5. The centroid is given as

$$\text{Centroid} = \frac{\text{Sum of poles} - \text{Sum of zeros}}{P - Z} = \frac{(0-1-5)-(-1.5)}{3-1} = -2.25$$

6. **Break-away points:**
   The C.E. of the system is given as

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K(s+1.5)}{s(s+1)(s+5)} = 0$$

$$K = \frac{s(s^2+6s+5)}{s+1.5}$$

$$\frac{dK}{ds} = 0 \Rightarrow s^3 + 5.25s^2 + 9s + 3.75 = 0$$

$$\Rightarrow \quad s = 0.6,\ s = 2.3 + j0.89,\ s = -2.3 - j0.89.$$

   The valid break-away point is $B_1 = -0.6$

7. **Angle of departure:**
   Since there is no complex pole and zero, we need not find the angle of departure or arrival.

8. **Intersection points of the root locus branches with imaginary axis:**
   The C.E. of the system is given as

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K(s+1.5)}{s(s+1)(s+5)} = 0$$

$$\Rightarrow \quad s(s^2 + 6s + 5) + K(s + 1.5) = 0$$

$$\Rightarrow \quad s^3 + 6s^2 + (5 + K)s + 1.5K = 0$$

s^3	1	$5+K$	0
s^2	6	$1.5K$	0
s^1	$(30+4.5)/6$	0	0
s^0	$1.5K$	0	0

   For a stable system, $1.5K > 0$, $(30 + 4.5K)/6 > 0$

$$\Rightarrow \quad K > 0,\ K < -6.67$$

   Since the value of K is negative, there is no crossing point on the imaginary axis. In other words for any positive value of K, the root locus will not cross the imaginary axis.
   The complete root locus plot is as shown in Figure (m).

