If, now, the constant magnitude loci (*M* circles) and constant phase angle loci (*N* circles) are transferred to the gain-phase plot, the resultant chart is called Nichol's chart.

If we superimpose the gain-phase plot of an open-loop t.f. on Nichol's chart, we get very easily the closed-loop frequency response. The magnitude is expressed in decibels, while the phase angle is in degrees. Nichol's chart gives the points of intersection of the gain-phase plot of an open-loop t.f., which are symmetrical about the −180° axis. The *M* and *N* loci are repeated for every 360°, and there is symmetry at every 180° interval. The *M* loci are centred about the critical point (0 dB, −180°)

Using Nichol's chart, the closed-loop frequency response can be determined graphically from the locus of the open-loop system. When the Nichol's plot of $G(j\omega)$ is sketched on Nichol's chart, the locus of $G(j\omega)$ will cut the *M* and *N* contours at various points. The cutting point of the locus of $G(j\omega)$ with the *M* contour gives the magnitude of closed-loop system corresponding to a frequency same as that of $G(j\omega)$ at that point. The cutting point of locus of $G(j\omega)$ and *N* contour gives the phase of closed loop corresponding to a frequency same as that of $G(j\omega)$ at that point. The magnitude *M* and phase angle $\alpha (N = \tan \alpha)$ of closed-loop system are tabulated. The closed-loop frequency response can be plotted on a semi-log graph sheet using the tabulated values. The closed-loop frequency response consists of two plots, namely magnitude versus ω and phase angle versus ω. Figure 12.23 shows Nichol's chart.

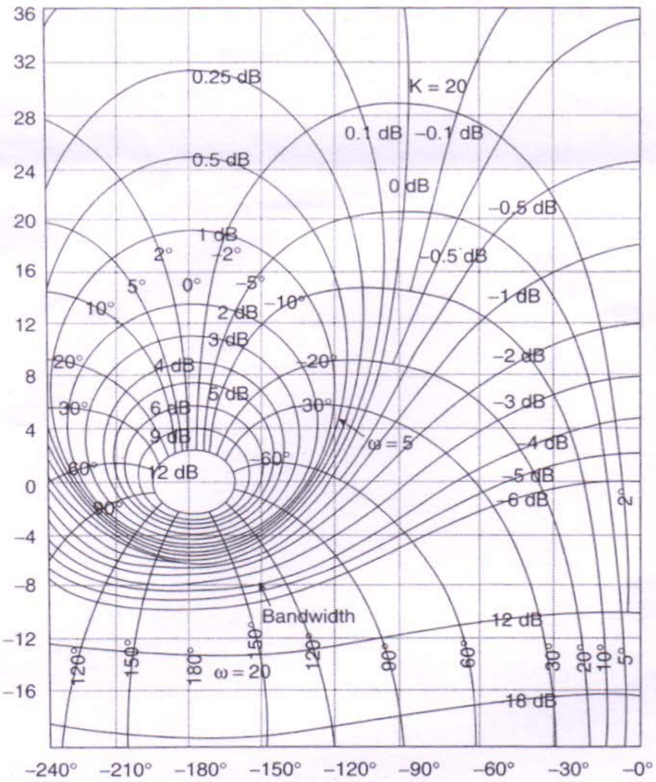


Figure 12.23 Nichol's chart

Nichol's chart is used for determining:

1. The complete closed-loop frequency response.
2. The value of resonant peak M_r of closed-loop system with given value of $G(j\omega)$.
3. The frequency ω_r corresponding to M_r for the closed-loop system.
4. Other frequency and time domain specifications if M_r and ω_r are known.
5. The 3-dB bandwidth of the closed-loop system.
6. The value of K for the given value of M_r.

ADDITIONAL SOLVED PROBLEMS

ASP-1: Sketch the polar plot for the t.f.

$$G(s) = \frac{12}{s^2(s+1)(s+2)}.$$

Solution:

Let the t.f. of the system is given as

$$G(s) = \frac{12}{s^2(1+s)(2+s)}.$$

To get sinusoidal t.f., substitute $s = j\omega$ in $G(s)$

$$G(j\omega) = \frac{12}{(j\omega)^2(1+j\omega)(2+j\omega)}$$

$$M = |G(j\omega)| = \frac{12}{\omega^2\sqrt{1+\omega^2}\sqrt{4+\omega^2}}$$

$$\phi = \angle G(j\omega) = 180 - \tan^{-1}\omega - \tan^{-1}\omega/2$$

ω	M	ϕ
0	∞	$-180°$
∞	0	$-270°$

Next, let us determine whether $G(j\omega)$ intersects any of the axis of $G(j\omega)$-plane.

$$G(j\omega) = \frac{-12}{\omega^2(1+j\omega)(2+j\omega)}$$

$$= \frac{-12(1-j\omega)(2-j\omega)}{\omega^2(1+j\omega)(2+j\omega)(1-j\omega)(2-j\omega)}$$

$$= \frac{-12(2-j\omega-j2\omega-\omega^2)}{\omega^2(1+\omega^2)(4+\omega^2)}$$

$$= \frac{-12(2-\omega^2)}{\omega^2(1+\omega^2)(4+\omega^2)} + \frac{12.j3\omega}{\omega^2(1+\omega^2)(4+\omega^2)}$$

Let us assign I.P. of $G(j\omega) = 0$.

$$\Rightarrow \qquad \frac{36\omega}{\omega^2(1+\omega^2)(4+\omega^2)} = 0 \Rightarrow \omega = \infty$$

Let us assign R.P. of $G(j\omega) = 0$

$$\Rightarrow \qquad \frac{12(2-\omega^2)}{\omega^2(1+\omega^2)(4+\omega^2)} = 0 \Rightarrow 2 - \omega^2 = 0$$

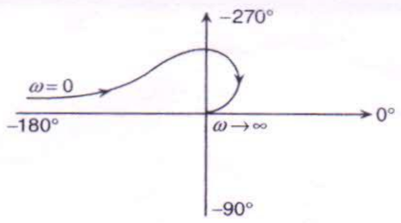
$$\Rightarrow \qquad \omega = \sqrt{2}$$

$$G(j\sqrt{2}) = \frac{12}{(j\sqrt{2})^2(1+j\sqrt{2})(2+j\sqrt{2})}$$

$$= \frac{-12}{2(-2+j2\sqrt{2}+j\sqrt{2}+2)}$$

$$= \frac{-6}{j3\sqrt{2}} = j\sqrt{2}$$

The polar plot is as shown in the following figure.



