent frame
    - 2) **resend all outstanding frames either when window gets full or when the timer of first frame expires**



## Example [ lost frame in Go-Back-N with time-out ]

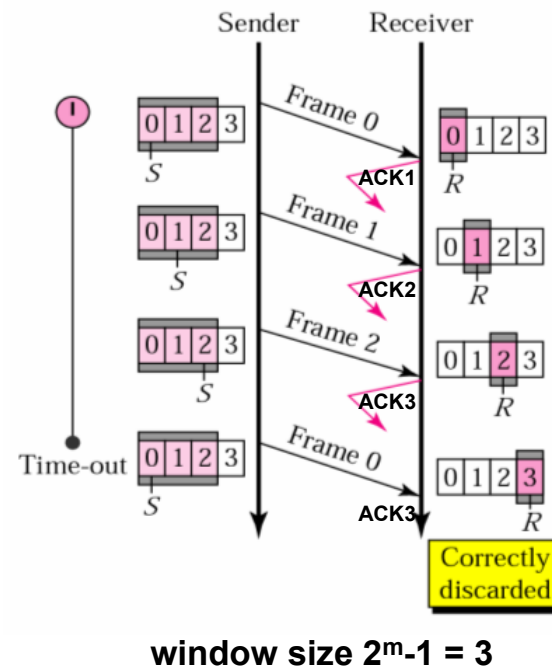
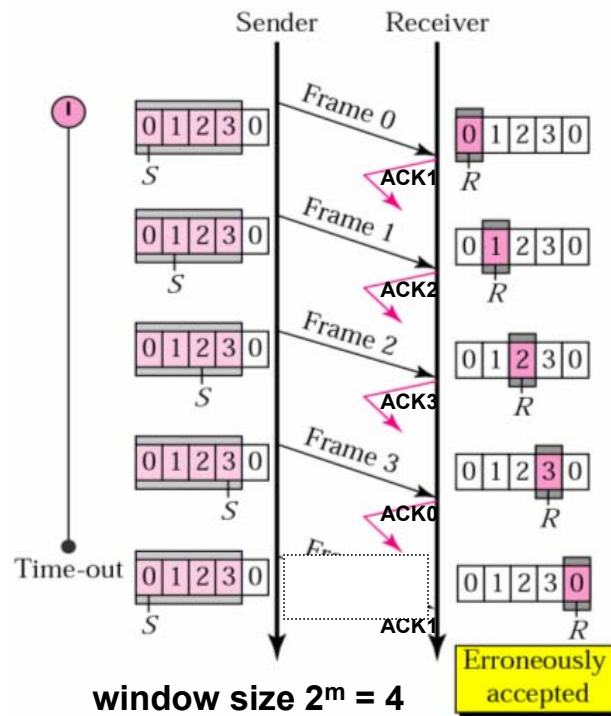


### Note:

- **ACKs number always defines the number of the next expected frame !!!**
- in Go-Back-N, receiver does not have to acknowledge each frame received – it can send one cumulative ACK for several frames

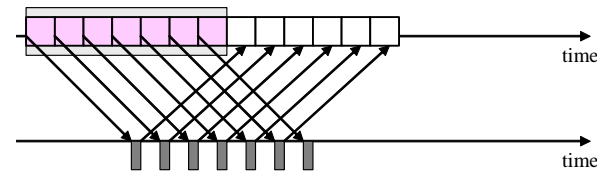
## Sequence Numbers and Window Size

- m bits allotted within a header for seq. numbers  
 $\Rightarrow 2^m$  possible sequence numbers
- how big should the sender window be!?
- $W > 2^m$  cannot be accepted – multiple frames with same seq. # in the window  $\Rightarrow$  ambiguous ACKs
- $W = 2^m$  can still cause some ambiguity – see below
- **$W = 2^m - 1$  acceptable !!!**



## Go-Back-N Efficiency

- completely efficient if  $W_s$  is large enough to keep channel busy, and if channel is error free



- in case of error-prone channel, with  $P_f$  frame loss probability, **time to deliver a frame is:**

- $t_{\text{frame}}$  - if 1<sup>st</sup> transmission succeeds – prob.  $(1-P_f)$
  - $t_{\text{frame}} + \frac{1}{1-P_f} \cdot W_s \cdot t_{\text{frame}}$  - if 1<sup>st</sup> transmission does NOT succeed – prob.  $P_f$

average # of frame/window (re)transmission until a successful transmission

- total average time required to transmit a frame:**

$$t_{\text{GBN}} = (1-P_f) \cdot t_{\text{frame}} + P_f \cdot \left( t_{\text{frame}} + \frac{1}{1-P_f} \cdot W_s \cdot t_{\text{frame}} \right) = t_{\text{frame}} + \frac{P_f}{1-P_f} \cdot W_s \cdot t_{\text{frame}}$$

