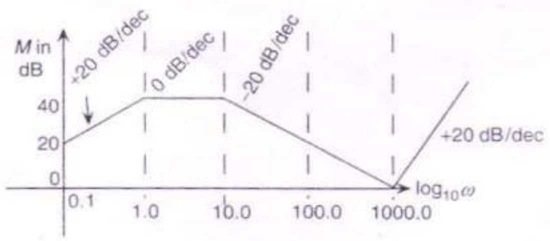
Problem-11.8: Find the t.f. of the system whose Bode's diagram is given as follows:



Solution:

The corner frequencies are at $\omega = 1.0$, $\omega = 10.0$ and $\omega = 1000.0$ rad/s.

$\omega = 1.0$ rad/s ------ corresponds to a pole.

$\omega = 10.0$ rad/s------ corresponds to a pole.

$\omega = 100.0$ rad/s------ corresponds to a zero of second order.

At $\omega = 1.0$ rad/s the magnitude $M = 40$ dB

$$40 = 20\log_{10} k + 20\log_{10}\omega$$

At $\omega = 1$,

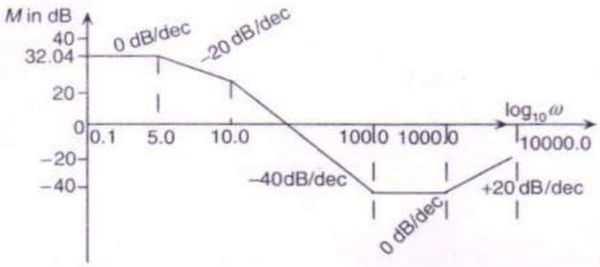
$$40 = 20\log_{10} k$$

$\therefore$
$$K = 100.$$

Therefore, the t.f. is

$$G(s) = \frac{100s(1+s/1000)^2}{(1+s/10)(1+s)} = \frac{100s(1+0.001s)^2}{(1+0.1s)(1+s)}.$$

Problem-11.9: Find the closed-loop t.f. of a system from the Bode's plot given in the figure. The plot represents $|G(j\omega)H(j\omega)|$ in dB along the y-axis with frequency ω on the x-axis. Assume unity feedback.



Solution:

The corner frequencies are at $\omega = 5$ rad/s, 10 rad/s, 100 rad/s and 1000 rad/s.

$\omega = 5$ and 10 rad/s ------ correspond to poles.

$\omega = 100$ and 1000.0 rad/s ------ correspond to zero.

$\omega = 100$ rad/s ------ double zero.

$\omega = 1000.0$ rad/s ------ simple zero since slope changes to 0 from −40 dB/dec and 0 dB to 20 dB/dec, respectively.

$$G(s)H(s) = \frac{K(1+0.01s)^2(1+0.001s)}{(1+0.2s)(1+0.1s)}$$

To evaluate k

At $\omega = 0.1$ rad/s, the contribution from the poles and zeros is negligible, and hence, we have

$$20\log|G(j\omega)H(j\omega)| = 20\log_{10} k = 32.04\text{dB}$$
$$\log_{10} k = 1.6020$$
$$k = 40$$

$$\therefore G(s)H(s) = \frac{40(1+0.01s)^2(1+0.001s)}{(1+0.2s)(1+0.1s)}.$$

The closed-loop t.f. is

$$\frac{G(s)H(s)}{1+G(s)H(s)} = \frac{40(1+0.01s^2)(1+0.001s)}{(1+0.2s)(1+0.01s)+40(1+0.01s^2)(1+0.001s)}.$$

11.12 ADVANTAGES OF BODE PLOT

The various advantages of Bode plot are

1. It shows both high and low frequency characteristics of a t.f. in a single diagram.
2. The Bode plot can be easily constructed using some valid approximations.
3. The relative stability of the system can be studied by calculating gain margin and phase margin.
4. It is possible to determine various other frequency domain specifications such as cut-off frequency and bandwidth.
5. Transfer function of the system can be obtained from Bode plot.
6. Using Bode plot, it is possible to compensate the system to obtain the desired response.
7. The value of system gain K can be designed for required specifications of GM and PM from Bode plot.
8. Experimentally, it is possible to draw a Bode plot without knowing the t.f.

ADDITIONAL SOLVED PROBLEMS

ASP-1: Sketch the Bode plot and determine the gain crossover frequency (ω_{gc}), phase crossover frequency (ω_{pc}), gain margin (GM), and phase margin (PM). Find the stability of the system for the given t.f.

$$G(s) = \frac{10}{s(s+1)(1+0.02s)}.$$

Solution:

Method 1 The open-loop t.f. is given as

$$G(s) = \frac{10}{s(s+1)(1+0.02s)}$$

To get the sinusoidal t.f., substitute $s = j\omega$ in $G(s)$.

$$G(j\omega) = \frac{10}{j\omega(j\omega+1)(1+0.02j\omega)}.$$

Phase angle $\phi = -90 - \tan^{-1}\omega - \tan^{-1}0.02\omega$

Phase plot

Frequency (ω)	Phase angle (ϕ)
0	−90
0.1	−95.82
1	−136.0
10	−185.6
15	−192.884
50	−223.8

Magnitude plot

The type of the gain t.f. is 1, i.e., $m = 0$, $n = 1$ and hence, we have

1. Initial slope = −20 dB/dec
2. Gain value in dB or intersection point on dB axis

$$= 20 + 20\log k = 20 + 20\log 10 = 40\text{dB}$$

The corner frequencies are 1 and 50 rad/s

Factor	Corner frequency in rad/s	Slope in dB/s	Change in slope in dB/s	Asymptotic log-magnitude characteristic
$\frac{1}{j\omega}$	None	−20	−20	Straight line of constant slope −20 dB/dec originating at $\omega = 1$.
$\frac{1}{1+j\omega}$	2	−20	−20 − 20 = −40	Straight line of constant slope −20 dB/dec originating at $\omega = 2$.
$\frac{1}{1+j0.02\omega}$	20	−20	−40 − 20 = −60	Straight line of constant slope −20 dB/dec originating at $\omega = 20$.

From the Bode plot, as shown in the figure, we have

1. Gain crossover frequency = 3.2 rad/s
2. Phase crossover frequency = 8 rad/s
3. Gain margin = 15 dB
4. Phase margin = 17°

Stability Analysis

Both GM and PM are positive, and hence, the system is stable (or) if GCF < PCF, the system is stable.

Method 2 Let us consider using the renewed method of plotting Bode's plot directly from the resultant plot.