ASP-2: Draw the polar plot for the t.f. $G(s) = \dfrac{1}{(1+s)^3}$

Solution:

The t.f. of the system is given as

$$G(s) = \frac{1}{(1+s)^3} = \frac{1}{(1+s)(1+s)(1+s)}.$$

To get sinusoidal t.f., substitute $s = j\omega$ in $G(s)$.

$$G(j\omega) = \frac{1}{(1+j\omega)(1+j\omega)(1+j\omega)}$$

$$M = |G(j\omega)| = \frac{1}{\sqrt{1+\omega^2}\,\sqrt{1+\omega^2}\,\sqrt{1+\omega^2}}$$

$$\phi = \angle G(j\omega) = -\tan^{-1}\omega - \tan^{-1}\omega - \tan^{-1}\omega = -3\tan^{-1}\omega$$

ω	**M**	ϕ
0	1	0
∞	0	–270°

Next, let us determine whether $G(j\omega)$ intersects any of the axis of $G(j\omega)$-plane.

$$G(j\omega) = \frac{1}{(1+j\omega)^3}$$

$$= \frac{(1-j\omega)^3}{(1+j\omega)^3(1-j\omega)^3}$$

$$= \frac{1+j\omega^3 - j3\omega - 3\omega^2}{(1+\omega^2)^3}$$

$$= \frac{1-3\omega^2}{(1+\omega^2)^3} + \frac{j\omega(\omega^2-3)}{(1+\omega^2)^3}$$

Let us assign I.P. of $G(j\omega) = 0$

$$\Rightarrow \qquad \frac{\omega\left(\omega^2-3\right)}{\left(1+\omega^2\right)^3} = 0 \Rightarrow \omega\,(\omega^2-3) = 0$$

$$\Rightarrow \qquad \omega = 0,\ \omega = \sqrt{3}$$

$$G(j0) = \frac{1}{1\times1\times1} = 1$$

$$G(j\sqrt{3}) = \frac{1}{\left(1+j\sqrt{3}\right)^3} = \frac{1}{1-j3\sqrt{3}+j3\sqrt{3}-9} = \frac{-1}{8} = -0.125.$$

Let us assign R.P. of $G(j\omega) = 0$

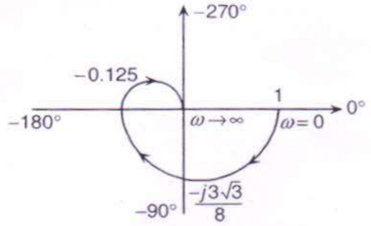
$$\Rightarrow \qquad \frac{1-3\omega^2}{\left(1+\omega^2\right)3} = 0 \Rightarrow 1-3\omega^2 = 0$$

$$\Rightarrow \qquad \omega = \frac{1}{\sqrt{3}}$$

$$G(j/\sqrt{3}) = \left(\frac{1}{1+j/\sqrt{3}}\right)^3 = \frac{3\sqrt{3}}{(j+\sqrt{3})^3}$$

$$= \frac{3\sqrt{3}}{-j+3\sqrt{3}-3\sqrt{3}+j9} = \frac{3\sqrt{3}}{j8} = \frac{-j3\sqrt{3}}{8} = -j0.649.$$

The polar plot is as shown in the following figure.

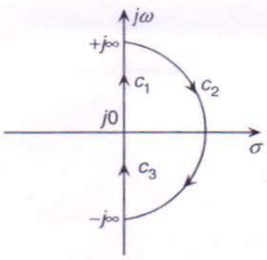


ASP-3: By Nyquist stability criterion, determine the stability of closed-loop system, when open-loop t.f. is given as $G(s)H(s) = \dfrac{3}{(s+1)(s+2)}$.

Solution:

Given that $G(s)H(s) = \dfrac{3}{(s+1)(s+2)}$

1. Number of poles in the right half of the s-plane, $P = 0$.
2. For stability, $N = -P = 0$
3. As there is no pole at the origin, the Nyquist contour is as shown in the figure, which contains Sections c_1, c_2 and c_3.



4. **Mapping of Section c_1:** In Section c_1, $\omega \to 0$ to ∞, that is, the mapping of Section c_1 gives polar plot of $G(j\omega)H(j\omega)$ in (u–v)-plane.

To get the sinusoidal t.f., substitute $s = j\omega$ in $G(s)H(s)$.

$$G(j\omega)H(j\omega) = \frac{3}{(1+j\omega)(2+j\omega)}$$

$$M = |G(j\omega)H(j\omega)| = \frac{3}{\sqrt{1+\omega^2}\sqrt{4+\omega^2}}$$

$$\phi = \underline{|G(j\omega)H(j\omega)} = -\tan^{-1}\omega - \tan^{-1}\omega/2$$

ω	M	ϕ
0	1.5	0
∞	0	−180°

$$G(j\omega)H(j\omega) = \frac{3}{(1+j\omega)(2+j\omega)}\frac{(1-j\omega)(2-j\omega)}{(1-j\omega)(2-j\omega)}$$

$$= \frac{3[2-\omega^2 + j(-2\omega-\omega)]}{(1+\omega^2)(4+\omega^2)}$$

$$= \frac{3(2-\omega^2)}{(1+\omega^2)(4+\omega^2)} - j\frac{3\omega\times 3}{(1+\omega^2)(4+\omega^2)}.$$

Let us assign I.P. of $G(j\omega)H(j\omega) = 0$

i.e.,
$$\frac{9\omega}{(1+\omega^2)(4+\omega^2)} = 0 \Rightarrow \omega = 0$$

$\therefore$
$$G(j\omega)H(j\omega) = \frac{3\times 2}{1\times 4} = 1.5$$

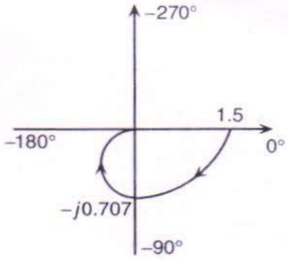
Let us assign R.P. of $G(j\omega)H(j\omega) = 0$

i.e.,
$$\frac{3(2-\omega^2)}{(1+\omega^2)(4+\omega^2)} = 0 \Rightarrow \omega^2 = 2$$

$\Rightarrow$
$$\omega = \sqrt{2}$$

$\therefore$
$$G(j\omega)H(j\omega) = -j\times\frac{3\sqrt{2}\times 3}{3\times 6} = -\frac{j}{\sqrt{2}} = -j0.707.$$

Thus, the mapping of Section c_1 in $(u–v)$-plane gives the following figure.