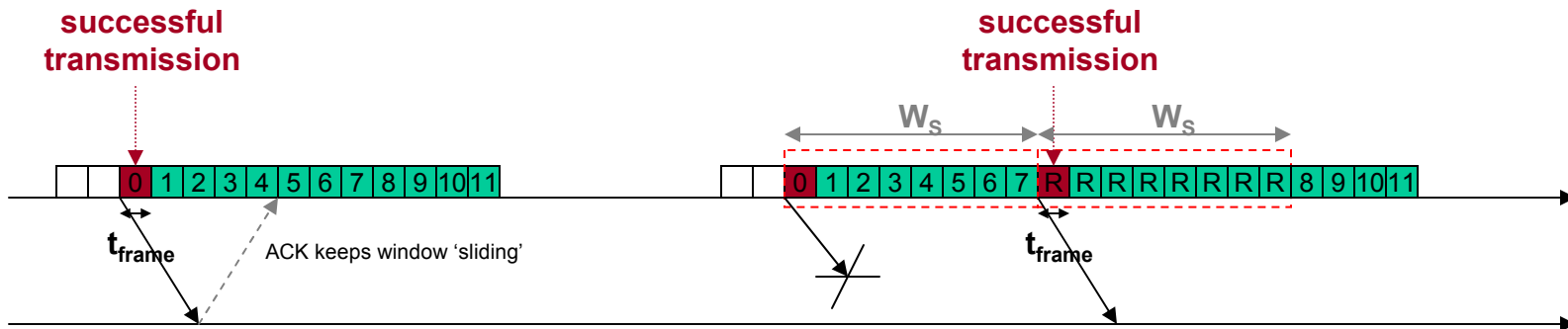
- transmission efficiency**

$$\eta_{\text{GBN}} = \frac{n_f - n_{\text{header}}}{t_{\text{GBN}} \cdot R} = \frac{1 - \frac{n_{\text{header}}}{n_f}}{1 + (W_s - 1)P_f} (1 - P_f)$$

(\*\*)



What is total average time required to transmit a frame, assuming  $P_f$ ?



1<sup>st</sup> attempt successful:  $t_{GBN} = t_{frame}$

2<sup>nd</sup> attempt successful:  $t_{GBN} = t_{frame} + W_s \cdot t_{frame}$

average case:  $t_{GBN} = t_{frame} + E[\# \text{ of transmissions in error}] \cdot W_s \cdot t_{frame}$

$$E[\# \text{ of transmissions in error}] = \frac{P_f}{1 - P_f}$$

$$t_{GBN} = t_{frame} + \frac{P_f}{1 - P_f} W_s \cdot t_{frame}$$

$\Rightarrow$

$$\eta_{GBN} = \frac{n_f - n_{header}}{R} = \frac{1 - \frac{n_{header}}{n_f}}{1 + (W_s - 1)P_f} (1 - P_f)$$

## Example [ Stop-and-Wait vs. Go-Back-N ]

$n_f = 1250 \text{ bytes} = 10000 \text{ bits}$

$n_{ACK} = n_{header} = 25 \text{ bytes} = 200 \text{ bits}$

Compare S&W with GBN efficiency for random bit errors with  $p_b = 0, 10^{-6}, 10^{-5}, 10^{-4}$  and bandwidth-delay product  $R \cdot 2 \cdot (t_{prop} + t_{proc}) = 1 \text{ Mbps} \cdot 100 \text{ ms} = 100000 \text{ bits} = 10 \text{ frames} \rightarrow$  use  $W_s = 11$ .

