Solution of First sessional Exam Vth Sem EC (Digital Signal Processing)

1 a) **Define digital signal processing**..

DSP is defined as changing or analysing information which discrete sequences of numbers.

b) **Explain the basic elements required for realization of digital system**.

Adder,scale changer, delay element

c) **What is the fundamental time period of the signal x(t)=sin15πt**.

T= 2/15 sec

d) **Draw a transformation matrix of size 4x4 and explain the properties of twiddle factor**

$$W = \begin{bmatrix} \omega^0 & \omega^0 & \omega^0 & \omega^0 \\ \omega^0 & \omega^1 & \omega^2 & \omega^3 \\ \omega^0 & \omega^2 & \omega^4 & \omega^6 \\ \omega^0 & \omega^3 & \omega^6 & \omega^9 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{bmatrix}$$

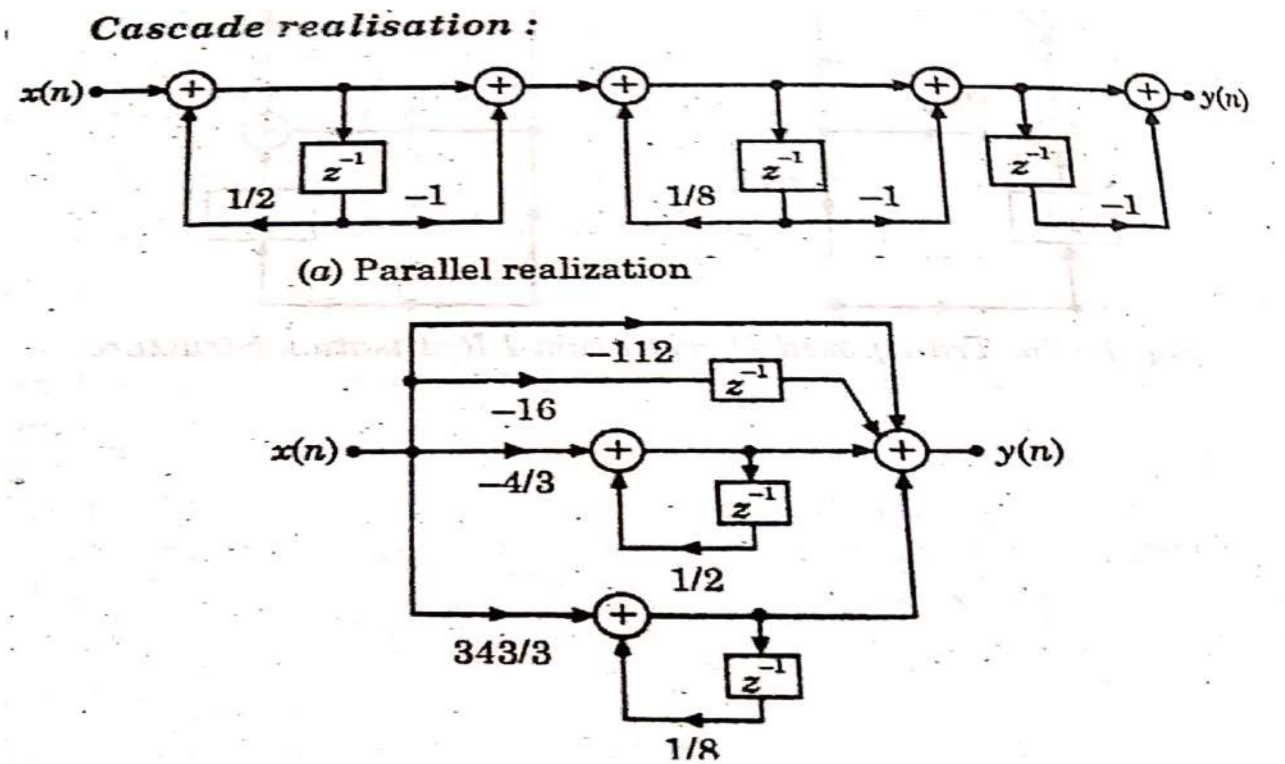
where $\omega = e^{-\frac{2\pi i}{4}} = -i$. or twiddle facor

e) **Differentiate between IIR and FIR filters.**

FIR	IIR
1. It is having linear phase	1. It is having no linear phase.
2. No of necessary multiplications are more.	2. Less no. of multiplications are required.
3. It is a stable filter.	3. Stability depend upon the system.
4. Probability of overflow error is very less.	4. More probability of overflow error in case of direct form.
5. Sensitivity to filter coefficient quantization is low.	5. High sensitivity to filter co-efficient quantization.
6. FIR cannel simulate prototype analog filter.	6. It can simulate prototype analog filter.