5. **Mapping of Section c_2:** The mapping of Section c_2 from s-plane to $(u–v)$-plane is obtained by substituting $s = \lim_{R\to\infty} Re^{j\theta}$ in $G(s)H(s)$ and varying θ from +90° to –90°. Since $s \to Re^{j\theta}$ and $R\to\infty$, $1 + sT \approx sT$

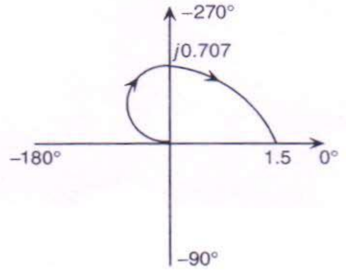
$$G(s)G(s) = \frac{3}{(s+1)(s+2)} = \frac{1.5}{(1+s)(1+0.5s)} = \frac{1.5}{s\times 0.5s} = \frac{3}{s^2}$$

$$G(s)H(s)\Big|_{s=\lim_{R\to\infty} Re^{j\theta}} = \frac{3}{\lim_{R\to\infty} R^2 e^{j2\theta}} = 0e^{-j2\theta}$$

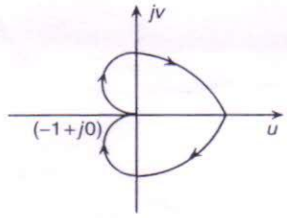
$$= 0\underline{|-180^\circ} \text{ to } 0^\circ \text{ to } +180^\circ.$$

From the afore-mentioned discussion, we can say that Section c_2 in s-plane is mapped into a circular arc of zero radius (i.e., a point) in the $(u–v)$-plane and not required to be analysed.

6. **Mapping of Section c_3:** In Section c_3, $\omega \to -\infty$ to 0, that is, mapping of Section c_3 gives the inverse polar plot of $G(j\omega)H(j\omega)$, as shown in the following figure.



7. **Complete Nyquist plot:** The complete Nyquist plot in $G(s)H(s)$ or $(u–v)$-plane can be obtained by combining the mapping of individual sections as shown in the following figure.



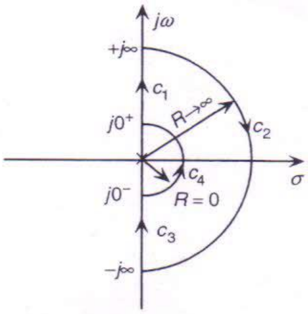
8. The Nyquist plot does not encircle the critical point $(-1 + j0)$ (i.e., $N = 0$). Since the given t.f. does not contain any poles in the right half of the s-plane, the closed-loop system is stable.

ASP-4: The open-loop t.f. of a system is given as $G(s)H(s) = \dfrac{1+4s}{s^2(1+s)(1+2s)}$. Determine the stability of closed-loop system by using Nyquist criterion. If the closed-loop system is not stable, then find the number of closed-loop poles lying on the right half of the s-plane.

***Solution*:**

Given that $G(s)H(s) = \dfrac{1+4s}{s^2(1+s)(1+2s)}$.

1. Number of poles in the right half of the s-plane, $P = 0$.
2. For stability, $N = -P = 0$.
3. As there are two poles at origin, the Nyquist contour is as shown in the figure, which contains Sections c_1, c_2, c_3 and c_4.



4. **Mapping of Section c_1:** In Section c_1, $\omega \to 0$ to ∞, that is, the mapping of Section c_1 gives the polar plot of $G(j\omega)H(j\omega)$ in $(u\text{–}v)$-plane.

   To get sinusoidal t.f., substitute $s = j\omega$ in $G(s)H(s)$.

$$G(j\omega)H(j\omega) = \frac{1 + j4\omega}{(j\omega)^2(1 + j\omega)(1 + j2\omega)}.$$

$$M = |G(j\omega)H(j\omega)| = \frac{\sqrt{1 + 16\omega^2}}{\omega^2\sqrt{(1 + \omega^2)(1 + 4\omega^2)}}$$

$$\phi = \underline{|G(j\omega)H(j\omega)} = -180^\circ - \tan^{-1}\omega - \tan^{-1}2\omega + \tan^{-1}4\omega$$

ω	M	ϕ
0	1.5	–180°
∞	0	–270°

$$G(j\omega)H(j\omega) = \frac{1 + j4\omega}{-\omega^2(1 - 2\omega^2 + 3j\omega)}$$

$$= \frac{-(1 + j4\omega)}{\omega^2(1 - 2\omega^2 + j3\omega)} \times \frac{(1 - 2\omega^2 - j3\omega)}{(1 - 2\omega^2 - j3\omega)}$$

$$= \frac{-[1 + 10\omega^2 + j(\omega - 8\omega^3)]}{\omega^2[(1 - 2\omega^2)^2 + 9\omega^2]}$$

$$= \frac{-(1 + 10\omega^2)}{\omega^2[(1 - 2\omega^2) + 9\omega^2]} - \frac{j(\omega - 8\omega^3)}{\omega^2[(1 - 2\omega^2) + 9\omega^2]}$$

Let us assign I.P. of $G(j\omega)H(j\omega) = 0$

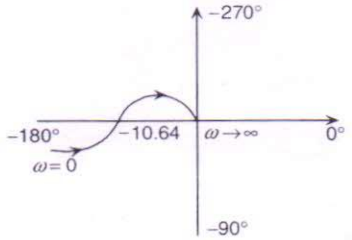
i.e.,
$$\frac{\omega - 8\omega^3}{\omega^2[(1 - 2\omega^2) + 9\omega^2]} = 0$$

$\Rightarrow$
$$\omega(1 - 8\omega^2) = 0$$

$$\omega = 0,\ \omega = \frac{1}{2\sqrt{2}}.$$

$$\therefore \quad G(j\omega)H(j\omega) = -\frac{\left(1+10\times\frac{1}{8}\right)}{\frac{1}{8}\left[\left(1-2\times\frac{1}{8}\right)^2+\frac{9}{8}\right]} = -10.64.$$

Thus, the mapping of Section c_1 in $(u\text{–}v)$-plane is as given in the following figure.