Efficiency	$p_b=0$	$p_b=10^{-6}$	$p_b=10^{-5}$	$p_b=10^{-4}$
S&W	8.9%	8.8%	8.0%	3.3%
GBN	98%	88.2%	45.4%	4.9%

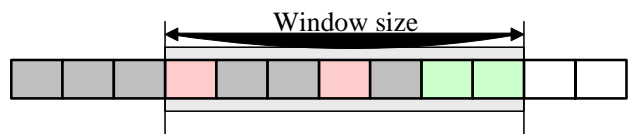
- Go-Back-N provides significant improvement over Stop-and-Wait for large delay-bandwidth product
- Go-Back-N becomes inefficient as error rate increases

## **(3) Selective Repeat ARQ**

# Selective Repeat ARQ

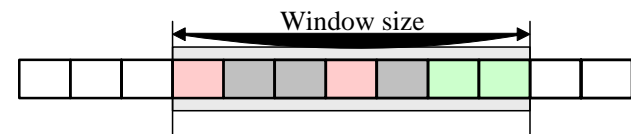
## Selective Repeat ARQ

- Go-Back-N is NOT suitable for 'noisy links' – in case of a lost/damaged frame a whole window of frames need to be resent
  - excessive retransmissions use up the bandwidth and slow down transmission
- Selective Repeat ARQ overcomes the limitations of Go-Back-N by adding 2 new features
  - (1) receiver window > 1 frame, so that out-of-order but error-free frames can be accepted
  - (2) retransmission mechanism is modified – only individual frames are retransmitted
- Selective Repeat ARQ is used in TCP !!!



Legend for sender window:  
Grey: already ACKed  
Red: sent, not yet ACKed  
Green: usable not yet sent  
White: not usable

sender window of size  $W_s$



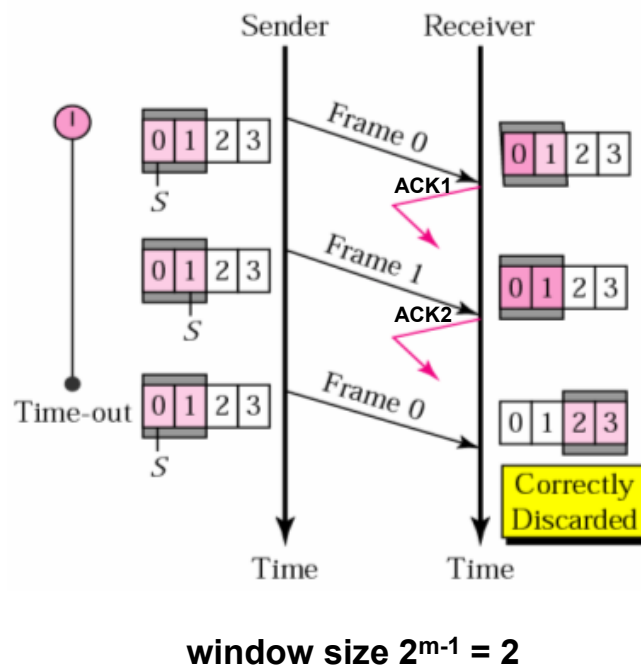
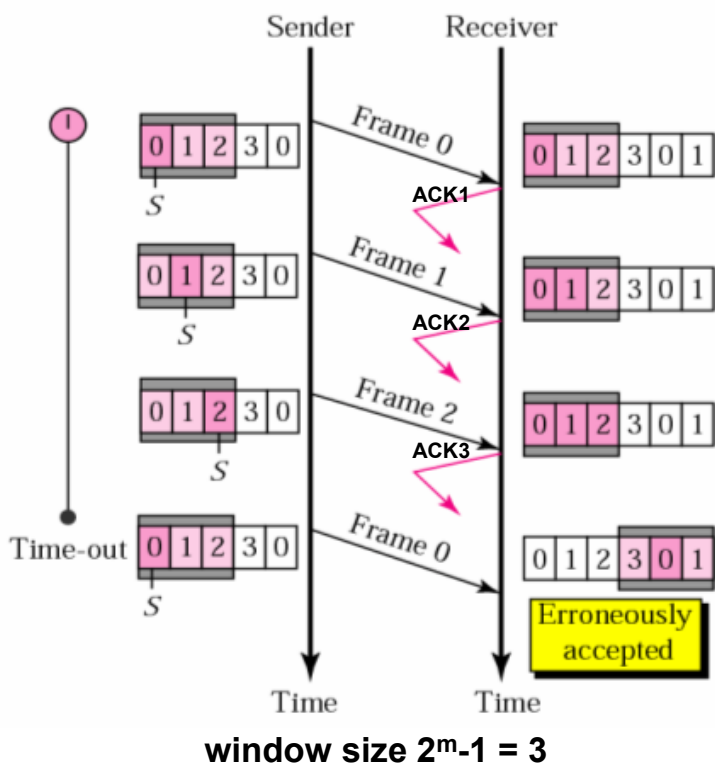
Legend for receiver window:  
Grey: out of order buffered but already ACKed  
Red: expected, not yet received  
Green: acceptable (within window)  
White: not usable

receiver window of size  $W_r$



## Window Sizes ( $W_S$ and $W_R$ )

- $m$  bits allotted within a header for sequence numbers  $\Rightarrow 2^m$  possible sequence numbers
- how big should the windows be!?
- $W_S$  and  $W_R = 2^m - 1$  cannot be accepted due to possible ambiguity as shown below
- $W = 2^m / 2 = 2^{m-1}$  acceptable !!!