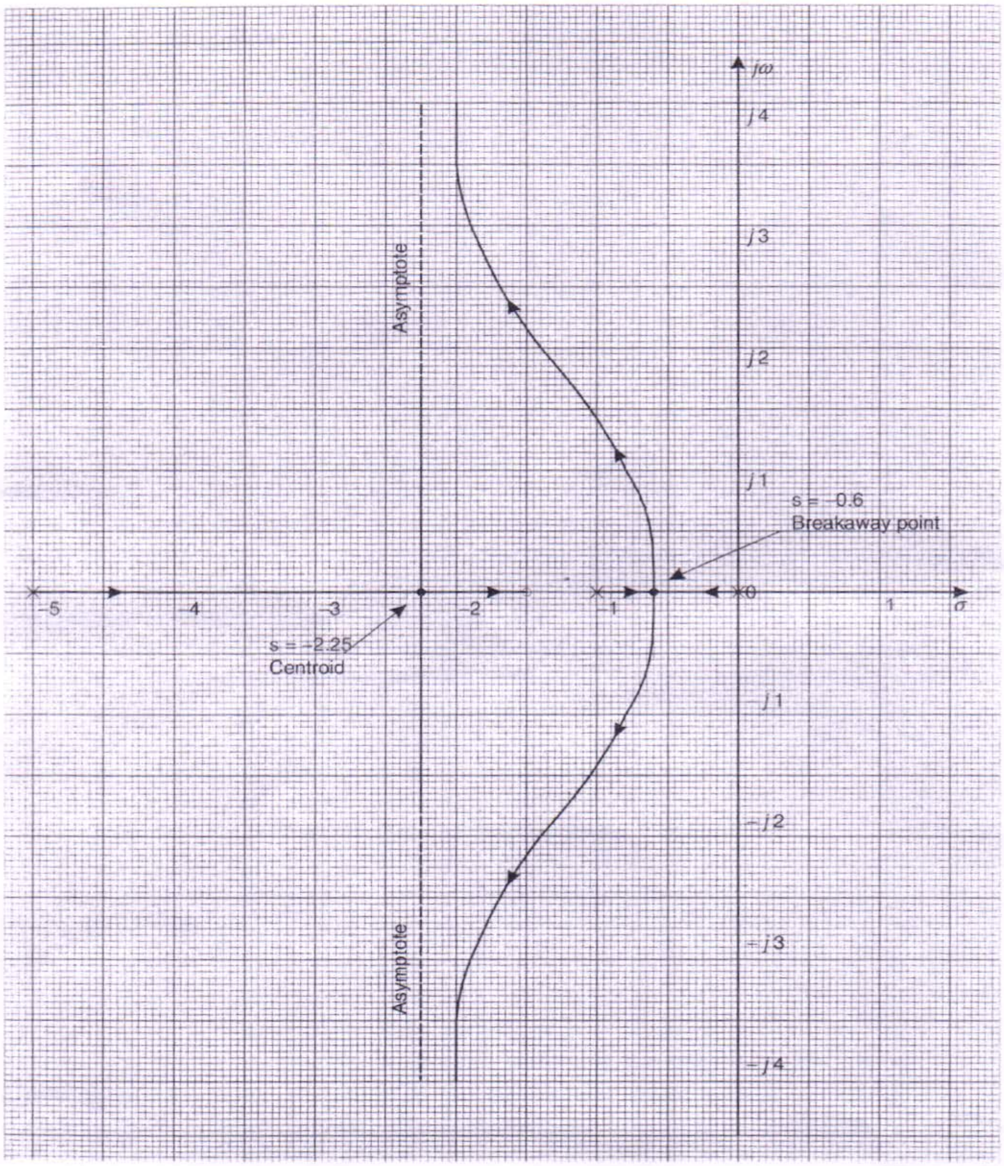

Figure (m) Root locus plot for ASP-7

ASP-8: Sketch the root locus plot of a unity feedback system whose OLTF is $G(s) = \dfrac{s}{(s^2+4)(s+2)}$.

Solution:

Given that

$$G(s) = \frac{s}{(s^2+4)(s+2)}.$$

1. Root locus is symmetrical about real axis.
2. The root locus plot starts from a pole at $K = 0$ and terminates at an open-loop zero, that is, at $K = \infty$.
3. The poles are given as
   $s = -2, s^2 + 4 = 0$
   $P_1 = -2, P_2 = j2, P_3 = -j2,$
   The zeros are given as
   $s = 0$
   Number of poles = $P = 3$
   Number of zeros = $Z = 1$
   Number of root locus branches = $N = P = 3$
   Number of asymptotic lines = $n = P - Z = 2$
4. The angle of asymptotic lines with negative real axis

$$\phi = \frac{(2q+1)180}{P-Z}, \text{ where } q = 0, 1$$
$$= 180/2, 3 \times 180/2$$
$$= 90, 270.$$

5. The centroid is given as

$$\text{Centroid} = \frac{\text{Sum of poles} - \text{Sum of zeros}}{P-Z} = \frac{(0-2-j2+j2)-(0)}{3-1} = -1.$$

The complete root locus is as shown in the below figure.

ASP-9: Sketch the root locus plot of the system whose OLTF is given as $G(s)H(s) = \frac{K}{s(s+4)(s^2+8s+32)}$.

Hence, find the value of K so that the system has a damping factor 0.707.

Solution:

Given that

$$G(s)H(s) = \frac{K}{s(s+4)(s^2+8s+32)}.$$

1. Root locus is symmetrical about real axis.
2. The root locus plot starts from a pole at $K = 0$ and terminates at an open-loop zero, that is, at $K = \infty$.
3. The poles are given as
   $s = 0, s = -4, s^2 + 8s + 32 = 0$
   $P_1 = 0, P_2 = -4, P_3 = -4 + j4, P_4 = -4 - j4,$
   Number of poles = $P = 4$
   Number of zeros = $Z = 0$
   Number of root locus branches = $N = P = 4$
   Number of asymptotic lines = $n = P - Z = 4$
4. The angle of asymptotic lines with negative real axis

$$\phi = \frac{(2q+1)180}{P-Z}, \text{ where } q = 0, 1, 2, 3$$
$$= 180/4, 3 \times 180/4, 5 \times 180/4, 7 \times 180/4$$
$$= 45, 135, 225, 315.$$