5. **Mapping of Section c_2:** Mapping of Section c_2 from s-plane to $(u\text{–}v)$-plane is obtained by assigning $s = \lim_{R\to\infty} Re^{j\theta}$ in $G(s)H(s)$ and varying θ from $+\pi/2$ to $-\pi/2$. Since $s \to Re^{j\theta}$ and $R \to \infty$, $1 + T \approx sT$.

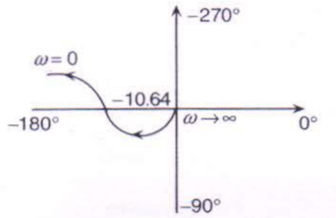
$$G(s)H(s) = \frac{1+4s}{s^2(1+s)(1+2s)} = \frac{4s}{s^2\times s\times 2s} = \frac{2}{s^3}$$

$$\therefore \quad G(s)H(s)\Big|_{s=\lim_{R\to\infty} Re^{j\theta}} = \frac{2}{\lim_{R\to\infty} R^3 e^{j3\theta}} = 0e^{-j3\theta}$$

$$= 0\underline{|-270^\circ} \text{ to } 0^\circ \text{ to } +270^\circ.$$

From the afore-mentioned discussion, we can say that Section c_2 in s-plane is mapped into a circular area of zero radius (i.e., a point) in the $(u\text{–}v)$-plane and not required to be analysed.

6. **Mapping of Section c_3:** In Section c_3, $\omega \to -\infty$ to 0, that is, the mapping of Section c_3 gives the inverse polar plot of $G(j\omega)H(j\omega)$, as shown in the following figure.



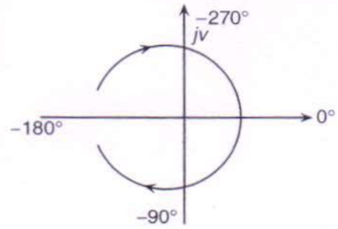
7. **Mapping of Section c_4:** Mapping of Section c_4 from s-plane to $(u\text{–}v)$-plane can be obtained by substituting $s = \lim_{R\to 0} Re^{j\theta}$ in $G(s)H(s)$ and varying θ from $-\pi/2$ to $+\pi/2$. Since $s = Re^{j\theta}$ and $R = 0$, $1 + sT = 1$.

$$G(s)H(s) = \frac{1+4s}{s^2(1+s)(1+2s)} = \frac{1}{s^2\times 1\times 1} = \frac{1}{s^2}.$$

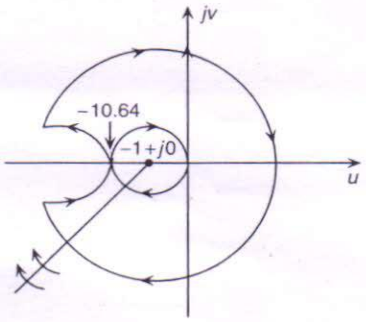
$$G(s)H(s)\Big|_{s=\lim_{R\to 0} Re^{j\theta}} = \frac{1}{\lim_{R\to 0} Re^{j\theta}} = \infty e^{-j2\theta}$$

$$= \infty\underline{|+180^\circ} \text{ to } 0^\circ \text{ to } -180^\circ.$$

From the afore-mentioned discussion, we can say that Section c_4 in the s-plane is mapped into a circle of infinite radius with argument varying from +180° to −180°, as shown in the following figure.



8. **Complete Nyquist plot:** The complete Nyquist plot in $G(s)H(s)$ or $(u–v)$-plane can be obtained by combining the mapping of individual sections as shown in the following figure.



9. The number of encirclements of $(-1 + j0)$ are
   $N = +2$ (clockwise encirclements)
   However, for stability, $N = 0$.
   ∴ The closed-loop system is unstable.
   According to the mapping theorem, we have

$$N = Z - P$$

$$2 = Z - 0 \Rightarrow Z = 2$$

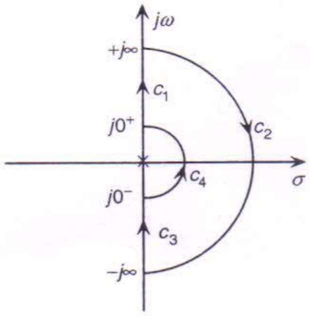
Actually, there are 2 zeros of $1 + G(s)H(s)$ encircled by Nyquist path, that is, 2 closed-loop poles are there in the right half of the s-plane, due to which the closed-loop system is unstable.

ASP-5: Sketch the Nyquist plot of the system whose open-loop t.f. is given as $G(s)H(s) = \dfrac{5}{s(1+0.5s)}$ and hence determine the gain margin.

Solution:

Given that $G(s)H(s) = \dfrac{5}{s(1+0.5s)}$.

1. Number of poles in the right half of the s-plane, $P = 0$.
2. For stability, $N = -P = 0$.
3. As there is one pole at origin, the Nyquist contour is chosen as shown in the figure, which contains Sections c_1, c_2, c_3 and c_4.