## Selective Repeat Efficiency

- completely efficient if  $W_s$  is large enough to keep channel busy, and if channel is error free
  - of course, sequence number space must be 2X sequence number space of Go-Back-N
- in case of error-prone channel, total average time required to transmit a frame:

$$t_{SR} = \frac{t_{frame}}{1 - P_f} = \frac{n_f}{R \cdot (1 - P_f)}$$

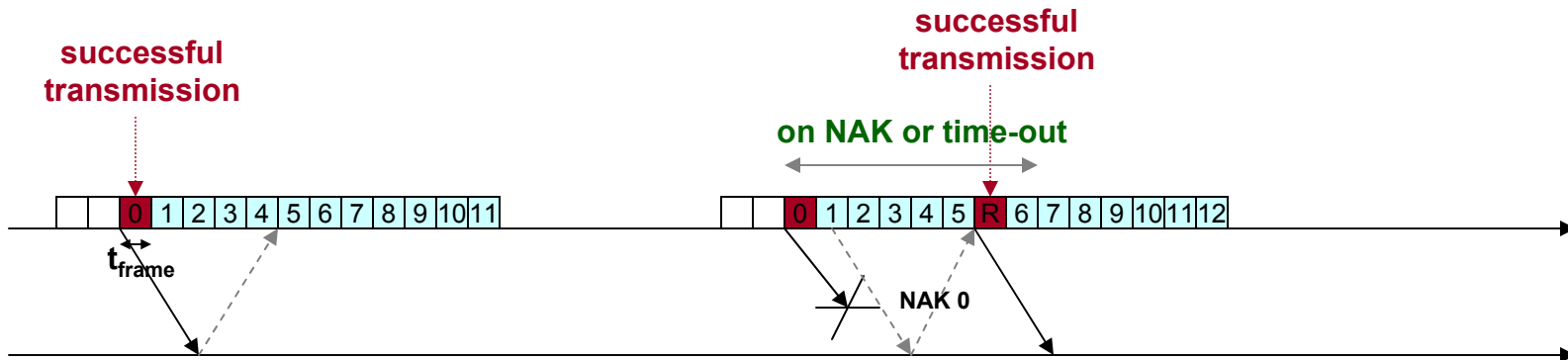
- transmission efficiency

$$\eta_{SR} = \frac{R_{eff}}{R} = \frac{n_f - n_{header}}{R \cdot t_{SR}} = \left(1 - \frac{n_{header}}{n_f}\right) \cdot (1 - P_f)$$

(\*\*\*)

# Selective Repeat ARQ (cont.)

What is total average time required to transmit a frame, assuming  $P_f$ ?



1<sup>st</sup> attempt successful:  $t_{\text{SR}} = t_{\text{frame}}$

2<sup>nd</sup> attempt successful:  $t_{\text{SR}} = t_{\text{frame}} + t_{\text{frame}}$

average case:  $t_{\text{SR}} = t_{\text{frame}} + \text{E}[\# \text{ of transmissions in error}] \cdot t_{\text{frame}}$

$$\frac{P_f}{1 - P_f}$$

$$t_{\text{SR}} = t_{\text{frame}} + \frac{P_f}{1 - P_f} \cdot t_{\text{frame}} = \frac{1}{1 - P_f} \cdot \frac{n_f}{R}$$

$\Rightarrow$

$$\eta_{\text{SR}} = \frac{\frac{n_f - n_{\text{header}}}{R}}{t_{\text{SR}}} = \left(1 - \frac{n_{\text{header}}}{n_f}\right) (1 - P_f)$$