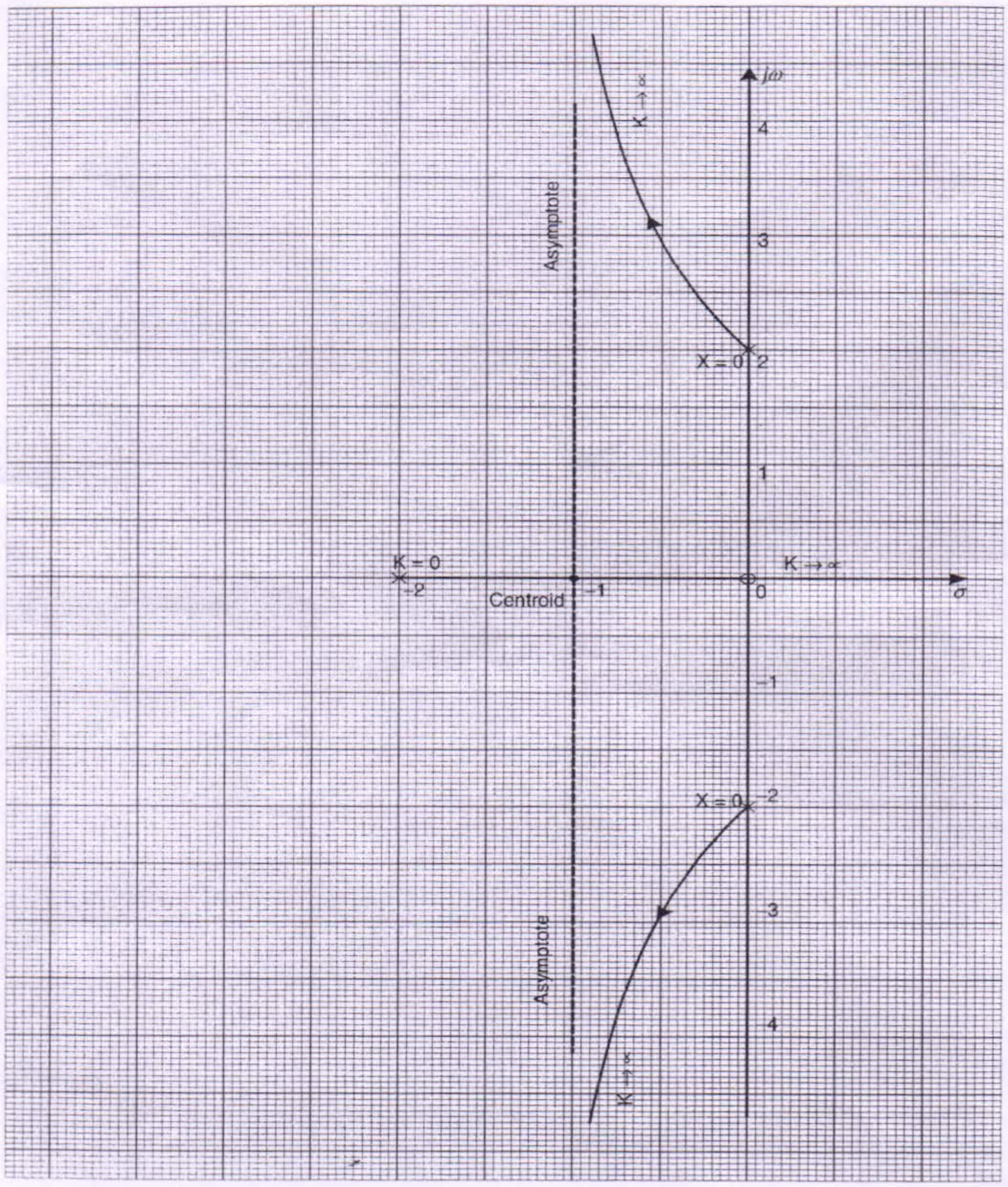

Figure (n) Root locus for ASP-9

5. The centroid is given as

$$\text{Centroid} = \frac{\text{Sum of poles} - \text{Sum of zeros}}{P - Z} = \frac{(0 - 4 - 4 + j4 - 4 - j4) - (0)}{4 - 0} = -3.$$