4. **Mapping of Section c_1:** In Section c_1, $\omega \to 0$ to ∞, that is, the mapping of Section c_1 gives the polar plot of $G(j\omega)H(j\omega)$ in $(u$–$v)$-plane.
   To get the sinusoidal t.f., substitute $s = j\omega$ in $G(s)H(s)$.

$$G(j\omega)H(j\omega) = \frac{5}{j\omega(1 + j0.5\omega)}.$$

$$M = |G(j\omega)H(j\omega)| = \frac{5}{\omega\sqrt{1 + 0.25\omega^2}}.$$

$$\theta = \angle G(j\omega)H(j\omega) = -90 - \tan^{-1} 0.5\omega.$$

ω	M	ϕ
0	∞	−90
∞	0	−180

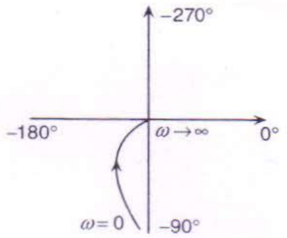
$$G(j\omega)H(j\omega) = \frac{5}{j\omega(1 + j0.5\omega)}$$

$$= \frac{5}{-0.5\omega^2 + j\omega} = -\frac{5}{(0.5\omega^2 - j\omega)}\frac{(0.5\omega^2 + j\omega)}{(0.5\omega^2 + j\omega)}$$

$$= \frac{-5(0.5\omega^2 + j\omega)}{0.25\omega^2 + \omega^2}$$

$$= \frac{-5(0.5\omega^2 + j\omega)}{1.25\omega^2} = \frac{-2.5}{1.25} - j\frac{5}{1.25\omega}.$$

There is no intersection point of the plot with the real or imaginary axis.
Thus, the mapping of Section c_1 in $(u$–$v)$-plane gives the following figure.



5. **Mapping of Section c_2:** The mapping of Section c_2 from the s-plane to $(u–v)$-plane is obtained by substituting $s = \lim_{R\to\infty} Re^{j\theta}$ in $G(s)H(s)$ and varying θ from $+\pi/2$ to $-\pi/2$. Since $s \to Re^{j\theta}$ and $R\to\infty$, $1 + sT \approx sT$

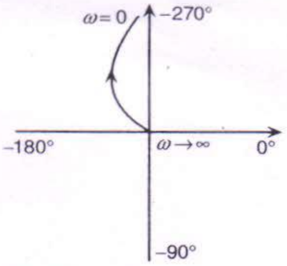
$$G(s)H(s) = \frac{5}{s(1+0.5s)} = \frac{5}{s\times 0.5s} = \frac{10}{s^2}$$

$$G(s)H(s)\Big|_{s=\lim_{R\to\infty} Re^{j\theta}} = \frac{10}{\lim_{R\to\infty} R^2 e^{j2\theta}} = 0e^{-j2\theta}$$

$$= 0\underline{|-180^\circ} \text{ to } 0^\circ \text{ to } +180^\circ$$

From the afore-mentioned discussion, we can say that Section c_2 in the s-plane is mapped into a circular of zero radius in the $(u–v)$-plane and not analysed.

6. **Mapping of Section c_3:** In Section c_3, $\omega \to -\infty$ to 0, that is, mapping of Section c_3 gives the inverse polar plot of
$G(j\omega)H(j\omega)$ as in the following figure.



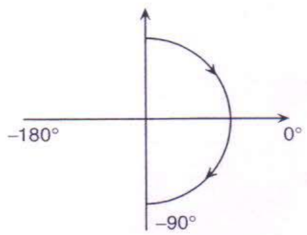
7. **Mapping of Section c_4:** Mapping of Section c_4 from s-plane to $(u–v)$-plane can be obtained by substituting $s = \lim_{R\to 0} Re^{j\theta}$ in $G(s)H(s)$ and varying θ from $-\pi/2$ to $+\pi/2$. Since $s = Re^{j\theta}$ and $R = 0$, $1 + sT = 1$.

$$G(s)H(s) = \frac{5}{s(1+0.5s)} = \frac{5}{s\times 1} = \frac{5}{s}$$

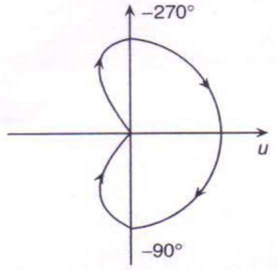
$$G(s)H(s)\Big|_{s=\lim_{R\to 0} Re^{j\theta}} = \frac{5}{\lim_{R\to\infty} Re^{j2\theta}} = \infty e^{-j\theta}$$

$$= \infty\underline{|90^\circ} \text{ to } 0^\circ \text{ to } -90^\circ$$

From the afore-mentioned discussion, we can say that Section c_4 in the s-plane is mapped into a circle of infinite radius with argument varying from +90° to −90° as shown in the following figure.



8. **Complete Nyquist plot:** The complete Nyquist plot in (u–v)-plane can be obtained by combining the mapping of individual section as in the following figure.



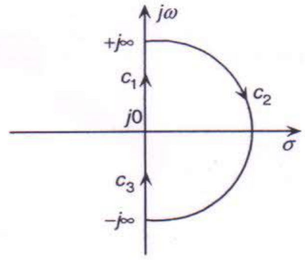
9. The Nyquist plot does not encircle the critical point $(-1+j0)$, that is, $N = 0$. Since the given t.f. does not contain any poles in the right half of the s-plane, the closed-loop system is stable.
   As $\omega_{pc} = 0$ for any increase in gain, ω_{pc} cannot approach ω_{gc}, and hence $G.M = +\infty$dB

ASP-6: For a feedback control system, sketch the Nyquist plot whose open-loop t.f. is given as $G(s)H(s) = \dfrac{40}{(s+4)(s^2+2s+2)}$. Find the gain margin and stability from Nyquist plot.

Solution:

Given that $G(s)H(s) = \dfrac{40}{(s+4)(s^2+2s+2)}$.

1. Number of poles in the right half of the s-plane, $P = 0$.
2. For stability $N = -P = 0$.
3. As there is no pole at the origin, the Nyquist contour is chosen as shown in the figure, which contains Sections c_1, c_2 and c_3.



4. **Mapping of Section c_1:** In Section c_1, $\omega \to 0$ to ∞, that is, the mapping of Section c_1 gives the polar plot of $G(j\omega)H(j\omega)$ in $(u–v)$-plane.

To get the sinusoidal t.f., substitute $s = j\omega$ in $G(s)H(s)$.

$$G(j\omega)H(j\omega) = \frac{40}{(j\omega+4)(2-\omega^2+j2\omega)}$$

$$M = |G(j\omega)H(j\omega)| = \frac{40}{\sqrt{\omega^2+16}\sqrt{(2-\omega^2)^2+4\omega^2}}$$

$$\theta = \underline{|G(j\omega)H(j\omega)} = -\tan^{-1}\omega/4 - \tan^{-1}\left[\frac{2\omega}{2-\omega^2}\right]$$

Note: A quadratic factor contributes an angle of ±180° as $\omega \to \infty$ and contributes an angle of 0° as $\omega \to 0$.