# Stop-and-Wait vs. Go-Back-N vs. Selective Repeat 33

## Performance Comparison

- assume  $n_{ACK}$  and  $n_{header}$  are negligible relative to  $n_f$ , and

$$\frac{2(t_{prop} + t_{proc})R}{n_f} = L = W_s - 1$$

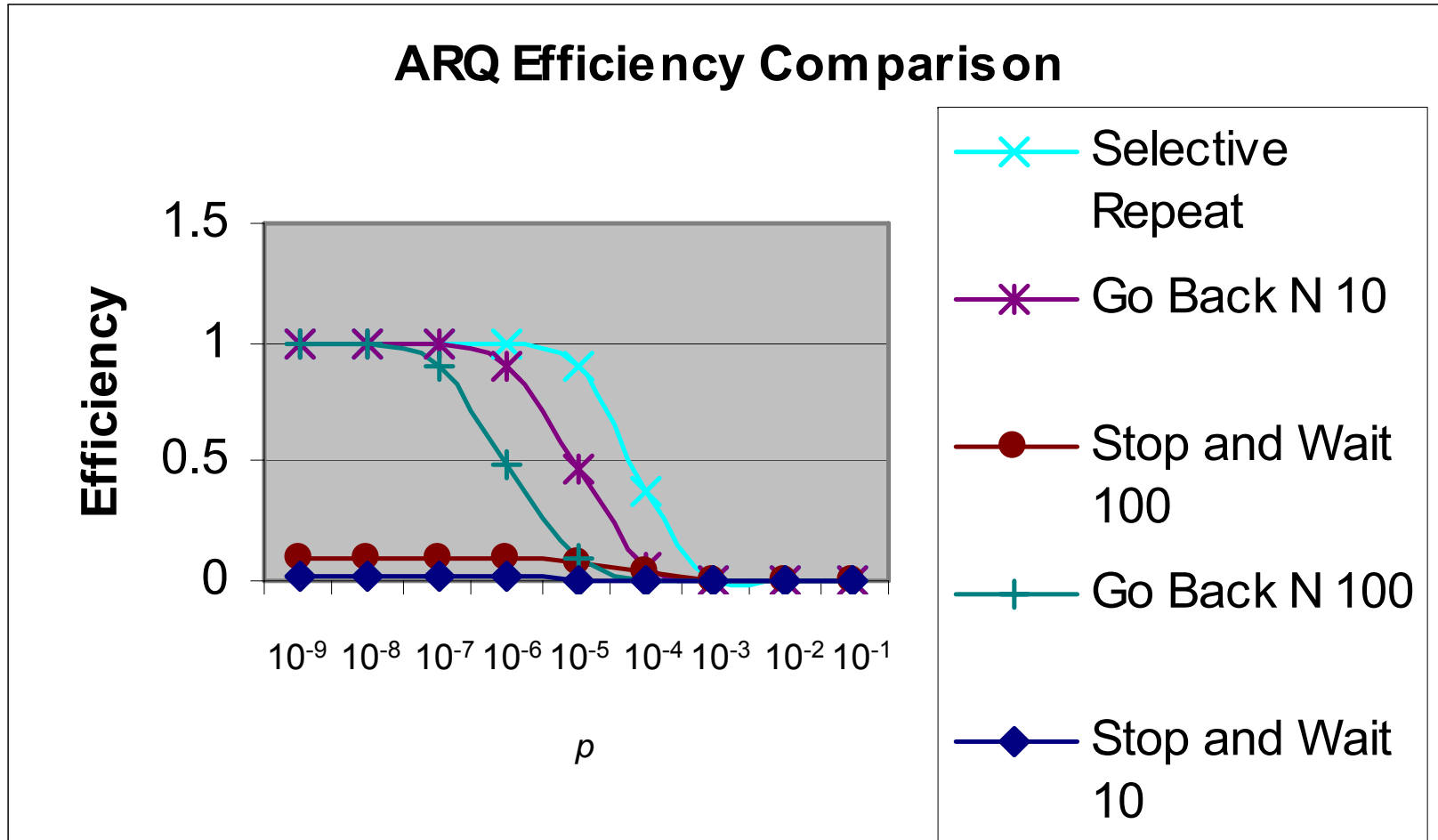
←  $W_s$  is for 1 less than the number of frames currently in transit

size of the "pipe" in multiples of frames

- efficiencies of three ARQ techniques are

$$\left. \begin{aligned} \eta_{SW} &= \frac{1}{1+L} \cdot (1-P_f) \\ \eta_{GBN} &= \frac{1}{1+LP_f} (1-P_f) \\ \eta_{SR} &= (1-P_f) \end{aligned} \right\} \eta_{SW} < \eta_{GBN} < \eta_{SR}$$

- for  $0 < P_f < 1$ , Selective Repeat provides best performance
- for  $P_f \rightarrow 0$  Go-Back-N as good as Selective Repeat



Delay-Bandwidth product = 10, 100