Consider $$G(s) = \frac{10}{s(1+s)(1+0.025)}.$$

Substitute $s = j\omega$,

$$G(j\omega) = \frac{10}{j\omega(1+j\omega)(1+0.025)}.$$

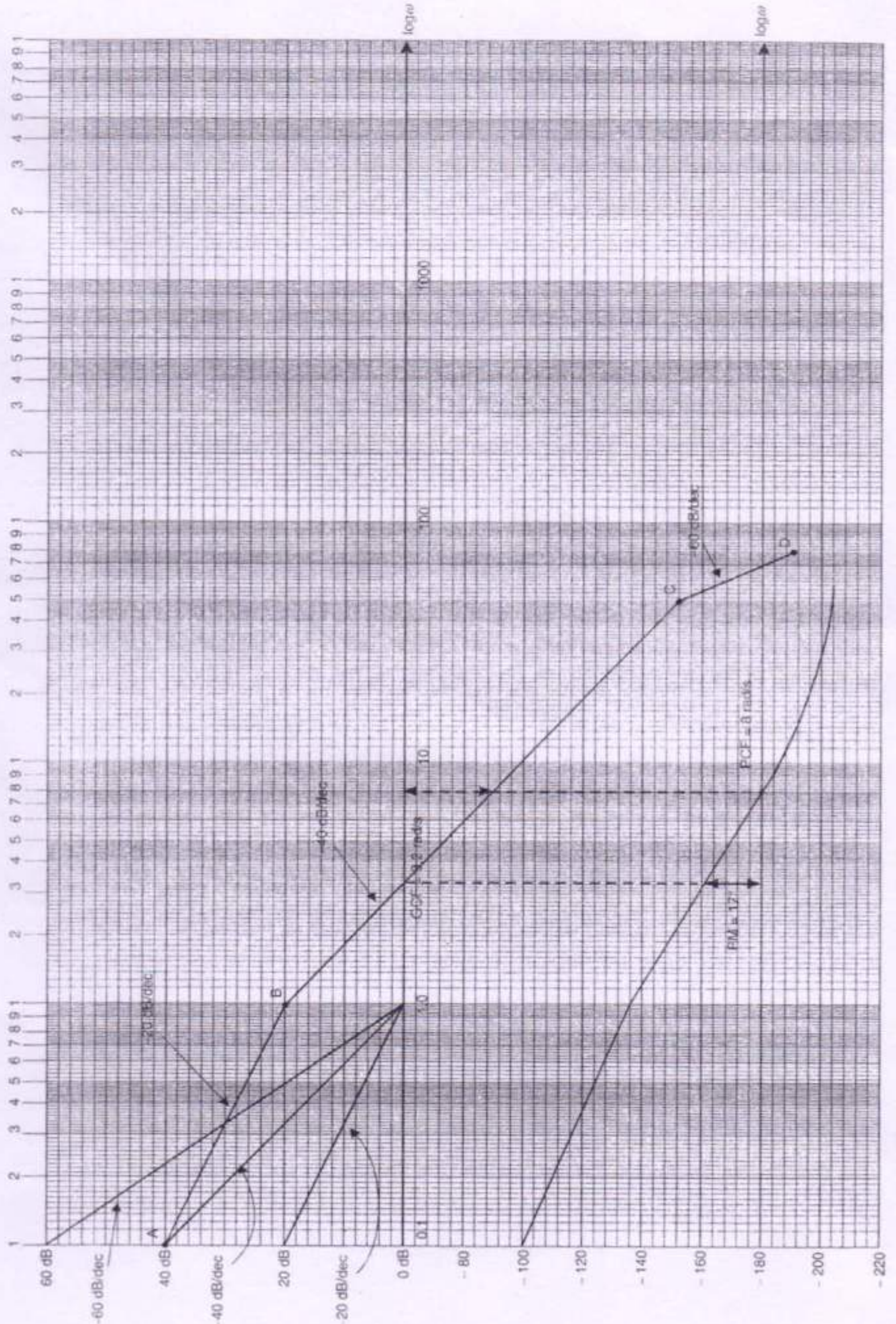


Figure (e) Bode plot for ASP-1

The corner frequencies are at 1, 50.

The starting point for the magnitude plot is calculated as

$$= 20\log k - (m-n)20.$$

$$= 20\log 10 + 20$$

$$= 20 + 20 = 40 \text{ dB}$$

The magnitude is

$$20\log|G(j\omega)| = 20\log 10 - 20\log\omega - 20\log\sqrt{1+\omega^2} - 20\log\sqrt{1+(0.020)^2}$$

1. The starting point is 40 dB with a slope of −20 dB/dec as there is a pole due to s^{-1} factor in the given t.f. Let us assume that this as point *A*. From point *A*, move a unit distance horizontally equal to 1 dec, go vertically downwards till 20 dB magnitude, draw a line from point *A* till 20 dB magnitude for −20 dB/dec slope line, as there is a pole due to the factor s^{-1} up to the corner frequency 1.0 rad/s. Let us assume this point as *B*, as shown in Figure (e).
2. From this corner frequency 1.0 rad/s, as the next corner frequency 50 rad/s corresponds to a pole, the resultant slope is −20 − 20 = −40 dB/dec. Move again a decade distance horizontally from this point *B* and go vertically downwards up to 40 dB magnitude for −40 dB/dec slope line and join *B* with this point till the next corner frequency is 50 rad/s; let us assume it as *C*.
3. For the corner frequencies greater than 50 rad/s, as this frequency is contributed by a pole in the above t.f., the resultant slope is −40 − 20 = −60 dB/dec. Hence, from point *C*, move a decade distance horizontally and go vertically downwards for 60 dB magnitude. Draw a line from *C* to reach this 60 dB magnitude point and assume this as *D*.
4. The phase angle is drawn on similar grounds for various values of ω in the same semi-log sheet, so as to find
   gain crossover frequency = 3.2 rad/s
   phase crossover frequency = 8 rad/s
   gain margin = 15 dB
   phase margin = 17°

ASP-2: Draw the Bode plot for the system having open-loop t.f.

$$G(s) = \frac{50}{(s+1)(s+2)}, H(s) = 1.$$

Solution:

The open-loop t.f. is given as

$$G(s)H(s) = \frac{50}{(s+1)(s+2)} = \frac{50}{2(1+s)(1+s/2)} = \frac{25}{(1+s)(1+0.5s)}$$

To get the sinusoidal t.f., substitute $s = j\omega$ in $G(s)H(s)$.

$$G(j\omega)H(j\omega) = \frac{25}{(1+j\omega)(1+j0.5\omega)}.$$

The phase angle is given as

$$\phi = -\tan^{-1}\omega - \tan^{-1}0.5\omega.$$

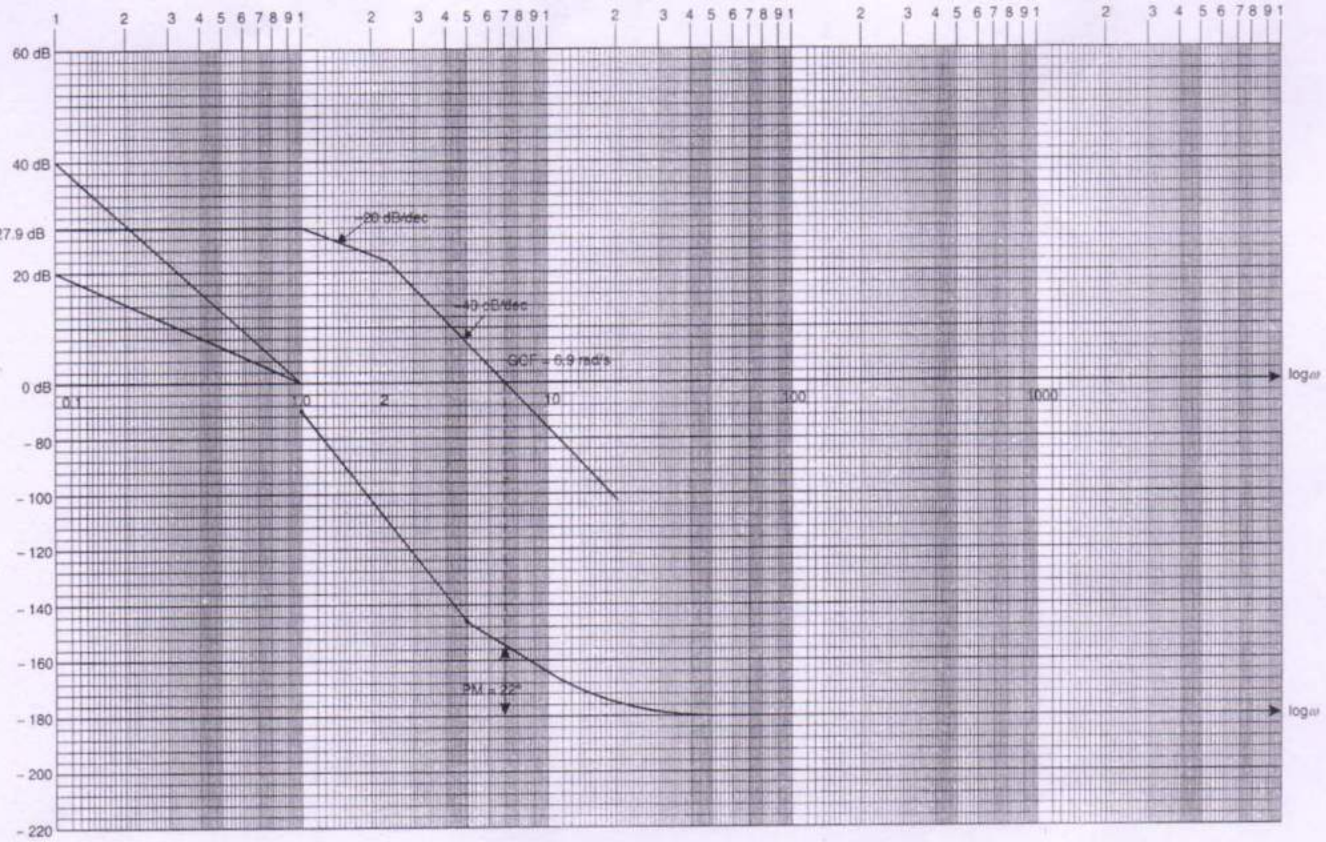


Figure (f) Bode plot for ASP-2

Phase plot

Frequency (ω)	Phase angle (ϕ)
0.1	−8.57
1	−71.56
2	−108.43
10	−162.9
25	−173.13
75	−177.7
100	−178.2

Magnitude plot

The type of the given t.f. is 0, i.e., $m = 0$, $n = 0$ and hence,

1. Initial slope = 0 dB/dec
2. Gain value in dB or intersection point on dB axis

$$20\log k = 20\log 25 = 27.9\text{dB}$$

The corner frequencies are 1 and 2, as shown in Figure (f).