ω	M	ϕ
0	5	0°
∞	0	–270°

$$G(j\omega)H(j\omega) = \frac{40}{(4+j\omega)(2-\omega^2+j2\omega)} \times \frac{(4-j\omega)(2-\omega^2-2j\omega)}{(4-j\omega)(2-\omega^2-2j\omega)}.$$

$$= \frac{40(4-j\omega)(2-\omega^2-j2\omega)}{(16+\omega^2)[(2-\omega^2)^2+4\omega^2)]}$$

$$= \frac{40[8-4\omega^2-j8\omega-j2\omega+j\omega^3-2\omega^2]}{(16+\omega^2)(4+\omega^4)}$$

$$= \frac{40(8-6\omega^2)}{(16+\omega^2)(4+\omega^4)} + \frac{j(\omega^3+10\omega)}{(16+\omega^2)(4+\omega^4)}.$$

Let us assign I.P. of $G(j\omega)H(j\omega) = 0$.

$$\text{i.e.,} \quad \frac{\omega^3-10\omega}{(16+\omega^2)(4+\omega^4-3\omega^2)} = 0$$

$$\Rightarrow \quad \omega(\omega^2-10) = 0$$

$$\omega = 0, \omega = \sqrt{10}\text{, i.e., } \omega_{pc} = \sqrt{10}.$$

$$\therefore \quad G(s)H(s) = \frac{40(8-60)}{(16+10)(4+100)} = -0.769\text{, i.e., } Q = -0.769.$$

Let us assign R.P. of $G(j\omega)H(j\omega) = 0$.

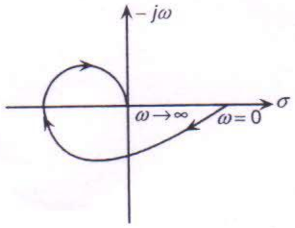
$$\text{i.e.,} \quad \frac{8-6\omega^2}{(16+\omega^2)(4+\omega^4)} = 0$$

$$\Rightarrow \quad 8-6\omega^2 = 0 \Rightarrow \omega = \sqrt{4/3}.$$

$$G(j\omega)H(j\omega) = \frac{j\sqrt{\frac{4}{3}}\left[\frac{4}{3}-10\right]}{\left[16+\frac{4}{3}\right]\left[4+\frac{16}{9}\right]}.$$

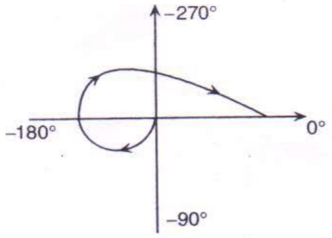
$$= \frac{j1/54\times(-26)\times 9}{52\times 52} = -j0.1$$

Thus, the mapping of Section c_1 in $(u–v)$-plane is shown in the following figure.

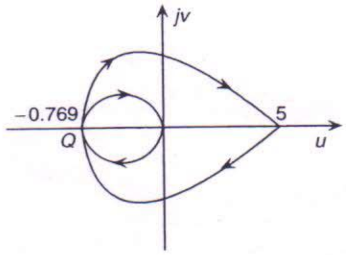


5. **Mapping of Section c_2:** The mapping of Section c_2 from s-plane to $(u–v)$-plane gives a point and need not be considered.

6. **Mapping of Section c_3:** In Section c_3, $\omega\to\infty$ to 0, that is, mapping of Section c_3 gives the inverse polar plot of $G(s)H(s)$, as given in the following figure.



7. **Complete Nyquist plot:** The complete Nyquist plot in $(u–v)$-plane can be obtained by combining the mapping of individual section as in the following figure.



8. The Nyquist plot does not encircle the critical point $(-1 + j0)$, that is, $N = 0$. Since the given t.f. does not contain any poles in the right half of the s-plane, the closed-loop system is stable.

$$\text{GM} = \frac{1}{|0Q|} = \frac{1}{0.769} = 1.3.$$

$$\text{GM in dB} = 20\log\frac{1}{|0Q|}$$

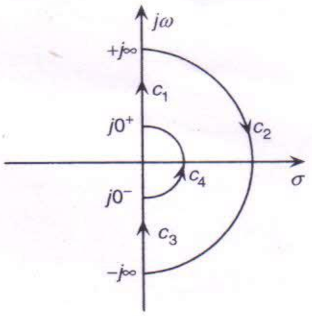
$$= 20\log\left(\frac{1}{0.769}\right) = 2.27\text{dB}$$

ASP-7: Draw the Nyquist plot for the system whose open-loop t.f. is $G(s)H(s) = \dfrac{K}{s(s+2)(s+10)}$. Determine the range of K for which the closed-loop system is stable.

***Solution*:**

Given that $G(s)H(s) = \dfrac{K}{s(s+2)(s+10)}$.

1. Number of poles in the right half of the s-plane, $P = 0$.
2. For stability, $N = -P = 0$.
3. As there is one pole at origin, the Nyquist contour is chosen as shown in the figure, which contains Sections c_1, c_2, c_3 and c_4.



4. **Mapping of Section c_1:** In Section c_1, $\omega \to 0$ to ∞, that is, the mapping of Section c_1 gives the polar plot of $G(j\omega)H(j\omega)$ in $(u\text{–}v)$-plane.
   To get the sinusoidal t.f., substitute $s = j\omega$ in $G(s)H(s)$

$$\therefore \quad G(j\omega)H(j\omega) = \frac{K}{j\omega(j\omega+2)(j\omega+10)}$$

$$M = |G(j\omega)H(j\omega)| = \frac{K}{\omega\sqrt{\omega^2+4}\sqrt{\omega^2+100}}$$

$$\theta = \underline{|G(j\omega)H(j\omega)} = -90 - \tan^{-1}\omega/2 - \tan^{-1}\omega/10$$