2a) Obtain the cascade and parallel structure for the system function H(z) given below

$$H(Z) = \frac{(1-z^{-1})^3}{(1-0.5z^{-1})(1-0.25z^{-1})}$$



2c) Compute the Circular convolution of two discrete time sequences

$x_1(n) = \{1, 2, 1, 2\}$ and $x_2(n) = \{ 3, 2, 1, 4 \}$.

$$y(n) = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 3+4+1+8 \\ 6+2+2+4 \\ 3+4+1+8 \\ 6+2+2+4 \end{bmatrix} = \begin{bmatrix} 16 \\ 14 \\ 16 \\ 14 \end{bmatrix}$$

Circular convolution $= \{16, 14, 16, 14\}$

2d) Find the Discrete-Fourier Transform (DFT) of the sequence x (n) = (1, 1, 0, 0)

and find the IDFT of x (k) = (1, 0, 1, 0)

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-j2\pi nk/N}$$

Here, given that N = 4, therefore, we have

or $$X(k) = \sum_{n=0}^{3} x(n)e^{-j2\pi nk/4}$$

Now,

$$X(0) = x(0) + x(1) + x(2) + x(3)$$

or $$X(0) = 1 + 1 + 0 + 0 = 2.0$$

$$X(1) = x(0) + x(1)\, e^{-j\pi/2} = 1 - j$$

$$X(2) = x(0) + x(1)\, e^{-j\pi} = 1 - 1 = 0.0$$

$$X(3) = x(1)\, e^{-j3\pi/2} = 1 + j$$

Also, we know that the IDFT is expressed as

$$y(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{j2\pi k/N}$$

Here, since $N = 4$, therefore, we have

$$y(n) = \frac{1}{N}\sum_{k=0}^{3} Y(k)e^{j2\pi nk/4}$$

$$y(0) = \frac{1}{4}[Y(0) + Y(1) + Y(2) + Y(3)] = \frac{1}{4}(1 + 0 + 1 + 0) = \frac{1}{2}$$

$$y(1) = \frac{1}{4}[1 + 0 + e^{j\pi} + 0] = (1 - 1)/4 = 0.0$$

$$y(2) = \frac{1}{4}[1 + 0 + e^{j2\pi} + 0] = \frac{1}{2}$$

$$y(3) = \frac{1}{4}[1 + 0 + e^{j3\pi} + 0] = 0.0$$

2d) Obtain the ladder structure for the system function H (z) given below.

$$H(Z) = \frac{2 + 8z^{-1} + 6z^{-2}}{1 + 8z^{-1} + 12Z^{-2}}$$

z^{-2}	6	8	2
z^{-2}	12	8	1
z^{-1}	4	3/2	
z^{-1}	7/2	1	
1	5/14	0	
1	1		

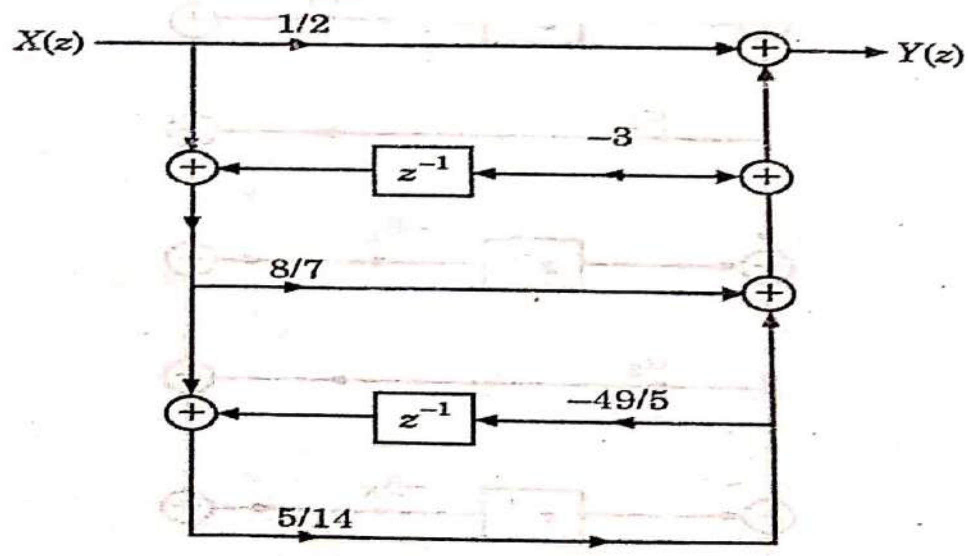
The parameters for the ladder structure will be