Factor	Corner frequency in rad/s	Slope in dB/dec	Change in slope in dB/dec	Asymptotic log-magnitude characteristic
25	-	0	-	Straight line of constant slope of 0 dB/dec starting from 20 log 25 = 27.9 dB point
$\frac{1}{1+j\omega}$	$\omega_1 = 1$	−20	0 − 20 = −20	Straight line of constant slope 20 dB/dec originating at ω_1 = 1 rad/s
$\frac{1}{1+j0.5\omega}$	$\omega_2 = 2$	−20	−20 − 20 = −40	Straight line of constant slope 20 dB/dec originating at ω_2 = 2 rad/s

ASP-3: The open-loop t.f. of a unity feedback system is

$$G(s) = \frac{2(s+0.25)}{s^2(s+1)(s+0.5)} = \frac{1+4s}{s^2(1+s)(1+2s)}.$$

Determine 1. GM and PM 2. GCF and PCF 3. Stability

Solution:

The open-loop t.f. is given as $G(s) = \frac{1+4s}{s^2(1+s)(s+2s)}$.

To get the sinusoidal t.f., substitute $s = j\omega$ in $G(s)$.

i.e.,
$$G(s) = \frac{1+j4\omega}{j\omega^2(1+j\omega)(1+j2\omega)}$$

The phase angle is given as

$$\phi = -180 - \tan^{-1}\omega - \tan^{-1}2\omega + \tan^{-1}4\omega.$$

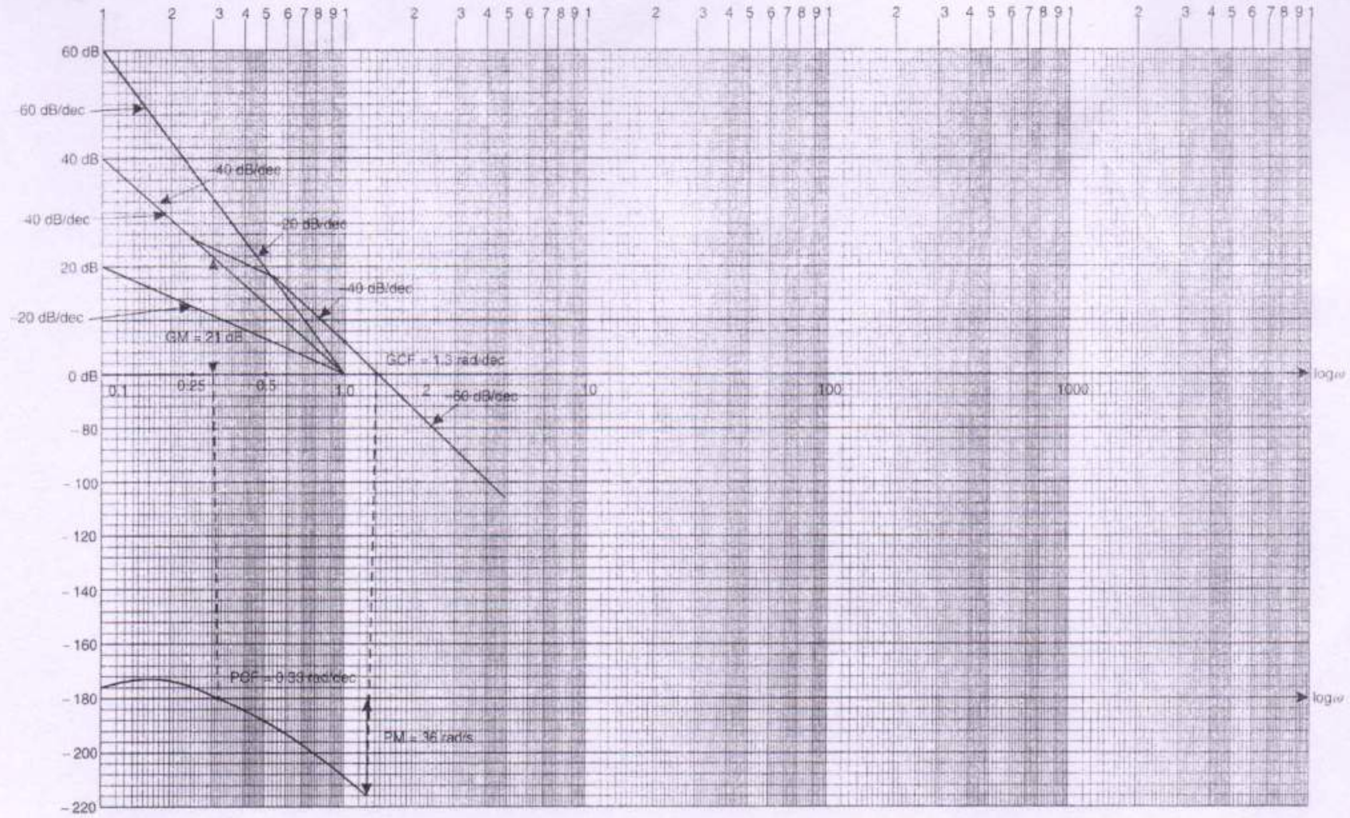


Figure (g) Bode plot for ASP-3

Phase plot

Frequency (ω)	Phase angle (ϕ)
0.1	−175.2
0.5	−188.2
1	−212.4
10	−262.8
15	−265.2
25	−267.2

Magnitude plot

The type of the given system is 2, i.e., $m = 0$, $n = 2$ and hence

1. Initial slope = −40 dB/dec
2. Gain value in dB or intersection point on dB axis

$$40 + 20\log k = 40 + 20\log 1 = 40\text{dB}$$

The corner frequencies are 0.25, 0.5 and 1 rad/s.

Factor	Corner frequency in rad/s	Slope in dB/dec	Change in slope dB/dec	Asymptotic log-magnitude characteristic
$\frac{1}{(j\omega^2)}$	None	−40	-	Straight line of constant slope of −40 dB/s passing through $\omega = 1$
$1 + j4\omega$	$\omega_1 = 0.25$	+20	−40 + 20 = −20	Straight line of constant slope +20 dB/dec originating at $\omega_1 = 0.25$
$\frac{1}{1+j2\omega}$	$\omega_2 = 0.5$	−20	−20 − 20 = −40	Straight line of constant slope −20 dB/dec originating at $\omega_2 = 0.5$
$\frac{1}{1+j\omega}$	$\omega_3 = 1$	−20	−40 − 20 = −60	Straight line of constant slope −60 dB/dec originating at $\omega_3 = 1$.

The Bode plot is drawn as shown in Figure (g).

From the Bode plot, we have

1. The gain crossover frequency (GCF) = 0.3 rad/s
   The phase crossover frequency (PCF) = 0.33 rad/s
2. The gain margin (GM) = 21 dB
   The phase margin (PM) = −36°

Stability Analysis

Since GCF > PCF, the system is unstable.

ASP-4: Sketch the Bode plot for the following t.f. and determine the system gain k for the GCF to be 5 rad/s,

$$G(s) = \frac{ke^{-0.1s}}{s(1+s)(1+0.1s)}.$$

Solution:

Let us assume that the gain value $k = 1$

The open-loop t.f. is given as

$$G(s) = \frac{e^{-0.1s}}{s(1+s)(1+0.1s)}.$$

To get the sinusoidal t.f., substitute $s = j\omega$ in $G(s)$.

i.e., $$G(s) = \frac{e^{-0.1j\omega}}{j\omega(1+j\omega)(1+0.1\omega)}.$$

Magnitude of $e^{-0.1j\omega} \Rightarrow \left|e^{-0.1j\omega}\right| = 1$

The phase angle of $e^{-0.1j\omega}$ is given as

$$\angle e^{-j0.1\omega} = -0.1\omega \text{ rad } [\because \text{ if } x = e^{-j\theta} \text{ the angle of } x, \text{ i.e., } \angle x = -\theta].$$

$$= -0.1\omega \times \frac{180}{\pi}$$

$$= -5.73\ \omega \text{ degrees.}$$

The phase angle of the open-loop t.f. is given as

$$\phi = -90 - 5.73\omega - \tan^{-1}\omega - \tan^{-1}0.1\omega.$$

Phase Plot

Frequency (ω)	Phase angle (ϕ)
0.1	–97
0.5	–122
1	–146
2	–176
2.5	–186.5
5	–223
10	–276

Magnitude plot

The type of the given t.f. is 1, i.e., $m = 0$, $n = 1$ and hence, we have

1. Initial slope = –20 dB/dec
2. Gain value in dB or intersection point on dB axis

$$20 + 20 \log k = 20 + 20 \log 1 = 20 \text{ dB}$$

The corner frequencies are 1 and 10 rad/s.