The complete root locus is as shown in Figure (n).

6. **Break-away points:**
   The C.E. of the system is given as

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K}{s(s+4)(s^2+8s+32)} = 0$$

$$K = -s(s+4)(s^2+8s+32)$$

$$K = -(s^4 + 12s^3 + 64s^2 + 128s)$$

$$\frac{dK}{ds} = 0 \Rightarrow 4s^3 + 6s^2 + 128s + 128 = 0$$

$$\Rightarrow \quad s^3 + 9s^2 + 32s + 32 = 0.$$

   After solving the above-mentioned equation, the valid break-away point is $B_1 = -1.57$.
7. **Intersection points of the root locus branches with imaginary axis:**
   The C.E. of the system is given as

$$1 + G(s)H(s) = 0$$

$$s^4 + 12s^3 + 64s^2 + 128s + K = 0$$

s^4	1	64	K
s^3	12	128	0
s^2	53.33	K	0
s^1	$128 - 0.225K$	0	0
s^0	K	0	0

   For a stable system, $K > 0$, $128 - 0.225K > 0$

$$\Rightarrow \quad 0 < K < 569.$$

   For a stable system, the maximum value of K is 569. For $K > 569$, the roots lie on the RHS of the s-plane, and hence, $K = 569$ is the value for marginal stability.

   The auxiliary equation is $53.33s^2 + K = 0$

$$\Rightarrow \quad 53.33s^2 + 569 = 0$$

$$\Rightarrow \quad s = \pm j\,3.266.$$

8. **Angle of departure:**

$$\phi_{p3-(-4+j4)} = 180 - (\phi_{p1} + \phi_{p2} + \phi_{p4})$$

$$= 180 - (135 + 90 + 90) = -135^\circ$$

$$\phi_{p4-(-4-j4)} = +135^\circ$$

   The complete root locus plot is as shown in Figure (o).