ω	M	ϕ
0	∞	−90°
∞	0	−270°

$$G(s)H(s) = \frac{K}{s \times 2(1+s/2)10(1+s/10)}$$

$$= \frac{0.05K}{s(1+0.5s)(1+0.1s)}$$

$$G(j\omega)H(j\omega) = \frac{0.05K}{j\omega(1+j0.5\omega)(1+j0.1\omega)}$$

$$= \frac{0.05K}{j\omega(1+j0.6\omega-0.05\omega^2)}$$

$$= \frac{0.05K}{-0.6\omega^2 + j\omega(1-0.05\omega^2)}$$

$$= \frac{0.05K[-0.6\omega^2 - j\omega(1-0.05\omega^2)]}{[-0.6\omega^2 + j\omega(1-0.05\omega^2)][-0.6\omega^2 - j\omega(1-0.05\omega^2)]}$$

$$= \frac{-0.05K \times 0.6\omega^2}{0.36\omega^4 + \omega^2(1-0.05\omega^2)^2} - \frac{j0.05K\omega(1-0.05\omega^2)}{0.36\omega^4 + \omega^2(1-0.05\omega^2)^2}$$

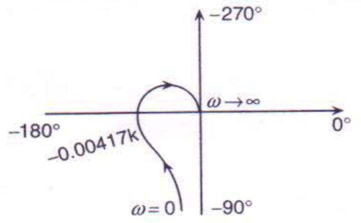
Let us assign I.P. of $G(j\omega)H(j\omega) = 0$.

i.e., $$1 - 0.05\omega^2 = 0 \Rightarrow \omega \frac{1}{\sqrt{0.05}} = 4.47$$

i.e., $$\omega_{pc} = 4.47 \text{ rad/s.}$$

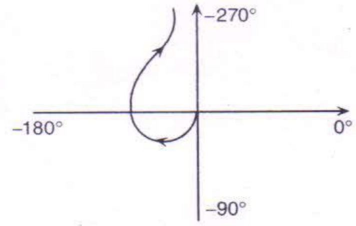
$$\therefore \quad G(j\omega)H(j\omega) = \frac{-0.05K \times 0.6 \times 4.47^2}{0.36 \times 4.47^2 + 4.47^2(1-0.05\times 4.47^2)^2} = -0.00417K.$$

Thus, the mapping of Section c_1 in the $(u\text{–}v)$-plane gives the following figure.



5. **Mapping of Section c_2:** Thus, mapping of Section c_2 from the s-plane to $(u–v)$-plane can be obtained by substituting $s = \lim_{R\to\infty} Re^{j\theta}$ in $G(s)H(s)$, which gives a circular area of zero radius (i.e., a point). Therefore, it need not be considered.

6. **Mapping of Section c_3:** In Section c_3, $\omega \to -\infty$ to 0, that is, the mapping of Section c_3 gives the inverse polar of $G(j\omega)H(j\omega)$ as given in the following figure.



7. **Mapping of Section c_4:** The mapping of Section c_4 from s-plane to $(u–v)$-plane can be obtained by substituting $s = \lim_{R\to 0} Re^{j\theta}$ in $G(s)H(s)$ and varying θ from $-\pi/2$ to $+\pi/2$. Since $s = Re^{j\theta}$ and $R \to 0$, $1+sT \approx 1$.

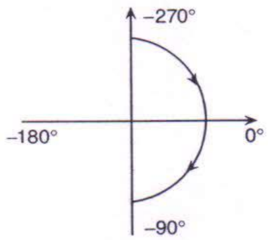
$$\therefore \quad G(s)H(s) = \frac{K}{s(s+2)(s+10)}$$

$$= \frac{0.05K}{s(1+0.05s)(1+0.1s)} = \frac{0.05K}{s\times 1\times 1} = \frac{0.05K}{s}$$

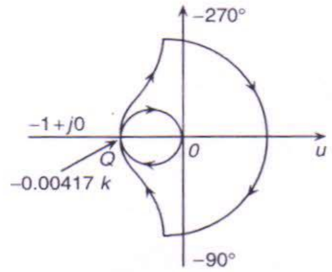
$$G(s)H(s)\Big|_{s=\lim_{R\to 0} Re^{j\theta}} = \frac{0.05K}{\lim_{R\to 0} Re^{j\theta}} = \infty e^{-j\theta}$$

$$= \infty\angle{+90^\circ} \text{ to } 0^\circ \text{ to } -90^\circ.$$

From the afore-mentioned discussion, we can say that Section c_4 in the s-plane is mapped into a circle of infinite radius with argument varying from +90° to –90°, as shown in the following figure.



8. **Complete Nyquist plot:** The complete Nyquist plot in (u–v)-plane can be obtained by combining the mapping of individual section as shown in the following figure.



9. Now, for absolute stability, $N = 0$, that is, $(-1 + j0)$ point should be located on the left side of point Q.

i.e.,
$$|0Q| < 1$$
⇒
$$|0.0041K| < 1$$
$$K < \frac{1}{0.00417} < 240$$

∴ The range of value of K for stability is

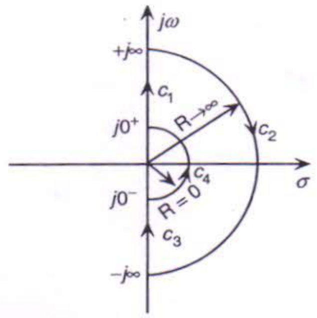
$$0 < K < 240$$

ASP-8: Sketch the Nyquist plot for a system with $G(s)H(s) = \frac{10(s+3)}{s(s-1)}$ and also determine the closed-loop stability.

Solution:

Given that $G(s)H(s) = \frac{10(s+3)}{s(s-1)}$.

1. Number of poles in the right half of the s-plane, $P = 0$.
2. For stability, $N = -P = -1$
3. As there is one pole at origin, the Nyquist contour is chosen as shown in the figure, which contains Sections c_1, c_2, c_3 and c_4.



4. **Mapping of Section c_1:** In Section c_1, $\omega \to 0$ to ∞, that is, the mapping of Section c_1 gives the polar plot of $G(j\omega)H(j\omega)$ in $(u–v)$-plane.