Factor	Corner frequency in rad/s	Slope in dB/dec	Change in slope dB/dec	Asymptotic log-magnitude characteristic
$\frac{1}{j\omega}$	None	–20	-	Straight line of constant slope of –20 dB/s passing through $\omega = 1$
$e^{-j0.1\omega}$	None	-	-	Coincide with 0 dB line
$\frac{1}{1+j\omega}$	$\omega_1 = 0.25$	–20	–20 – 20 = –40	Straight line of constant slope –20 dB/dec originating at $\omega_1 = 1$
$\frac{1}{1+j0.1\omega}$	$\omega_2 = 0.5$	–20	–40 – 20 = –60	Straight line of constant slope –20 dB/dec originating at $\omega_2 = 10$

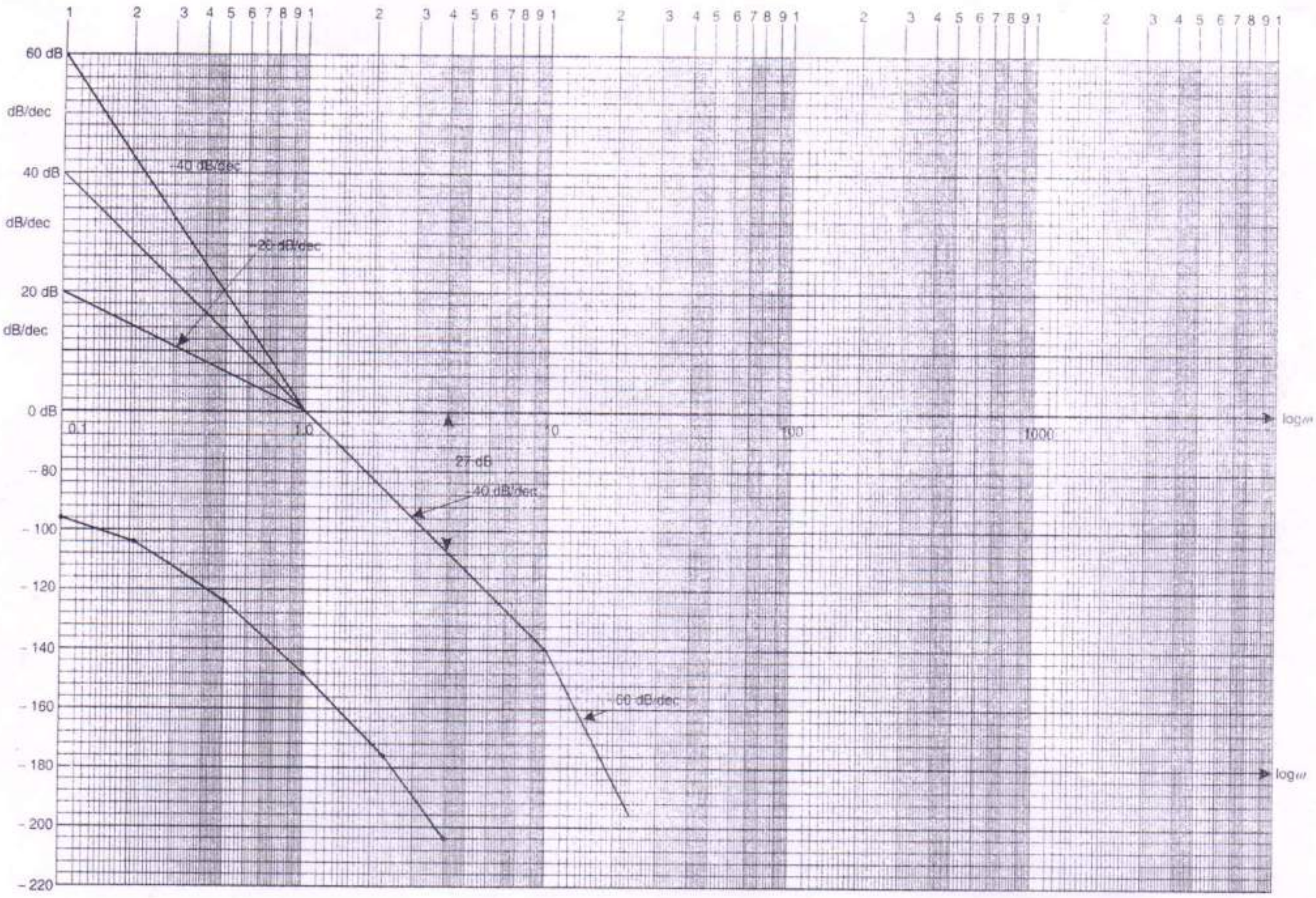


Figure (h) Bode plot for ASP-4

If the GCF is required to be 5 rad/s, then the corresponding magnitude plot is lifted by 27 dB, as shown in Figure (h).

$$\therefore \qquad 20\log k = 27$$

$$\log k = \frac{27}{20} = 1.35$$

$$\Rightarrow k = 10^{1.35} = 22.4.$$

ASP-5: Sketch the Bode plot for the following t.f. and determine phase margin and gain margin $G(s) = \dfrac{75(1+0.2s)}{s(s^2+16s+100)}$.

Solution:

The t.f. of the system is given as

$$G(s) = \frac{75(1+0.2s)}{s(s^2+16s+100)}$$

$$G(s) = \frac{75(1+0.2s)}{s\times 100\left[\frac{s^2}{100}+\frac{16s}{100}+1\right]}$$

$$G(s) = \frac{75(1+0.2s)}{s(1+0.01s^2+0.16s)}.$$

To get sinusoidal t.f., substitute $s = j\omega$ in $G(s)$; then, we have

$$G(j\omega) = \frac{0.75(1+0.2j\omega)}{j\omega(1-0.01\omega^2+j0.16\omega)}.$$

The phase angle is given as

$$\phi = \angle G(j\omega) = \tan^{-1}0.2\omega - 90 - \tan^{-1}\left(\frac{0.16\omega}{1-0.001\omega^2}\right)\text{for } \omega \le \omega_n$$

$$= \phi - 90 - \phi_2$$

$$= \phi - 90 - (\phi_2 + 180)\,\text{for } \omega > \omega_n.$$

Phase plot

Frequency (rad/s)	−90	$-\tan^{-1}0.2\omega$ (ϕ_1)	$-\tan^{-1}\frac{0.16\omega}{1-0.01\omega^2}$ (ϕ_2)	ϕ
0.1	−90	+1.145	−0.916	−89.771
1	−90	+11.31	−9.18	−87.87
5	−90	+45	−46.84	−91.84
10	−90	+63.43	−90	−116.57
15	−90	75.56	+62.487 − 180	−131.97
20	−90	75.9	+46.847 − 180	−147.253
25	−90	78.69	+37.30 − 180	−154.01
40	−90	82.87	+23.106 − 180	−164.02
50	−90	84.29	+18.43 − 180	−167.20
100	−90	87.13	+9.18 − 180	−173.69
500	−90	90	+1.833 − 180	−178.167

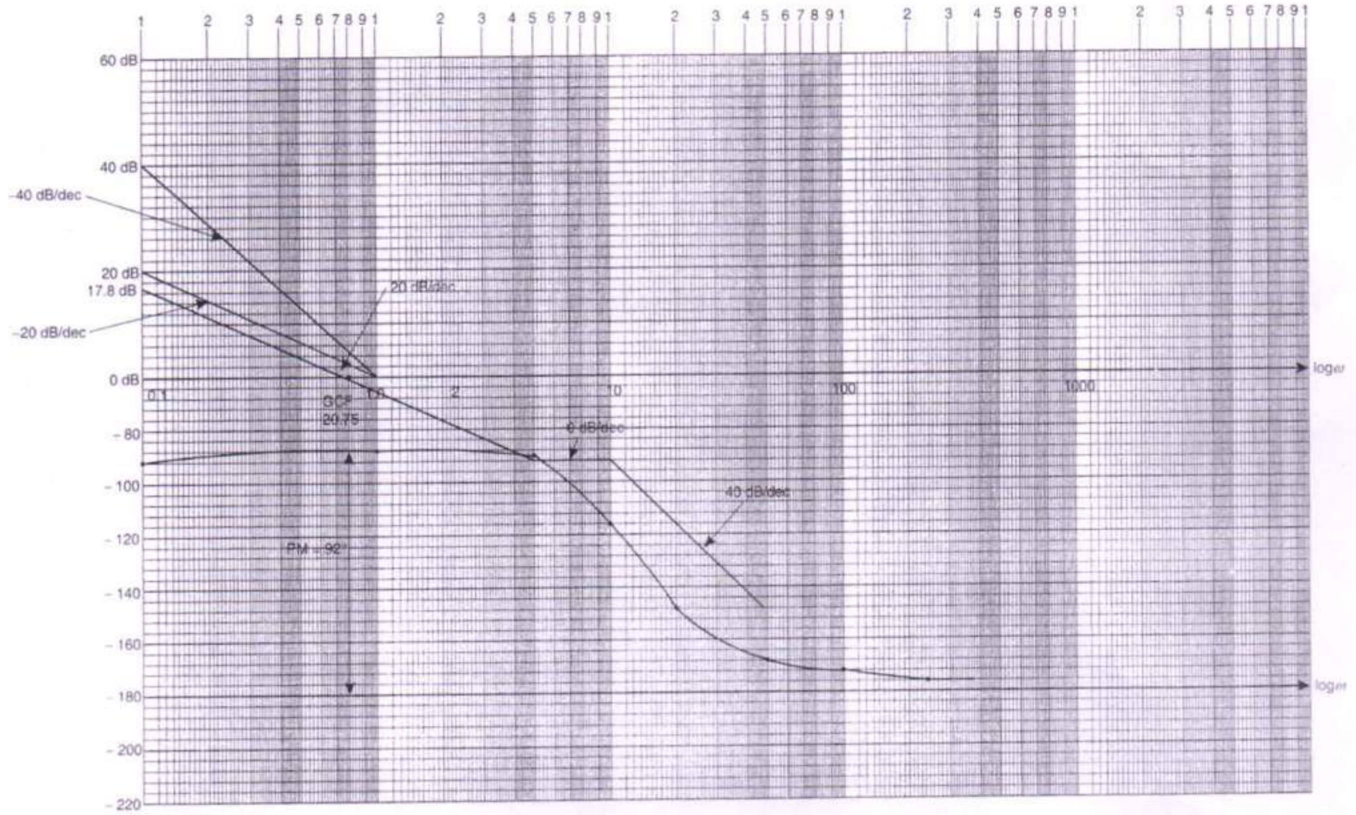


Figure (i) Bode plot for ASP-5

Magnitude plot

Since the system is of Type 1, we have

1. Initial slope = −20 dB/dec
2. Intersection point on the dB axis

$$= 20 + 20\log k$$
$$= 20 + 20\log 0.75$$
$$= 17.5\text{dB}.$$

The corner frequencies are 5 and 10 rad/s.