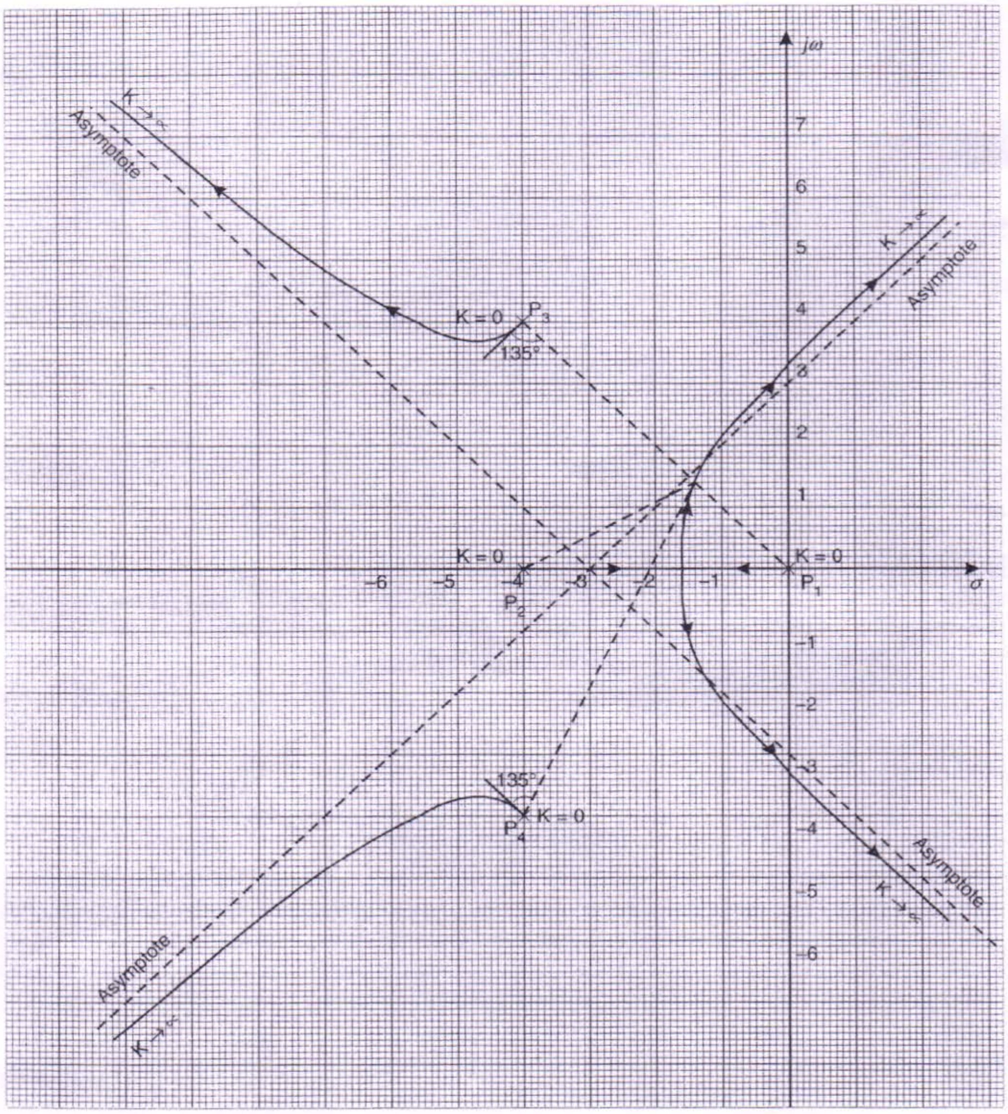

Figure (o) Root locus plot for ASP-9

Since $\xi = 0.707 \Rightarrow \cos\theta = \xi = 0.707 \Rightarrow \theta = 45°$

From the origin, a line at an angle $\theta = 45°$ is drawn, which intersects the root locus plot at $s = -1.35 + j1.35$. As this point lies on the root locus, the following condition is satisfied.

$$|G(s)H(s)| = 1$$

$$\left|\frac{K}{s(s+4)(s^2+8s+32)}\right| = 1$$

$$\left|\frac{K}{(-1.35+j1.35)(-1.35+j1.35+4)[(-1.35+j1.35)^2+8(-1.35+j1.35)+32]}\right|=1$$

$$\Rightarrow \qquad \frac{k}{130}=1 \Rightarrow K=130.$$

ASP-10: The OLTF of a unity feedback system is $G(s)=\dfrac{K(s+a)}{s(s+b)}$.

1. Prove that break-away and break-in points will exist only when $|a|>|b|$.
2. Prove that the complex points on the root locus form a circle with centre $(-a, 0)$ and radius $\sqrt{a^2-ab}$.

Solution:

Given that

$$G(s)H(s)=\frac{K(s+a)}{s(s+b)}.$$

There are two open-loop poles, and hence, the number of branches are 2 and start at $P_1=0$ and $P_2=-b$, respectively. One of the branches terminates at infinity and other terminates at zero $Z_1=-a$.