   To get sinusoidal t.f., substitute $s = j\omega$ in $G(s)H(s)$.

$$\therefore \quad G(j\omega)H(j\omega) = \frac{10(3+j\omega)}{j\omega(j\omega-1)}.$$

$$M = |G(j\omega)H(j\omega)| = \frac{10\sqrt{9+\omega^2}}{\omega\sqrt{1+\omega^2}}$$

$$\phi = \underline{|G(j\omega)H(j\omega)} = -90 + \tan^{-1}\left(\frac{\omega}{3}\right) - \tan\left(\frac{-\omega}{1}\right)$$

$$= -90 + \tan^{-1}\left(\frac{\omega}{3}\right) - \left(180^\circ - \tan^{-1}\omega\right)$$

$$= -270^\circ + \tan^{-1}\left(\frac{\omega}{3}\right) + \tan^{-1}\omega$$

ω	**M**	ϕ
0	∞	–270°
∞	0	–90°

$$G(j\omega)H(j\omega) = \frac{-10(3+j\omega)}{j\omega(1-j\omega)} = \frac{-\omega(3+j\omega)}{j\omega+\omega^2}$$

$$= \frac{-10(3+j\omega)(\omega^2-j\omega)}{(\omega^2+j\omega)(\omega^2-j\omega)}$$

$$= \frac{-10(3\omega^2 - j3\omega + j\omega^3 + \omega^2)}{(\omega^4+\omega^2)}$$

$$= \frac{-40\omega^2}{\omega^4+\omega^2} - \frac{10j(\omega^3-3\omega)}{\omega^4+\omega^2}$$

Let us assign I.P. of $G(j\omega)H(j\omega) = 0$.

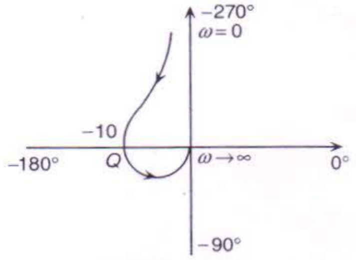
i.e., $$\frac{10j(\omega^3-3\omega)}{\omega^4+\omega^2} = 0$$

$\Rightarrow$ $$\omega^3 - 3\omega = 0$$

$\Rightarrow$ $$\omega = 0,\ \omega = \sqrt{3} \ \therefore \text{ i.e } \omega_{pc} = \sqrt{3}$$

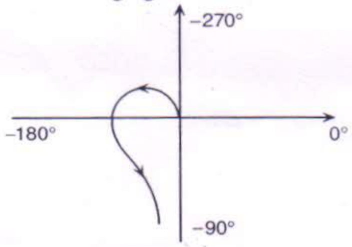
$$\therefore\ G(j\omega)H(j\omega) = \frac{-40\times 3}{9+3} = -10 \ \therefore \text{ i.e., } Q = -10.$$

Thus, the mapping of Section c_1 in $(u\text{–}v)$-plane gives the following figure.



5. **Mapping of Section c_2:** The mapping of section from s-plane to $(u\text{–}v)$-plane can be obtained by substituting $s = \lim_{R\to\infty} Re^{i\theta}$ in $G(s)H(s)$, which gives a circular area of zero radius (i.e., a point). Therefore, it is not considered.

6. **Mapping of Section c_3:** In Sectionc_3, $\omega \to -\infty$ to 0, that is, mapping of Section c_3 gives the inverse polar plot of $G(j\omega)H(j\omega)$ as in the following figure.



7. **Mapping of Section c_4:** Mapping of Section c_4 from s-plane to $(u\text{–}v)$-plane can be obtained by substituting $s = \lim_{R\to 0} Re^{j\theta}$ in $G(s)H(s)$ and varying θ from $-\pi/2$ to $+\pi/2$. Since $s = Re^{j\theta}$ and $R\to 0$, $1+sT \approx 1$.

$$\therefore \qquad G(s)H(s) = \frac{10(s+3)}{s(s-1)} = \frac{10\times 3\left(1+\frac{s}{3}\right)}{s\times -1(1-s)} = \frac{10\times 3\times 1}{s\times -1} = \frac{30}{s\times(-1)}$$

$$G(s)H(s)\Big|_{s-\lim\limits_{R\to 0} Re^{j\theta}} = \frac{30}{\lim\limits_{R\to 0} Re^{i\theta}\times(-1)}$$

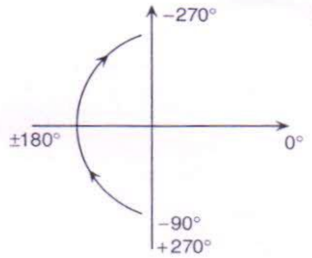
$$= \frac{30}{\lim\limits_{R\to 0} Re^{j(\theta-180^\circ)}} = \infty e^{j(\theta-180^\circ)}$$

$$= \infty\angle{+270} \text{ to } 0^\circ \text{ to } +90^\circ.$$

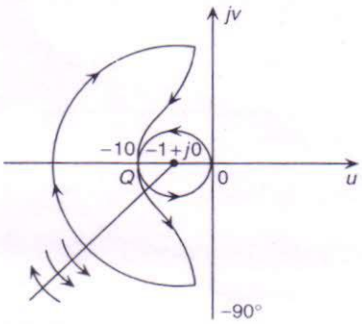
Note:

(−1) introduces −180° phase angle in the phase angle expression.

From the afore-mentioned discussion, we can say that Section c_4 in the s-plane is mapped into a circle of infinite radius with argument varying from +270° to +90°, as shown in the following figure.



8. **Complete Nyquist plot:** The complete Nyquist plot in (u–v)-plane can be obtained by combining the mapping of individual section as shown in the following figure.



9. The number of encirclements of $(-1 + j0)$ are
$N = 1 - 2 = -1$ (anticlockwise).
Since the number of anticlockwise encirclements is equal to the number of open-loop poles on the right half of the s-plane, the closed-loop system is stable.

REVIEW QUESTIONS

1. Explain Nyquist stability criterion clearly.
2. Discuss the effect of adding poles and zeros to Nyquist plot.
3. Define gain margin and phase margin. Explain the significance of these terms to find the closed-loop system stability.
4. Discuss the significance of constant M and N circles to find the closed-loop system stability.
5. Explain the polar plot with an example.
6. Explain how Nyquist contour is selected for stability analysis.
7. What are the advantages of Nyquist method?
8. How can closed-loop stability be predicted from open-loop t.f.?
9. Why Nyquist path does not contain L.H.S of s-plane? Explain.
10. Draw and explain polar plots for Type-0, -1 and -2 systems.
11. How are gain margin and phase margin calculated from Nyquist plot?
12. What are constant M circles? Explain.
13. What are constant N circles? Explain.
14. Show that the loci of constant phase angles are circles.