Factor	Corner frequency in rad/s	Slope in dB/dec	Change in slope in dB/dec	Asymptotic log-magnitude characteristic
$\frac{1}{j\omega}$	–	−20	−20	Straight line of slope of −20 dB/dec passing through = 1 rad/s
$1 + j0.2\omega$	$\omega_1 = 0.25$	+20	0	Straight line of slope + 20 dB/dec originating at $\omega_1 = 5$ rad/s
$\frac{1}{1 + 0.01\omega^2 + j0.16\omega}$	$\omega_2 = 0.5$	−40	40	Straight line of slope 0 dB/dec originating from −40 dB/dec

From the Bode plot shown in Figure (i), we have

1. phase margin = 92°
2. Gain margin = ∞

Stability

Both GM and PM are positive, and hence, the given system is stable.

Note: For the quadratic factor, the corner frequency is ω_n. In quadratic factors, the angle varies from 0° to 180°. However, the calculator calculates $\tan^{-1}$ only between 0° and 90°. Hence, a correction factor of 180° should be added to the phase angle after corner frequency ω_n.

ASP-6: Sketch the Bode plot for a unity feedback system characterised by the open-loop t.f. $G(s) = \dfrac{k(1+0.2s)(1+0.025s)}{s^3(1+0.001s)(1+0.005s)}$.

Show that the system is conditionally stable. Find the range of values of k for which the system is stable.

Solution:

Let us assume that $k = 1$

The open-loop t.f. is given as

$$G(s) = \frac{k(1+0.2s)(1+0.025s)}{s^3(1+0.001s)(1+0.005s)}.$$

To get the sinusoidal t.f., substitute $s = j\omega$ in $G(s)$.

$$G(j\omega) = \frac{(1+0.2j\omega)(1+0.025\omega)}{(j\omega)^3(1+j0.001\omega)(1+j0.005\omega)}.$$

The phase angle is given as

$$\phi = -270 + \tan^{-1} 0.2\omega + \tan^{-1} 0.025\omega - \tan^{-1} 0.001\omega - \tan^{-1} 0.005\omega.$$

Phase plot

Frequency in (rad/s)	Phase angle (ϕ)
0.1	-268°
0.8	-260°
1	-257°
3	-236°
10	-198°
15	-183°
30	-163°
60	-148°
100	-147°
300	-172
400	-182

Magnitude plot

Since the type of the system is 3, i.e., $m = 0$, $n = 3$ we have

1. Initial slope = −60 dB/dec.
2. Intersection point on dB axis = $60 + 20 \log k = 60$ dB.

The corner frequencies are 5, 40, 200 and 1000 rad/s

Factor	Corner frequency in rad/s	Slope in dB/dec	Change in slope dB/dec	Asymptotic log-magnitude characteristic
$\frac{1}{(j\omega)^3}$	–	−60	−60	A straight line of slope −60 dB/dec passing through $\omega = 1$.
$1 + j0.2\omega$	5	+20	−40	A straight line of slope +20 dB/dec originating from $\omega = 5$
$1 + j0.025\omega$	40	+20	−20	A straight line of slope +20 dB/dec originating from $\omega = 40$
$\frac{1}{1 + j0.005\omega}$	200	−20	−40	A straight line of slope −20 dB/dec originating from $\omega = 200$
$\frac{1}{1 + j0.005\omega}$	1000	−20	−60	A straight line of slope −20 dB/dec originating from $\omega = 1000$

From the graph shown in Figure (j), we know that the phase plot crosses the −180° line twice indicating that the system is conditionally stable.

$$20 \log k = 60$$

$$\log k = \frac{60}{20} = 3 \Rightarrow k = 10^3 = 1000$$

Further,

$$20 \log k = 98$$

$$\log k = \frac{98}{20} = 4.9$$

$$k = 10^{4.9} = 79432.8.$$

Hence, the condition for stability is $1000 < k < 79432.8$.

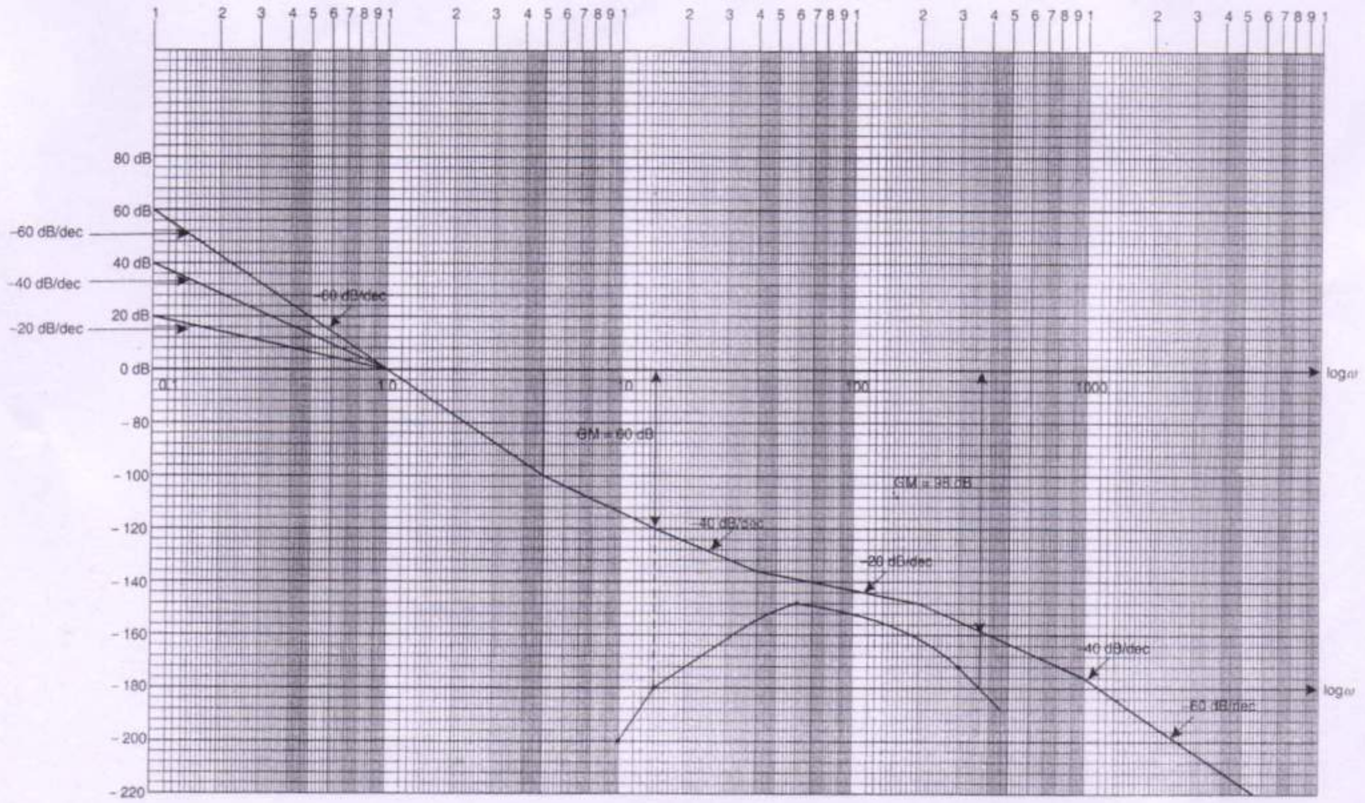


Figure (j) Bode plot for ASP-6

ASP-7: Determine the values of gain k for the following open-loop t.f. so that

1. The gain margin is 15 dB
2. Phase margin is 60°

$$G(s)H(s) = \frac{k}{s(1+0.1s)(1+s)}.$$

Solution:

The open-loop t.f. of the given system by assuming $k = 1$ is

$$G(s)H(s) = \frac{k}{s(1+0.1s)(1+s)} \text{ [substitute } k=1].$$

To get sinusoidal t.f., substitute $s = j\omega$ in $G(s)H(s)$.

$$G(j\omega)H(j\omega) = \frac{1}{j\omega(1+j0.1\omega)(1+j\omega)}$$

The phase angle is given as

$$\phi = -90 - \tan^{-1} 0.1\omega - \tan^{-1} \omega.$$

Phase plot

Frequency in (rad/s)	Phase angle (ϕ)
0.1	−96.2835
1	−140.7105
5	−195.25
10	−219.286
15	−232.4
20	−240.54
25	−245.90
30	−249.65
50	−257.54

Magnitude plot

Since the type of the system is 1, we have

1. The initial slope = −20 dB/dec
2. Intersection point on dB axis

$$= 20 + 20 \log k$$
$$= 20 + 20 \log 1 = 20 \text{ dB}$$

The corner frequencies are 1 and 10 rad/s as shown in Figure (k).