1. The C.E. is given as

$$1+G(s)H(s)=0$$

$$1+\frac{K(s+a)}{s(s+b)}=0$$

$$\Rightarrow \qquad K=-\frac{s(s+b)}{s+a} \qquad (1)$$

$$\frac{dK}{ds}=0 \Rightarrow s^2+2as+ab=0$$

$$\Rightarrow \qquad s=-a\pm\sqrt{a^2-ab}. \qquad (2)$$

Substituting the value of s in (1), we will get

$$K=-\frac{\left[-a+\sqrt{a(a-b)}\right]^2+b\left[-a+\sqrt{a(a-b)}\right]}{-a+\sqrt{a(a-b)}+a}$$

$$=\frac{a^2+a(a+b)-2a\sqrt{a(a-b)}+\left[-ab+\sqrt{a(a-b}\right]}{\sqrt{a(a-b)}}$$

$$=-\left[2\sqrt{a(a-b)}+b-2a\right].$$

From the above-mentioned equation of K, it is clear that for all values of a and b such that $|a|>|b|$, K is positive. Therefore, the break-away or break-in points will exists only when $|a|>|b|$.

1. If $|a|>|b|$, then break-away and break-in points exist. Let us choose a test point in the complex plane $s=x+jy$ that lies on the root locus. This point should satisfy angle criterion so that the point is a part of the root locus.

$$G(s)H(s)_{s=x+jy}=\frac{K(x+jy+a)}{(x+jy)(x+jy+b)}.$$

The phase angle is given as

$$\angle G(s)H(s)_{s=-x+jy} = \tan^{-1}\frac{y}{x+a} - \tan^{-1}\frac{y}{x} - \tan^{-1}\frac{y}{x+b} = -180^\circ$$

$$180 + \tan^{-1}\frac{y}{x+a} = \tan^{-1}\frac{y}{x} + \tan^{-1}\frac{y}{x+b}$$

By taking 'tan' on both sides, we have

$$\frac{\frac{y}{x} + \frac{y}{x+b}}{1 - \frac{y}{x}\times\frac{y}{x+b}} = \frac{\tan 180 + \frac{y}{x+a}}{1 - \tan 180 \times \frac{y}{x+a}}.$$

After simplifying the above-mentioned equation, we have

$$x^2 + 2xa + y^2 = -ab$$

By adding both sides a^2, we get

$$(x+a)^2 + y^2 = (a^2 - ab)$$

$$(x+a)^2 + (y+0)^2 = \left[\sqrt{a^2 - ab}\right]^2.$$

This is equation of a circle with centre $(-a, 0)$ and radius $\sqrt{a^2 - ab}$. Hence, it is proved.

REVIEW QUESTIONS

1. What do you understand by root locus plot?
2. Explain the constructional procedure of root locus plot.
3. Write a short note on angle and magnitude condition of root locus.
4. Explain the terms:
   (i) asymptotes (ii) centroid
   (iii) break-away points (iv) angle of departure
5. Define the term 'root locus' and state the rule for finding out the root locus on the real axis.
6. Explain the method of calculating the break-away points.
7. What is a root locus plot? Explain with a suitable example.
8. What are the features of root locus plot?
9. Show that the break-away and break-in points, if any, on the real axis for the root locus for $G(s)H(s) = \frac{KN(s)}{D(s)}$, where $N(s)$ and $D(s)$ are rational polynomials in S, and that they can be obtained by solving the equation $\frac{dK}{ds} = 0$.
10. Explain the stability of the system from the root locus plot in the following situations with suitable examples:
    (a) addition of open-loop poles.
    (b) addition of open-loop zeros.
11. What is break-away and break-in point? How are they determined?
12. What is centroid? How the centroid is calculated?
13. Open-loop t.f. of a unity feedback system is $G(s) = \frac{K(s+a)}{s(s+b)}$
    (a) Prove that break-away and break-in points will exist only when $|a| > |b|$
    (b) Prove that the complex points on the root locus form a circle with centre $(-a, 0)$ and radius $\sqrt{a^2 - ab}$