$$\alpha_0 = \frac{1}{2}, \qquad \beta_1 = 3, \qquad \alpha_1 = \frac{8}{7},$$

$$\beta_2 = \frac{49}{5}, \qquad \alpha_2 = \frac{5}{14}$$

Also, we have
$$H(z) = \frac{1}{2} + \cfrac{1}{3z^{-1} + \cfrac{1}{\cfrac{8}{7} + \cfrac{1}{(49/5)\,z^{-1} + \cfrac{1}{5/14}}}}$$

The resultant ladder structure



3a) Determine the circular convolution of following sequences and compare result with linear convolution: X (n) = (1.2.3.4) and H (n) = (1, 2, 1)

Section-C

Answer-3-a

x(n) = (1, 2, 3, 4)   h(n) = (1, 2, 1)

Circular Convolution.

1	2	3	4
1	2	1	0
0	0	0	0
2	3	4	1
6	8	2	4
4	1	2	3
12	12	8	8
y(3)	y(0)	y(1)	y(2)

y = {12, 8, 8, 12}

1	2	3	4	0	0
1	2	1	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
4	0	0	1	2	3
0	0	2	4	6	8
0	1	2	3	4	0
4	1	4	8	12	11
y(5)	y(0)	y(1)	y(2)	y(3)	y(4)

y = {1 4 8 12 11 4}

Linear Convolution

x(n) = (1, 2, 3, 4)   h(n) = (1, 2, 1)

$N_1 = 4$ $N_2 = 3$

3b) Compute 8- Point DFT of the sequence using radix-2 decimation-in frequency algorithm:

X (n) ={1/2, 1/2, 1/2, 1/2, 0, 0, 0, 0}

Please refer to class notes

4a) Determine 8- point DFT of the sequence x (n) = (1, 2, 3, 4)

Answer – 4.a

$$x(n) = (1, 2, 3, 4)$$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}} & -j & \frac{-1}{\sqrt{2}} - \frac{j}{\sqrt{2}} & -1 & \frac{-1}{\sqrt{2}} + \frac{j}{\sqrt{2}} & +j & \frac{+1}{\sqrt{2}} + \frac{j}{\sqrt{2}} \\ 1 & -j & -1 & j & 1 & -j & -1 & j \\ 1 & \frac{-1}{\sqrt{2}} - \frac{j}{\sqrt{2}} & j & \frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}} & -1 & \frac{+1}{\sqrt{2}} + \frac{j}{\sqrt{2}} & -j & \frac{-1}{\sqrt{2}} + \frac{j}{\sqrt{2}} \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & \frac{-1}{\sqrt{2}} + \frac{j}{\sqrt{2}} & -j & \frac{+1}{\sqrt{2}} + \frac{j}{\sqrt{2}} & -1 & \frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}} & +j & \frac{-1}{\sqrt{2}} - \frac{j}{\sqrt{2}} \\ 1 & +j & -1 & -j & 1 & +j & -1 & -j \\ 1 & +\frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}} & j & \frac{-1}{\sqrt{2}} + \frac{j}{\sqrt{2}} & -1 & \frac{-1}{\sqrt{2}} - \frac{j}{\sqrt{2}} & -j & \frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}} \end{bmatrix}$$

$$= \begin{bmatrix} 10 \\ 1 + 2(0.707 - j0.707) - 3j + 4(-0.707 - j0.707) \\ 2j - 2 \\ 1 + 2(-0.707 - j0.707) + 3j + 4(0.707 - j0.707) \\ -2 \\ 1 + 2(-0.707 + j0.707) - 3j + 4(0.707 + j0.707) \\ -2j - 2 \\ 1 + 2(0.707 + j0.707) + 3j + 4(-0.707 + j0.707) \end{bmatrix}$$