Factor	Corner frequency	Slope in dB/dec	Change in slope dB/dec	Asymptotic log-magnitude characteristic
$\frac{1}{j\omega}$	-	−20	−20	Straight line of slope of −20 dB/s passing through $\omega = 1$
$\frac{1}{1+j\omega}$	1	−20	−20 − 20 = −40	Straight line of slope of −20 dB/dec originating at $\omega_1 = 1$
$\frac{1}{1+0.1j\omega}$	10	−20	−40 − 20 = −60	Straight line of slope of −20 dB/s originating at $\omega_2 = 10$

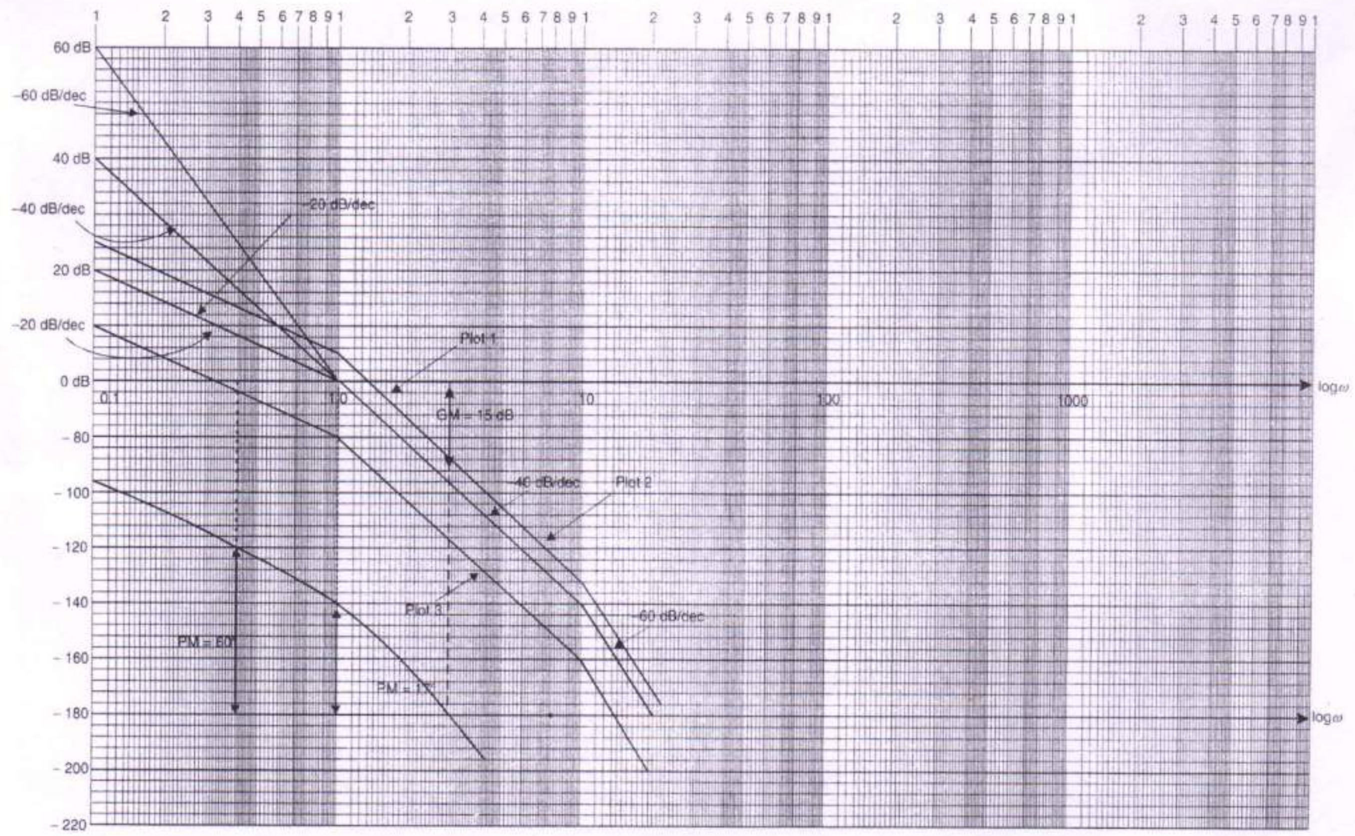
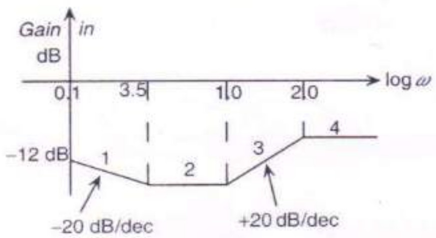


Figure (k) Bode plot for ASP-7

1. For the given gain margin 15dB, we have to lift the magnitude plot upwards as shown in the figure (the upwards lifted plot is given by Plot 2)
   From Plot 2, we have
$$20\log k = 25 - 20$$
$$\log k = 15/20 = 0.25$$
$$k = 10^{0.25} = 1.7778$$
2. For given phase margin 60°, we have to lift the magnitude plot downwards as shown in the figure (the downwards lifted plot is given by Plot 3)
   From Plot 3, we have
$$20 + 20\log k = 12$$
$$\log k = \frac{-8}{20} = -0.4.$$
$$k = 10^{-0.4} = 0.4$$

ASP-8: Find the t.f. of a system having Bode plot, as shown in the following figure.



***Solution*:**

For Part 1, at $\omega = 0.1$, dB = −12 and the slope of the curve is −20 dB/dec.

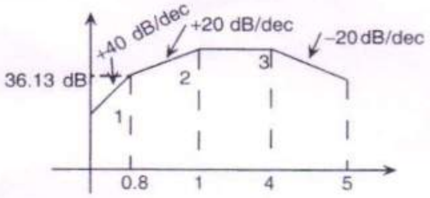
$\therefore$
$$\text{t.f.}_1 = \frac{k}{s}$$
$$\text{dB} = 20\log k - 20\log\omega$$
$$-12 = 20\log k - 20\log 0.1 = 20\log k + 20$$
$\Rightarrow$
$$20\log k = -32 \Rightarrow k = 0.025$$
$$\Rightarrow \text{t.f.}_1 = \frac{0.025}{s}.$$

At $\omega = 3.5$, the slope of the curve changes from −20 dB/s to 0 dB/dec, there is a zero at $\omega = 3.5$.
For Parts 1 and 2, we can write
$$\text{t.f.}_{12} = \frac{0.025(1 + s/3.5)}{s}.$$

At $\omega = 10$, the slope of the curve changes from 0 dB/dec to +20 dB/dec; there is a zero at $\omega = 10$.
For Parts 1, 2 and 3, we can write
$$\text{t.f.}_{123} = \frac{0.025(1 + s/3.5)(1 + s/10)}{s}.$$

At $\omega = 20$, the slope of the curve changes from +20 dB/dec to 0 dB/dec, there is a pole at $\omega = 20$.
For Parts 1, 2, 3 and 4, we can write
$$\text{t.f.} = \frac{0.0.25(1 + s/3.5)(1 + s/10)}{s(1 + s/20)} = \frac{0.025(1 + 0.285s)(1 + 0.1s)}{s(1 + 0.05s)}$$

ASP-9: Find the t.f. of a system having Bode plot as shown in the figure.