EXERCISE PROBLEMS

1. Check whether the point $s=-3+j5$ lies on the root locus of a system having $G(s)H(s)=\dfrac{K}{(s+1)(s+5)}$. Determine the corresponding value of K.
2. Sketch the root locus for the system having the OLTF, $G(s)H(s)=\dfrac{K(s+4)(s+5)}{(s+1)(s+3)}$.
3. A unity feedback system has an OLTF, $G(s)=\dfrac{K}{s^2(s+2)}$.
   1. By sketching a root locus plot, show that the system is unstable for all values of K.
   2. Add a zero to $s=-a(0\le a\le 2)$ and show that the addition of zero stabilises the system.
4. A feedback control system has an OLTF $G(s)H(s)=\dfrac{K}{s(s+3)(s^2+2s+2)}$. Find the root locus as K is varying from 0 to ∞.
5. Sketch the root locus for the system whose OLTF is given as $G(s)H(s)=\dfrac{K(s+4)(s+5)}{(s+3)(s+1)}$.
6. Sketch the root locus plot for all the values of K from 0 to ∞ for a negative feedback system whose OLTF is given as $G(s)H(s)=\dfrac{K(s+1)}{s^2(s+9)}$?
7. Sketch the root locus plot for all the values of K from 0 to ∞ for a negative feedback system whose OLTF $G(s)H(s)=\dfrac{K(s^2-4s+8)}{(s+1)(s-0.5)}$?
8. Using the root locus method, find gain (K) for the system having OLTF as $G(s)H(s)=\dfrac{K(s+2)(s+4)}{(s+1)(s+3)(s+5)}$ when $\xi=0.246$.
9. Consider a negative feedback system characterised by $G(s)H(s)=\dfrac{K(s+1)}{s(s+2)(s+3)(s+4)}$. Sketch the root locus plot for all values of K ranging from 0 to ∞. Further, find gain K for which the CLTF will have a pole on the real axis at −0.5.
10. Sketch the root locus plot for a closed-loop system having an OLTF $G(s)H(s)=\dfrac{K(s+1)(s+2)}{s(s-1)}$ for all values of K ranging from 0 to ∞. Find the range of values of K for closed-loop stability.

ANSWERS

1. Since $\phi=-180°$, the point $-3+j5$ lies on the root locus, $K=29$.
8. $K=33$
9. For closed-loop stability, $0<K<141$.
10. For closed-loop stability, $0.33<K<\infty$.

OBJECTIVE TYPE QUESTIONS

1. Intersection of root locus branches with imaginary axis can be determined by
   (a) Nyquist criterion (b) *R–H* criterion
   (c) Polar plot (d) Bode plot
2. The break-away point is determined by solving the equation
   (a) $\dfrac{dK}{ds}=0$ (b) $1+G(s)H(s)=0$
   (c) $\int\dfrac{dK}{ds}=0$ (d) $G(s)\,H(s)=0$