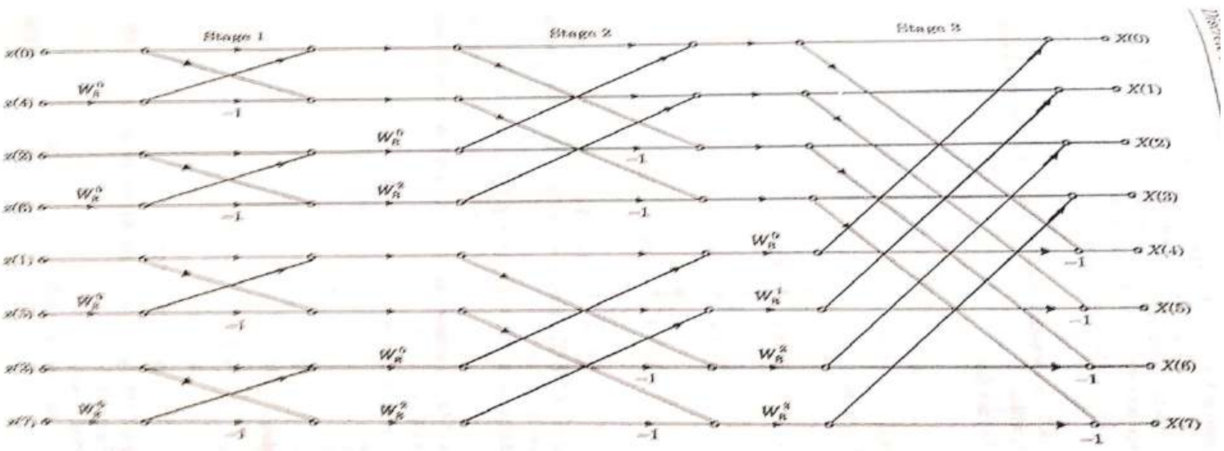
4b) How IIR filter Designing can be done by the use of following methods. Discuss each methods

(i) Impulse Invariance Method.

(ii) Bilinear Transformation Method

Please refer to class notes

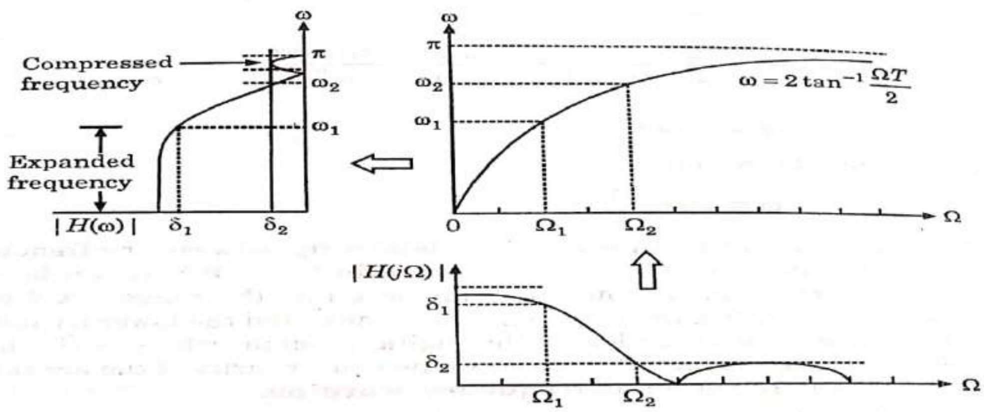
5a) Given x (n) $=2^n$ and N=8 find X (K) using Decimation In Time (DIT) FFT algorithm



5b) Convert following analog filters into digital filters. H(s) (s+0.1)/ ((s+0.1)^2 +9) using bilinear transformation. The digital filter should have a resonant frequency of w_r = π/4

We can obtain the sampling period

$$T = \frac{2}{\Omega_c}\tan\frac{\omega_r}{2} = \frac{2}{3}\tan\frac{\pi}{8} = 0.0276\text{s}$$



relationship Between ω and Ω

Now, making use of bilinear transformation, we have

$$H(z) = H_a(s)\Big|_{s=\frac{2}{T}\frac{(z-1)}{(z+1)}}$$

Thus, we write

$$H(z) = \frac{\frac{2}{T}\frac{(z-1)}{(z+1)} + 0.1}{\left[\frac{2}{T}\frac{(z-1)}{(z+1)} + 0.1\right]^2 + 9}$$

Simplifying, we get

$$H(z) = \frac{(2/T)(z-1)(z+1) + 0.1(z+1)^2}{[(2/T)(z-1) + 0.1(z+1)]^2 + 9(z+1)^2}$$

Putting $T = 0.276$ s, we have

$$H(z) = \frac{1 + 0.027z^{-1} - 0.973z^{-2}}{8.572 - 11.84z^{-1} + 8.177z^{-2}} \quad \textbf{Ans.}$$

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