Solution:

For Part 1

$$t.f._{1} = Ks^2$$

$$dB = 20\log k + 40\log\omega$$

At $\omega = 0.8$, dB = 36.13

$$36.13 = 20 \log k + 40 \log 0.5 = 20 \log k - 3.87$$

$$20 \log k \Rightarrow k = 10^2 = 100$$

$$\therefore t.f._{1} = 100\, s^2$$

At $\omega = 0.8$, the slope of the curve changes from 40 dB/dec to +20 dB/dec and there is a pole at $\omega = 0.8$.
For Parts 1 and 2, we can write

$$t.f._{12} = \frac{100s^2}{(1+s/0.8)}.$$

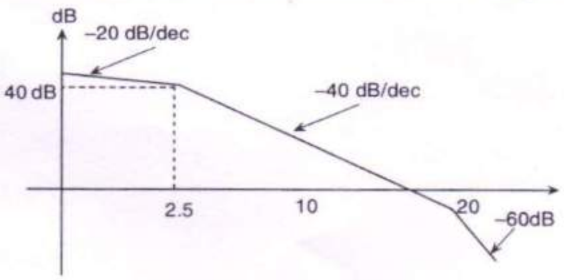
At $\omega = 1$, the slope of the curve changes from +20 dB/dec to 0 dB/dec and there is a pole at $\omega = 1$.
For Parts 1, 2 and 3, we can write

$$t.f._{123} = \frac{100s^2}{(1+s/0.8)(+s/1)}.$$

At $\omega = 4$, the slope of the curve changes from 0 dB/dec to –20 dB/dec and there is a pole at $\omega = 4$.
For Parts 1, 2, 3 and 4, we can write

$$t.f. = \frac{100s^2}{(1+s/0.8)(1+s/1)(1+s/4)} = \frac{100s^2}{(1+1.25s)(1+s)(1+0.25s)}.$$

ASP-10: Determine the t.f. whose approximate Bode plot is as shown in the figure.



Solution:

For Part 1

$$t.f._{1} = \frac{k}{s}.$$

The corner frequencies are 2.5 and 40.

At $$\omega = 2.5,\ \text{dB} = 40$$

$$40 = 20 \log k + 20 \log 2.5 = 20 \log k - 7.95$$

$$\Rightarrow \quad 20 \log k = 47.95 \Rightarrow k = 10^{2.398} = 250.$$

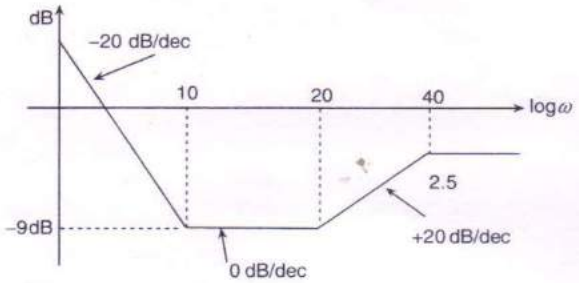
At $\omega = 2.5$ rad/s, the slope changes from −20 dB/dec to −40 dB/dec due to factor $\frac{1}{(1+s/2.5)}$.

At $\omega = 40$ rad/s, the slope changes from −40 dB/dec to −60 dB/dec due to factor $\frac{1}{(1+s/40)}$.

The t.f. is

$$G(s) = \frac{250}{s(1+s/2.5)(1+s/40)} = \frac{250}{s(1+0.46)(1+0.025s)}.$$

ASP-11: Find the t.f. of the system where asymptotic approximation is given in the figure.



Solution:

For Part 1

$$\text{t.f.}_1 = \frac{k}{s}.$$

The corner frequencies are 10, 20 and 40.

At $$\omega = 10,\ \text{dB} = -9$$

$$\text{dB} = 20 \log k - 20 \log 10$$

$$-9 = 20\log k - 20 \Rightarrow 20 \log k = 11 \Rightarrow \omega = 10^{11/20} = 3.5$$

$$\text{t.f.}_1 = 3.55/s$$

At $\omega = 10$ rad/s, the slope changes from −20 dB/dec to 0 dB/dec due to factor $(1 + s/\omega)$.
At $\omega = 20$ rad/s, the slope changes from 0 dB/s to +20 dB/dec due to factor $(1+ s/20)$. At $\omega = 40$ rad/s, the slope changes from +20 dB/dec to 0 dB/dec due to factor $\frac{1}{(1+s/40)}$.

The t.f. is given as

$$G(s) = \frac{3.55(1+s/10)(1+s/20)}{s(1+s/40)} = \frac{3.55(1+0.1s)(1+0.05s)}{s(1+0.025s)}.$$

REVIEW QUESTIONS

1. What is Bode plot? State the advantages of Bode plot.
2. State the steps for Bode plot.
3. Define gain margin and phase margin. Explain the significance of these terms to find the closed-loop system stability.
4. Define gain crossover frequency and phase crossover frequency. Explain the significance of these terms to find the system stability.
5. State the advantages and disadvantages of frequency response analysis.
6. Define the following terms:
   (i) Resonant frequency (ii) Cut-off rate
   (iii) Resonant peak (iv) Bandwidth
7. What do you mean by resonant peak? How is it related to the relative stability of a closed-loop system?
8. Explain the nature of Bode plot for
   (i) poles at origin (ii) simple pole (iii) simple zero
9. What is meant by corner frequency? What is its significance?
10. Obtain the correlation between the time response and the frequency response.

EXERCISE PROBLEMS

1. Sketch the Bode plot and determine GCF, PCF, GM and PM. Determine the stability of the system for the t.f. $G(s)H(s)=\dfrac{2(s+0.25)}{s^2(s+1)(s+0.25)}$.
2. Determine the value of gain K for the following open-loop t.f. so that
   (a) the gain margin is 20 dB (b) the phase margin is 60°
   $$G(s)H(s)=\frac{2(s+0.25)}{s^2(1+0.1s)(1+0.05s)}$$
3. Find the frequency domain specifications for a system having $G(s)=\dfrac{36}{s(s+8)}$ with unity feedback.
4. Sketch the Bode plot for the open-loop t.f. $G(s)=\dfrac{K}{s(1+sT)}$. Determine the stability of the system.
5. A unit-step input is applied to a unity feedback control system having t.f. $G(s)=\dfrac{K}{s(1+sT)}$. Determine (i) the values of K and T to have $M_p=20\%$, $\omega_r=6$ rad/s (ii) the resonant peak M_r.
6. A unity feedback has open-loop t.f. $G(s)=\dfrac{288(s+4)}{s(s+1)(S^2+4.8s+144)}$. Draw the Bode plot for the system.
7. Sketch the Bode plot for the forward path t.f. of a unity feedback system is given as $G(s)=\dfrac{10e^{-0.1s}}{s(1+0.1s)^2}$ and determine the stability of the system.
8. A unity feedback has open-loop t.f. $G(s)=\dfrac{K}{s(s+4)(s+10)}$. Draw the Bode plot for the system and find the value of K for a marginally stable system.
9. Draw the Bode plot for the open-loop t.f. given as $G(s)=\dfrac{4}{(s+2)(s+4)(s+5)}$ and determine GM and PM.

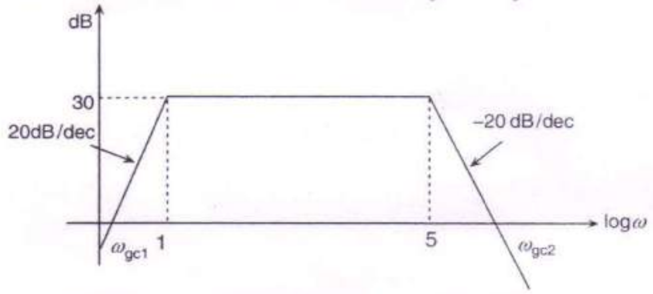
10. Sketch the Bode plot for the open-loop t.f. given as $G(s)H(s) = \dfrac{300(s^2+2s+4)}{s(s+10)(s+20)}$ and determine the stability of the system.

11. Sketch the Bode plot for the open-loop t.f. given as $G(s)H(s) = \dfrac{100K(s+5)(s+40)}{s^3(s+100)(s+200)}$ and determine the range of K for closed-loop stability.

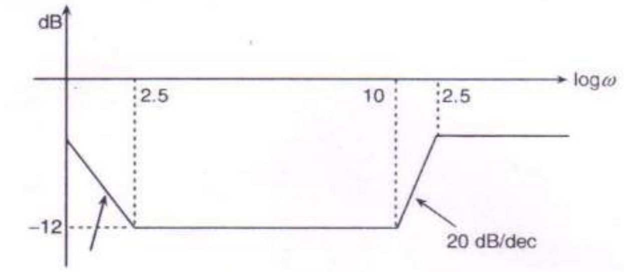
12. Sketch the Bode plot for the open-loop t.f. $G(s)H(s) = \dfrac{2e^{-sT}}{s(s+1)(1+0.5s)}$. Determine the maximum value of T for the system to be stable.

13. Sketch the Bode plot for the open-loop t.f. $G(s)H(s) = \dfrac{75e^{-sT}}{s(s^2+10s+100)}$. Determine the maximum value of T for the system to be stable.