3. The angle of asymptotic lines making with negative real axis is given as
(a) $\frac{2k\pi}{P-Z}$ (b) $\frac{(2k+1)\pi}{P-Z}$ (c) $\frac{(k+1)\pi}{P-Z}$ (d) $\frac{(2k+1)\pi}{P-Z}\cdot\frac{\pi}{2}$
4. When the root locus separates at a point between two open-loop poles, the point is called
(a) critical point (b) crossover point (c) centroid (d) break-away point
5. If the root locus branches cross the imaginary axis, the system becomes
(a) marginally stable (b) conditionally stable
(c) unstable (d) None
6. The intersection of the root locus branch with the imaginary axis can determined by using the equation
(a) $G(s)\ H(s) = 0$ (b) $1 - G(s)\ H(s) = 0$
(c) $1 + G(s)\ H(s) = 0$ (d) $\frac{dK}{ds} = 0$
7. The asymptotes to root locus branches intersect on the real axis at a point given as
(a) $\frac{\sum \text{Poles} - \sum \text{Zeros}}{(P-Z)}$ (b) $\frac{\sum \text{Poles} + \sum \text{Zeros}}{(P-Z)}$
(c) $\frac{\sum \text{Poles} - \sum \text{Zeros}}{(P+Z)}$ (d) None
8. In loop t.f., zeros are the same as
(a) zeros of $G(s)H(s)$ (b) zeros of $1 + G(s)\ H(s)$
(c) poles of $G(s)H(s)$ (d) poles of $1 + G(s)H(s)$
9. The value of K at which the root locus crosses the imaginary axis makes the system
(a) stable (b) under-damped
(c) marginally stable (d) unstable
10. The starting points of the root locus are
(a) closed-loop poles (b) closed-loop zeros
(c) open-loop poles (d) open-loop zeros
11. The ending points of the root locus are
(a) closed-loop poles (b) closed-loop zeros
(c) open-loop poles (d) open-loop zeros
12. The number of root loci branches for the OLTF $G(s)H(s) = \frac{K(s+5)(s+6)}{s(s+1)(s+3)(s^2+5s+6)}$ is
(a) 2 (b) 3 (c) 4 (d) 5
13. If the root locus branches do not cross the imaginary axes, the system is
(a) conditionally stable (b) inherently stable
(c) marginally stable (d) unstable
14. A unity feedback system has an OLTF $G(s)H(s) = \frac{K}{s(s^2+4s+13)}$. The centroid of the asymptotes of the root locus plot lies at
(a) –4 (b) –4/3 (c) –13 (d) –2
15. The OLTF of an unity feedback is given as $G(s) = \frac{5}{s(s+1)(s+3)(s+4)}$. The number of asymptotes of the root locus plot that tend to infinity is given as
(a) 1 (b) 2 (c) 3 (d) 4
16. The OLTF of an unity feedback system is given as $G(s) = \frac{K}{s(s+1)(s+3)}$. The break-away point of the root locus plot is given as
(a) –0.45 (b) –0.523 (c) –0.7 (d) None

17. A unity feedback system has an OLTF $G(s) = \frac{K}{s(s^2 + 4s + 13)}$. The angle of asymptotes are given as

(a) 450, 1,350, 2,250 (b) 600, 1,800, 3,000
(c) 900, 1,800, 2,700 (d) None

18. The OLTF of a feedback system is $\frac{K}{s(s+6)(s^2+4s+13)}$. The four branches of root locus originate at

(a) $-2, -3, -1 + j4, -1 - j4$ (b) $-1, -2, -3 + j4, -3 - j4$
(c) $0, -6, -2 + j3, -2 - j3$ (d) None

19. When the open-loop poles are added to the system, ———

(a) system becomes oscillatory (b) settling time increases
(c) there is a reduction in the range of K (d) All

20. When the open-loop zeros are added to the system, ———

(a) stability of the system increases (b) settling time increases
(c) range of K increases (d) All

21. The number of root loci of a unity feedback system having m number of poles and n number of zeros is

(a) m (b) n (c) $m - n$ (d) $n - m$

22. The root locus plot of the system having the loop t.f. $G(s)H(s) = \frac{K}{s(s+4)(s^2+4s+13)}$ has ——— break-away points

(a) 0 (b) 1 (c) 2 (d) 3

23. If the C.E. of the system is $s^3 + 2s^2 + (K+1)s + 3K = 0$, then the centroid is

(a) 0.5 (b) −0.5 (c) 0.25 (d) −0.25

24. The OLTF of a feedback system is the angle of departure of the root locus at $s = -1 + j$

(a) 90 (b) −90 (c) 180 (d) −180

25. Root locus technique is applicable to

(a) single-loop system (b) multiple loop system
(c) both (d) one

ANSWERS

1. b	**2.** a	**3.** b	**4.** d	**5.** c	**6.** c	**7.** a	**8.** a	**9.** c	**10.** c
11. d	**12.** d	**13.** b	**14.** b	**15.** d	**16.** a	**17.** b	**18.** d	**19.** c	**20.** d
21. c	**22.** d	**23.** a	**24.** d	**25.** c					