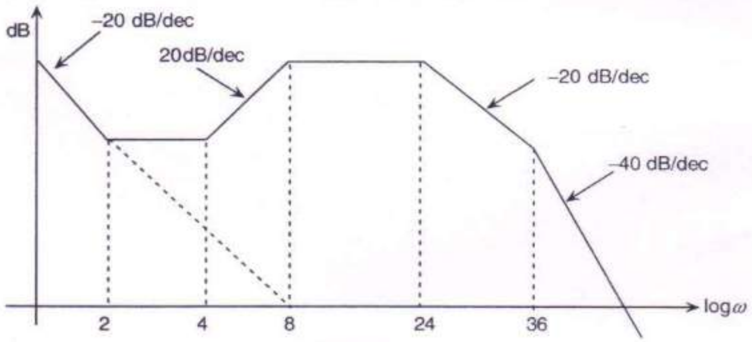
14. For the Bode plot as shown in the figure, find
(i) the open-loop t.f. (ii) ω_{gc1} and ω_{gc2}



15. Find the open-loop t.f. of a system whose approximate plot is as shown in the figure.



16. Find the open-loop t.f. of a system whose approximate plot is as shown in the figure.



ANSWERS

1. GCF = 1.24 rad/s, PCF = 0.4 rad/s, GM = −20, PM = 18°, Unstable
2. (a) $K = 2.65$ (b) $K = 10$
3. $\omega_r = 2$ rad /s, $M_r = 1.006$, BW = 6.7 rad/s
4. GM = ∞, PM = 47°, and the system is stable
5. (i) $K = 8.6$, $T = 0.14$ (ii) $M_r = 1.234$
7. GM = −5.3 dB, PM = 54°, and the system is unstable
8. For marginally stable system, GCF = PCF, $K = 400$
9. GM = 36 dB, PM = Very high and positive
10. GM = ∞, PM = 96°, the system is stable
11. $2820 < K < 48840$
12. $T = 7$ ms
13. $T = 2$ s
14. $\omega_{gc1} = 0.0316\text{rad/s}, \omega_{gc2} = 158\text{rad/s}$
15. $G(s)H(s) = \dfrac{0.63(1+0.4s)(1+0.1s)}{(1+0.04s)}$
16. $G(s)H(s) = \dfrac{8(1+0.5s)(1+0.0285s)}{s(1+125s)(1+0.042s)(1+0.028s)}$

OBJECTIVE TYPE QUESTIONS

1. The initial slope of the Bode plot for a t.f. having no poles at the origin is
   (a) −10 dB/decade (b) 0 dB/decade
   (c) 10 dB/decade (d) 24 dB/decade
2. The error at corner frequency due to the term $(1 + j\omega T)^{\pm N}$ is
   (a) $\pm 5N$ dB (b) $\pm 2N$ dB
   (c) $\pm 6N$ dB (d) $\pm 3N$ dB
3. The gain for drawing the Bode plot for the t.f. $G(s) = \dfrac{320(s+2)}{s(s+1)(s^2+8s+64)}$.
   (a) 10 (b) 5 (c) 16 (d) 160
4. The initial slope of the Bode plot gives an indication of the
   (a) type of the system (b) nature of the system time response
   (c) system stability (d) gain margin
5. With reference to the Bode plot, the corner frequencies for the t.f.
   $G(s)H(s) = \dfrac{20(1+0.5s)}{s(1+0.2s)(1+s)(s^2+2s+9)}$ are
   (a) 1, 2, 3 and 5 rad/s (b) 1, 2, 3 and 4 rad/s
   (c) 1, 3, 5 and 9 rad/s (d) 1, 2, 5 and 9 rad/s
6. The frequency at which the magnitude of the Bode plot crosses 0-dB axis gives
   (a) natural frequency (b) phase crossover frequency
   (c) gain crossover frequency (d) corner frequency
7. The phase angle at $\omega = \infty$ for the t.f. $G(j\omega) = \dfrac{90(1+j0.5\omega)}{j\omega(1+j\omega)(1+j2\omega)}$ is
   (a) 90° (b) −90° (c) 180° (d) −180°

8. For a stable system,
(a) both GM and PM positive
(b) both GM and PM are negative
(c) GM is positive and PM is negative
(d) GM is negative and PM is positive

9. For a Type I system, the intersection of the initial slope of the Bode plot with 0-dB axis gives
(a) steady-state error
(b) error constant
(c) phase margin
(d) crossover frequency

10. For a stable system,
(a) the gain crossover occurs earlier than phase crossover
(b) the phase crossover occurs earlier than gain crossover
(c) the gain crossover and phase crossover frequencies are very near to each other
(d) the gain crossover and phase crossover frequencies are coincident

11. The phase angle for the t.f. $G(j\omega) = \frac{1}{(j\omega T)^3}$ at corner frequency is
(a) 45°
(b) −90°
(c) −270°
(d) −135°

12. The C.E. of a closed-loop system is given as $S^2 + 4S + 16 = 0$, the resonant peak will be
(a) 2
(b) $2\sqrt{3}$
(c) 4
(d) $2\sqrt{2}$

13. (I) Nyquist criterion is in frequency domain
(II) Bode plot is in frequency domain
(III) Root locus plot is in time domain
(IV) *R–H* criterion is in time domain
(a) I, II and III are correct
(b) I, III and IV are correct
(c) I and II are correct
(d) All are correct

14. If the gain of the open-loop system is doubled, the gain margin
(a) is not effected
(b) gets doubled
(c) becomes half
(d) becomes one-fourth

15. The formula for resonant frequency ω_r is
(a) $\omega_r = \omega_n\sqrt{1-\xi^2}$
(b) $\omega_r = \omega_n\sqrt{1+\xi^2}$
(c) $\omega_r = \omega_n\sqrt{1-2\xi^2}$
(d) $\omega_r = \omega_n\sqrt{1+2\xi^2}$

16. The resonant peak M_r for a second-order system is unity, the damping ratio ξ for the system is
(a) 1.0
(b) 0.707
(c) 0.78
(d) 1.21

17. The term $G(s)H(s) = s$ on magnitude plot of Bode plot will have a slope
(a) 6 dB/octave
(b) −6 dB/octave
(c) 12 dB/octave
(d) 20 dB/dec

18. Frequency response of a system is defined as
(a) the transient response to a sinusoidal input
(b) the steady-state response to a sinusoidal input
(c) the transient response to a parabolic input
(d) the steady-state response to a unit step input

19. The Bode amplitude plot of a constant is a
(a) line with slope −40 dB/dec
(b) line with slope −20 dB/dec
(c) line with slope +20 dB/dec
(d) straight line parallel to frequency axis

20. An octave frequency range is given as
(a) $\frac{\omega_1}{\omega_2} = 2$
(b) $\frac{\omega_1}{\omega_2} = 4$
(c) $\frac{\omega_1}{\omega_2} = 8$
(d) $\frac{\omega_1}{\omega_2} = 10$

21. The value of 20 dB/dec is equal to
(a) 6 dB/octave (b) −6 dB/octave
(c) 12 dB/octave (d) −12 dB/octave

22. If the time constant of a first-order factor of a t.f. is T, the corner frequency is
(a) 10 T (b) $1/T$ (c) T (d) T^2

23. At low frequencies, the initial slope of the Bode plot for Type 3 system is
(a) −20 dB/dec (b) −60 dB/dec
(c) +20 dB/dec (d) +60 dB/dec

24. A unit change in $\log_{10}\omega$ corresponds to a frequency ratio of
(a) 2π (b) 3π (c) 10 (d) 100

25. The initial slope of the Bode plot for a Type 2 system is
(a) −12 dB/octave (b) −6 dB/octave
(c) −24 dB/octave (d) +24 dB/octave

26. The t.f. $G(s) = 1/s^2$ has a phase shift of
(a) 90° (b) −90° (c) −180° (d) 180°

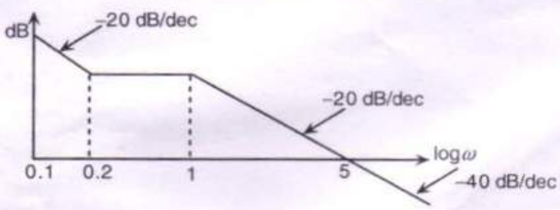
27. A system has 14 poles and 2 zeros. The slope of its highest frequency asymptote and its magnitude plot is
(a) −40 dB/dec (b) −240 dB/dec
(c) −280 dB/dec (d) −320 dB/dec

28. A decade frequency range is given as
(a) $\frac{\omega_1}{\omega_2} = 2$ (b) $\frac{\omega_1}{\omega_2} = 4$
(c) $\frac{\omega_1}{\omega_2} = 8$ (d) $\frac{\omega_1}{\omega_2} = 10$

29. For the second-order t.f. $T(s) = \frac{4}{s^2 + 2s + 4}$, the resonant peak will be
(a) 4 (b) $\frac{4}{3}$ (c) 2 (d) $\frac{2}{\sqrt{3}}$

30. For the second-order t.f. $T(s) = \frac{4}{s^2 + 2s + 4}$, the resonant frequency will be
(a) 2 (b) $\sqrt{2}$ (c) 3 (d) $\sqrt{3}$

31. The magnitude versus frequency plot is shown in the following figure:



The time constants are
(a) 1, 2 and 5 s (b) 0.2, 0.1 and 5 s
(c) 0.1, 0.4 and 0.8 s (d) None